LUI | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your barcode ID label here

Book of books used

External assessment 2025

Question and response book

Specialist Mathematics

Paper 2 — Technology-active

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator **permitted**.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 8 short response questions

Clear zone — margin trimmed off after completion of assessment

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Section 1

Instructions

- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- Choose the best answer for Questions 1–10.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Ensure you have filled an answer bubble for each question.

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Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has eight questions and is worth 50 marks.

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QUESTION 11 (4 marks)

The mass of checked bags that passengers take on a Brisbane–Sydney flight is normally distributed with a mean of 21.3 kg and a standard deviation of 4.2 kg.

A random sample of 16 checked bags was conducted.

a) Determine the probability that the mean mass of the checked bags for this sample exceeds 23 kg. *[2 marks]*

There is a 40% probability that the mean mass of the checked bags for this sample is within $\pm m$ kg of the population mean.

b) Determine the value of m. *[2 marks]*

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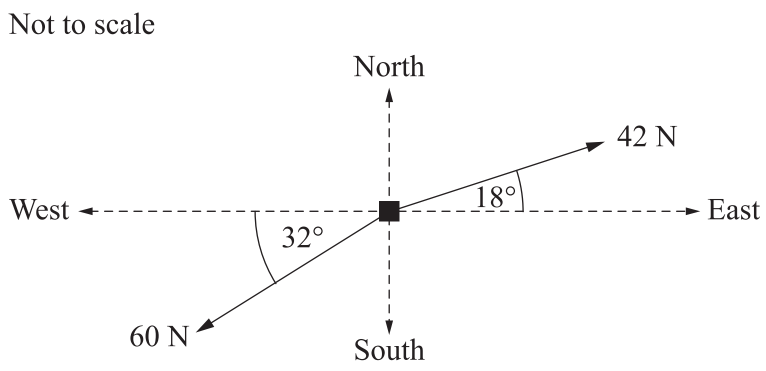
QUESTION 12 (7 marks)

A 10 kg object is travelling at ground level with a constant velocity.

At an instant, two forces of 60 N and 42 N act simultaneously on the object parallel to the ground in the directions shown.



Let unit vectors in the east and north directions be $\hat{i}$ and $\hat{j}$ respectively.

a) Determine the resultant force acting on the object expressed in Cartesian form. Simplify your answer. *[2 marks]*

b) Determine the acceleration of the object after the forces act. Leave your answer in Cartesian form. *[1 mark]*

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The object is initially travelling at 5 m s^{-1} in a northerly direction when the forces act.

c) Determine an expression for the object's velocity in terms of time, t, after the forces act. Leave your answer in Cartesian form. *[2 marks]*

d) Calculate the speed of the object after the forces have been acting for two seconds. *[2 marks]*

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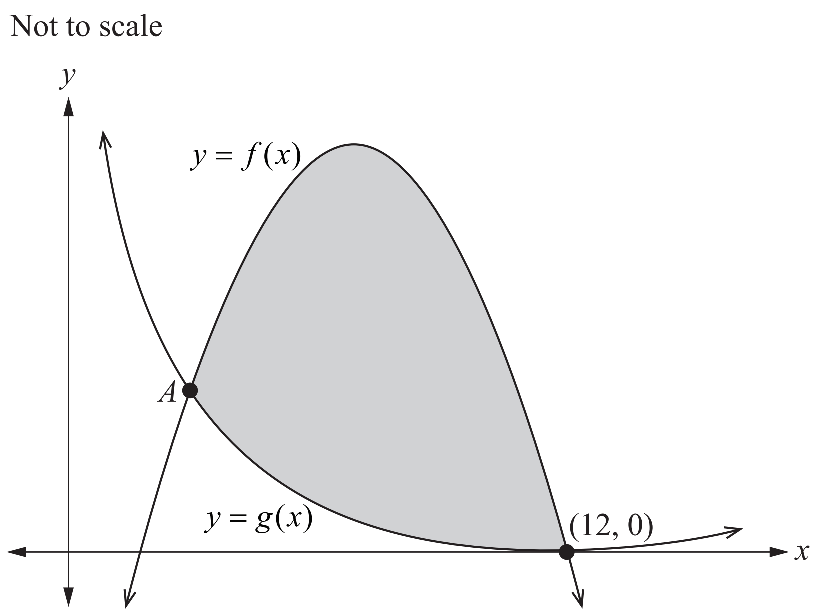
QUESTION 13 (7 marks)

The sketch shows sections of the functions $f(x) = -0.5x^2 + 7.5x - 18$ and $g(x) = 4\operatorname{cosec}\left(\frac{\pi x}{24}\right) - 4$.

