LUI | | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your barcode ID label here

Book ☐ of ☐ books used

External assessment 2025

Question and response book

Specialist Mathematics

Paper 1 — Technology-free

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are **not** permitted.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

Section 1

Instructions

- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- Choose the best answer for Questions 1–10.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Ensure you have filled an answer bubble for each question.

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Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 50 marks.

QUESTION 11 (5 marks)

Consider the system of linear equations represented using the augmented matrix shown.

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -6 \\ -2 & 1 & 1 & 1 \\ 0 & 0 & 4 & 4 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

Key: R_1 represents the row 1 values.

a) Modify the augmented matrix using the row operation shown. *[1 mark]*

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -6 \\ \square & \square & \square & \square \\ 0 & 0 & 4 & 4 \end{array}\right] \begin{array}{l} R_1 \\ R_2' = R_2 + 2R_1 \\ R_3 \end{array}$$

Key: $R_2' = R_2 + 2R_1$ indicates that the new row 2 values are equal to the sum of the existing row 2 and twice row 1 values.

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b) Given the row 1 values represent the equation $x - y - z = -6$, use your result from Question 11a) to determine the solution of the system of linear equations. *[3 marks]*

The system of linear equations is geometrically represented by three planes.

c) Use your result from Question 11b) to describe a geometrical interpretation of your solution of the system of linear equations. *[1 mark]*

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QUESTION 12 (6 marks)

A line, l, is given by the equation $\frac{x+1}{-2} = \frac{y-3}{2} = z - 2$.

a) Given the point $(-1, 3, a)$ lies on the line, determine the value of a. *[1 mark]*

b) Determine a vector, $\boldsymbol{d}$, in the direction of the line. *[1 mark]*

A plane, φ, is given by the equation $x - y + 4z = 8$.

c) Given the point $(b, b, -2b)$ lies on the plane, determine the value of b. *[1 mark]*

d) Determine a vector, $\boldsymbol{n}$, that is normal to the plane. *[1 mark]*

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e) Show that $\boldsymbol{d} \cdot \boldsymbol{n} = 0$. *[1 mark]*

Consider the statement: *The line, l, is perpendicular to the plane, φ.*

f) Use your result from Question 12e) to comment on the reasonableness of the statement. *[1 mark]*

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QUESTION 13 (5 marks)

The velocity (m s^{-1}) of a 3 kg object moving in a straight line is given by

$$v = 2\cos^{-1}\left(\frac{x}{3}\right),\ 0 \le x < 3$$

where x is its position (m) from the origin.

a) Determine the momentum (kg m s^{-1}) of the object when it is at the origin. *[2 marks]*

b) Determine the acceleration (m s^{-2}) of the object when it is at the origin. *[3 marks]*

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QUESTION 14 (5 marks)

A factory produces cans of juice. Each can has a labelled volume of 500 mL.

The factory manager conducted a random sample of 100 cans to assess whether the labelled volume was being met satisfactorily in production.

The mean volume of the sample was 498.9 mL with a sample standard deviation of 4.0 mL.

a) Based on this sample and using a z-value of 2, determine an approximate confidence interval for the population mean volume. *[3 marks]*

Industry regulation requires that the mean volume must meet or exceed the labelled volume.

b) Use your result from Question 14a) to state whether the current production meets the regulation. Justify your decision using mathematical reasoning. *[2 marks]*

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QUESTION 15 (5 marks)

a) Use integration by parts to show

$$\int x^2 e^{-x}\, dx = -e^{-x}\left(x^2 + 2x + 2\right) + c$$ *[3 marks]*

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The area of the bounded region between the graphs of $y = x^2e^{-x}$ and $y = x^2e^{-1}$ over the domain [0, 1] is given by

$$\int_0^1 x^2\left(e^{-x} - e^{-1}\right)\,dx$$

b) Use your result from Question 15a) to determine this area. Simplify your answer. *[2 marks]*

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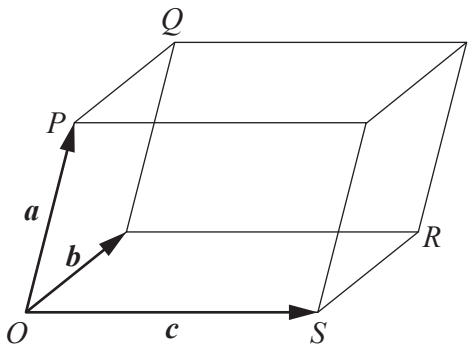
QUESTION 16 (5 marks)

A parallelepiped is a three-dimensional figure where all six faces are parallelograms. It can be defined by vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$, as shown. The origin O and points P, Q, R and S are vertices of the parallelepiped.



Use vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ to prove that the diagonal from P to R and the diagonal from Q to S bisect each other.

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QUESTION 17 (7 marks)

The radius of a cylinder decreases at a constant rate of 0.5 m s^{-1}, while maintaining a constant height of four metres.

Given that the cylinder has an initial volume of 100π m^3, determine the rate of change of the volume (m^3 s^{-1}) of the cylinder after four seconds.

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QUESTION 18 (6 marks)

An object is projected at an acute angle of θ below the horizontal, with an initial speed of 30 m s^{-1} from a position 90 m above ground level.

The object hits the ground 90 m horizontally from its projection point.

Use vector calculus to determine θ in its simplest form.

Assume that the magnitude of mean acceleration due to gravity on Earth is 10 m s^{-2} and that there is no air resistance.

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QUESTION 19 (6 marks)

Let $w \in C$ be any fifth root of unity, where $w \notin R$

Show that $w^3(1+w)\left(1+w^3\right) \in Z^-$

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Do not write outside this box.

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Write the question number you are responding to.

Do not write outside this box.