Formula book

Specialist Mathematics 2025

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Graph equations		
quadratic	$y = a(x-h)^2 + k$	$y = a(x-x_1)(x-x_2)$
cubic	$y = a(x-h)^3 + k$	$y = a(x-x_1)(x-x_2)(x-x_3)$
circle	$(x-h)^2 + (y-k)^2 = r^2$	
square root	$y = a\sqrt{x-h} + k$	
reciprocal	$y = \frac{a}{(x-h)} + k$	
exponential	$y = r^{(x-h)} + k$ (where $r > 0$)	
logarithmic	$y = \log_a(x-h) + k$ (where $a > 1$)	
trigonometric	$y = a\sin(b(x-h)) + k$	$y = a\cos(b(x-h)) + k$

Logarithms		
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$	
logarithmic laws and definitions	$\log_a(x) + \log_a(y) = \log_a(xy)$	$\log_a(x^n) = n\log_a(x)$
	$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$	$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$
	$\log_a(1) = 0$	$\log_a(a) = 1$

Calculus		
$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$		$\int_a^b f(x)dx \approx$ limit of sums $\sum_i f(x_i)\delta x_i$ $\int_a^b f(x)dx = F(b)-F(a)$
$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x)+\frac{d}{dx}g(x)$		$\int (f(x)+g(x))dx = \int f(x)dx + \int g(x)dx$ $\int k\,f(x)dx = k\int f(x)dx$
$\frac{d}{dx}x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1}+c$ for $n \neq -1$
$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}$	$\int \frac{1}{x}dx = \ln(x)+c$ for $x > 0$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\frac{d}{dx}\sin(f(x)) = f'(x)\cos(f(x))$	$\int \sin(x)dx = -\cos(x)+c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\frac{d}{dx}\cos(f(x)) = -f'(x)\sin(f(x))$	$\int \cos(x)dx = \sin(x)+c$
chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$	
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx}+v\frac{du}{dx}$	
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	
trapezoidal rule	$\int_a^b f(x)dx \approx \frac{w}{2}\left[f(x_0)+2(f(x_1)+f(x_2)+f(x_3)+\ldots f(x_{n-1}))+f(x_n)\right]$ where $w = \frac{b-a}{n}$	

Trigonometry	
cosine rule	$c^2 = a^2+b^2-2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	area $= \frac{1}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
binomial probability	$P(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
	standard deviation	$\sqrt{Var(X)}$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} x\, p(x)\,dx$
	variance	$Var(X) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\, p(x)\,dx$
Bernoulli distribution	mean	p
	variance	$p(1-p)$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
approximate margin of error	$z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	
complementary probability	$P(\overline{A}) = 1 - P(A)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A)P(B)$	
conditional probability	$P(A \cap B) = P(A \mid B)P(B)$	

Additional Calculus for Specialist Mathematics			
$\frac{d}{dx}\ln(x)=\frac{1}{x}$	$\int\frac{1}{x}dx=\ln\lvert x\rvert+c$ for $x\neq 0$		
$\frac{d}{dx}\ln(f(x))=\frac{f'(x)}{f(x)}$	$\int\frac{f'(x)}{f(x)}dx=\ln\lvert f(x)\rvert+c$ for $f(x)\neq 0$		
$\frac{d}{dx}\tan(x)=\sec^2(x)$	$\int\sec^2(x)dx=\tan(x)+c$		
$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right)=\frac{1}{\sqrt{a^2-x^2}}$	$\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c$		
$\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right)=\frac{-1}{\sqrt{a^2-x^2}}$	$\int\frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c$		
$\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right)=\frac{a}{a^2+x^2}$	$\int\frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$		
integration by parts	$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$		
volume of a solid of revolution	about the *x*-axis	$V=\pi\int_a^b[f(x)]^2\,dx$	
	about the *y*-axis	$V=\pi\int_a^b[f(y)]^2\,dy$	
Simpson's rule	$\int_a^b f(x)\,dx\approx\frac{w}{3}\left[f(x_0)+4[f(x_1)+f(x_3)+\ldots]+2[f(x_2)+f(x_4)+\ldots]+f(x_n)\right]$ where $w=\frac{b-a}{n}$		
simple harmonic motion	If $\frac{d^2x}{dt^2}=-\omega^2x$ then $x=A\sin(\omega t+\alpha)$ or $x=A\cos(\omega t+\beta)$		
	$v^2=\omega^2\left(A^2-x^2\right)$	$T=\frac{2\pi}{\omega}$	$f=\frac{1}{T}$
acceleration	$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$		

Additional Trigonometry for Specialist Mathematics	
Pythagorean identities	$\sin^2(A)+\cos^2(A)=1$ $\tan^2(A)+1=\sec^2(A)$ $\cot^2(A)+1=\operatorname{cosec}^2(A)$
angle sum and difference identities	$\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$ $\sin(A-B)=\sin(A)\cos(B)-\cos(A)\sin(B)$ $\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$ $\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B)$
double-angle identities	$\sin(2A)=2\sin(A)\cos(A)$ $\cos(2A)=\cos^2(A)-\sin^2(A)$ $=1-2\sin^2(A)$ $=2\cos^2(A)-1$
product identities	$\sin(A)\sin(B)=\frac{1}{2}(\cos(A-B)-\cos(A+B))$ $\cos(A)\cos(B)=\frac{1}{2}(\cos(A-B)+\cos(A+B))$ $\sin(A)\cos(B)=\frac{1}{2}(\sin(A+B)+\sin(A-B))$ $\cos(A)\sin(B)=\frac{1}{2}(\sin(A+B)-\sin(A-B))$