Two points of intersection at (12, 0) and point A are shown.



a) Determine the coordinates of point A. *[1 mark]*

Consider the shaded bounded region between the functions.

b) Determine an approximate area of this region using Simpson's rule with four intervals. Show evidence of the values substituted into this rule in your solution. *[3 marks]*

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c) State a definite integral that represents the area of the shaded bounded region. *[1 mark]*

d) Determine the value of your result from Question 13c). Give your answer to two decimal places. *[1 mark]*

e) Other than working to more decimal places, state a strategy involving Simpson's rule that could be used to improve the accuracy of your result from Question 13b). *[1 mark]*

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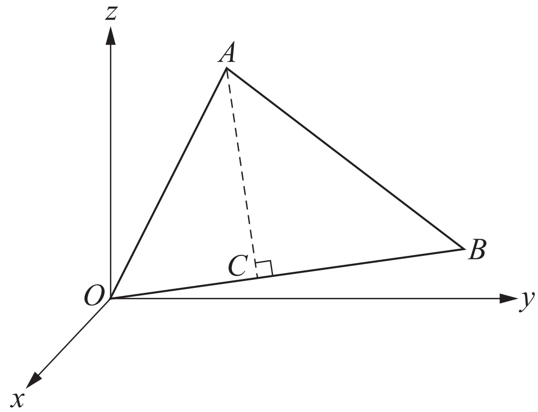
QUESTION 14 (8 marks)

The origin, O, is joined to points $A(1, 2, 5)$ and $B(-3, 4, 0)$ to form triangle OAB.

Point C is the point on OB such that AC is perpendicular to OB, as shown.

Not to scale



a) Given the length of side OB is 5 units, show that the vector projection of

$\overrightarrow{OA}$ on $\overrightarrow{OB}$ is $\frac{1}{5}\begin{pmatrix}-3\\4\\0\end{pmatrix}$. *[2 marks]*

b) Use your result from Question 14a) to determine the length of OC. *[1 mark]*

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c) Determine the length of side *OA*. *[1 mark]*

d) Use Pythagoras' theorem to determine the length of *AC*. *[1 mark]*

e) Use your result from Question 14d) to determine the area of triangle *OAB*. *[1 mark]*

f) Use a vector product method to verify your result from Question 14e). *[2 marks]*

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QUESTION 15 (6 marks)

De Moivre's theorem can be expressed as

$$\left(r\left(\cos(\theta)+i\sin(\theta)\right)\right)^n = r^n\left(\cos(n\theta)+i\sin(n\theta)\right)\ \forall n \in Z^+$$

Prove De Moivre's theorem using mathematical induction.

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QUESTION 16 (6 marks)

A certain population can be approximately modelled by the differential equation

$$\frac{dP}{dt} = 0.5P(1 - 0.2P)$$

where P is the population in millions and t is the number of years since 1 January 2025.

Given that the population on 1 January 2025 was estimated at 0.3 million, use a calculus approach to estimate the population on 1 January 2030.

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QUESTION 17 (5 marks)

A variable, X, is assumed to be normally distributed with $\mu = 24.311$ and $\sigma = 5.102$.

Two 90% confidence intervals for μ were calculated from two different random samples from X, with the second sample being smaller than the first sample by 60.

Both confidence intervals were calculated using the population standard deviation rather than their respective sample standard deviations.

The confidence interval produced from the first sample was (23.560, 25.498).

Determine the probability that the confidence interval produced from the second sample overlaps the confidence interval from the first sample.

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QUESTION 18 (7 marks)

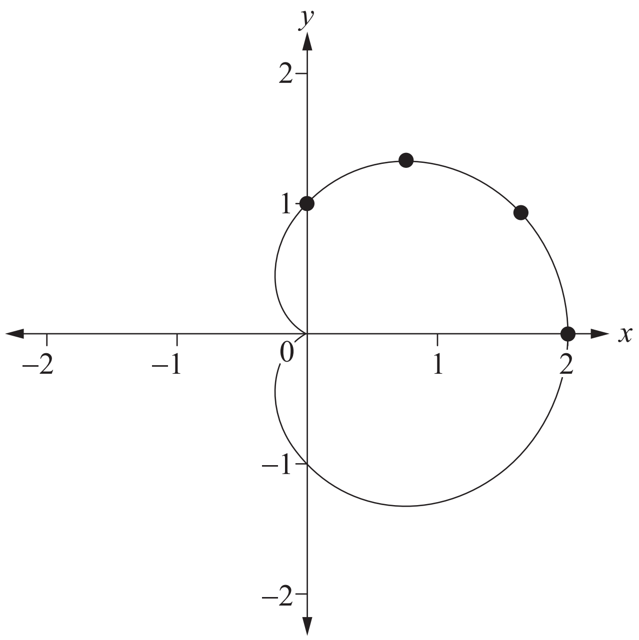
Polar curves are defined by points that are a variable distance of r units from the origin and dependent on the angle θ (in radians) measured from the positive x-axis.

Consider the polar curve $r = 1 + \cos(\theta)$.

A table of four polar coordinates on this curve is shown.

θ	r
0	2
$\frac{\pi}{6}$	$1+\frac{\sqrt{3}}{2}$
$\frac{\pi}{3}$	1.5
$\frac{\pi}{2}$	1

The graph shows the polar curve $r = 1 + \cos(\theta)$ for $0 \le \theta \le 2\pi$ on a Cartesian plane. The polar coordinates from the table have been plotted on the curve.



The length of a polar curve, L, from $\theta = a$ to $\theta = b$ can be determined using the rule

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$$

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Use a complete algebraic method to determine the length of the section of the given polar curve that lies above the *x*-axis.

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ADDITIONAL PAGE FOR STUDENT RESPONSES

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