Graph equations for Specialist Mathematics		
sphere	$(x-h)^2+(y-k)^2+(z-l)^2=r^2$	
circle	$(x-h)^2+(y-k)^2=r^2$	
ellipse	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	
hyperbola	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

Additional Statistics for Specialist Mathematics		
inclusion-exclusion principle	$\lvert A \cup B \rvert = \lvert A \rvert + \lvert B \rvert - \lvert A \cap B \rvert$ $\lvert A \cup B \cup C \rvert = \lvert A \rvert + \lvert B \rvert + \lvert C \rvert - \lvert A \cap B \rvert - \lvert A \cap C \rvert - \lvert B \cap C \rvert + \lvert A \cap B \cap C \rvert$	
permutation	${}^nP_r = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1)$	
combination	${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	
probability density function of the exponential distribution	$f(t) = \lambda e^{-\lambda t}$ for $t \geq 0,\ \lambda > 0$	
exponential random variable	mean	$\frac{1}{\lambda}$
	standard deviation	$\frac{1}{\lambda}$
sample means	mean	μ
	standard deviation	$\frac{\sigma}{\sqrt{n}}$
approximate margin of error	$E = z\frac{s}{\sqrt{n}}$	
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$	

Real and complex numbers		
complex number forms	$z = a + bi = r(\cos(\theta) + i\sin(\theta)) = r\operatorname{cis}(\theta)$	
modulus	$\lvert z \rvert = r = \sqrt{a^2 + b^2}$	
argument	$\operatorname{Arg}(z) = \theta,$ $\tan(\theta) = \frac{b}{a},\ \ -\pi < \theta \leq \pi,\ a \neq 0$	$\arg(z) = \operatorname{Arg}(z) + 2\pi n,\ n \in \mathbb{Z}$
product	$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	
quotient	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$	
De Moivre's theorem	$z^n = r^n \operatorname{cis}(n\theta)$	

Matrices		
commutative law for addition	$\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{B}+\boldsymbol{A}$	
additive identity	$\boldsymbol{A}+\boldsymbol{0}=\boldsymbol{A}$	
additive inverse	$\boldsymbol{A}+(-\boldsymbol{A})=\boldsymbol{0}$	
multiplicative identity	$\boldsymbol{A}\boldsymbol{I}=\boldsymbol{A}=\boldsymbol{I}\boldsymbol{A}$	
multiplicative inverse	$\boldsymbol{A}\boldsymbol{A}^{-1}=\boldsymbol{I}=\boldsymbol{A}^{-1}\boldsymbol{A}$	
left distributive law	$\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C})=\boldsymbol{A}\boldsymbol{B}+\boldsymbol{A}\boldsymbol{C}$	
right distributive law	$(\boldsymbol{B}+\boldsymbol{C})\boldsymbol{A}=\boldsymbol{B}\boldsymbol{A}+\boldsymbol{C}\boldsymbol{A}$	
determinant	If $\boldsymbol{A}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\boldsymbol{A})=ad-bc$	
multiplicative inverse matrix	$\boldsymbol{A}^{-1}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}=\dfrac{1}{\det(\boldsymbol{A})}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \det(\boldsymbol{A})\neq 0$	
linear transformations	dilation of factor *a* parallel to the *x*-axis and factor *b* parallel to the *y*-axis	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation of angle *θ* anticlockwise about the origin	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection in the line $y=x\tan(\theta)$	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Vectors		
magnitude	$\lvert \boldsymbol{a} \rvert = \left\lvert \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \right\rvert = \sqrt{a_1^{\,2} + a_2^{\,2}}$	$\lvert \boldsymbol{a} \rvert = \left\lvert \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right\rvert = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$
direction	$\tan(\theta) = \frac{y}{x},\ x \neq 0$	
unit vector	$\hat{\boldsymbol{n}} = \frac{\boldsymbol{n}}{\lvert \boldsymbol{n} \rvert}$	
scalar (dot) product	$\boldsymbol{a} \cdot \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \cos(\theta)$	
	$\boldsymbol{a} \cdot \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = a_1 b_1 + a_2 b_2$	$\boldsymbol{a} \cdot \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$
vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + t\,\boldsymbol{d}$	
parametric equations of a line	$x = a_1 + t\,d_1$ $y = a_2 + t\,d_2$ $z = a_3 + t\,d_3$	
Cartesian equation of a line	$\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$	
vector (cross) product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \sin(\theta)\, \hat{\boldsymbol{n}}$	
	$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$	
scalar projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert \boldsymbol{a} \rvert \cos(\theta) = \boldsymbol{a} \cdot \hat{\boldsymbol{b}}$	
vector projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert \boldsymbol{a} \rvert \cos(\theta)\, \hat{\boldsymbol{b}} = (\boldsymbol{a} \cdot \hat{\boldsymbol{b}})\, \hat{\boldsymbol{b}} = \left(\frac{\boldsymbol{a} \cdot \boldsymbol{b}}{\boldsymbol{b} \cdot \boldsymbol{b}} \right) \boldsymbol{b}$	
vector equation of a plane	$\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	

Physical constant	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8\ \text{m s}^{-2}$

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