LUI | | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your barcode ID label here

Book of books used

External assessment 2024

Question and response book

Specialist Mathematics

Paper 1 — Technology-free

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are **not** permitted.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

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Section 1

Instructions

- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- Choose the best answer for Questions 1–10.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Ensure you have filled an answer bubble for each question.

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Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 50 marks.

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QUESTION 11 (4 marks)

The vector equation of a straight line is given by $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + k\begin{pmatrix} -1 \\ 2 \end{pmatrix}$, where k is a scalar.

a) Express the equation of the line as a pair of parametric equations. *[1 mark]*

b) Use your result from Question 11a) to express the equation of the line as a Cartesian equation. *[1 mark]*

c) Determine the coordinates of the point that the line passes through when $k = 5$. *[1 mark]*

d) Determine the value of k when the line intersects the y-axis. *[1 mark]*

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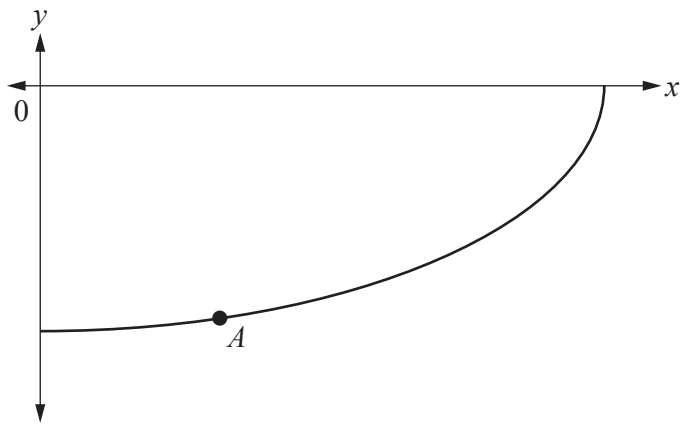
QUESTION 12 (7 marks)

Point A lies on a section of the ellipse $3x^2 + y^2 = 10$ as shown.

The coordinates of A are $\left(\sqrt{2}, y_1\right)$.



a) Determine the value of y_1. *[2 marks]*

b) Use implicit differentiation to determine an expression for $\frac{dy}{dx}$ in terms of x and y. *[2 marks]*

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c) Use your results from Questions 12a) and 12b) to determine the gradient of the tangent to the curve at A. *[1 mark]*

Consider the region between the given section of the ellipse, the x-axis and the lines $x = 0$ and $x = \sqrt{2}$.

d) Determine the volume of the solid of revolution formed by rotating this region about the x-axis. Express your answer in simplest form. *[2 marks]*

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QUESTION 13 (4 marks)

$P(z) = a z^2 - i z + 1 - 3i$ and $Q(z) = z^2 + 3i z + 2a$, where $a \in C$, have the same remainder when divided by $z - i$.

Use the remainder theorem to determine the value of a.

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QUESTION 14 (5 marks)

The displacement (cm) of a particle from the origin as it travels in two-dimensional space at time t for $0 \le t < \frac{\pi}{2}$ seconds is given by

$$\boldsymbol{r} = \left(2\sec\left(t\right) - 1\right)\hat{\boldsymbol{i}} + \tan\left(t\right)\hat{\boldsymbol{j}}$$

a) Express the path of the particle as a pair of parametric equations. *[1 mark]*

A general Cartesian form of a hyperbola with centre (h, k) is

$$\frac{\left(x-h\right)^2}{a^2} - \frac{\left(y-k\right)^2}{b^2} = 1, \text{ where } a, b \neq 0.$$

b) Use a suitable Pythagorean identity to show that the path of the particle can be expressed in this general Cartesian form. *[3 marks]*

c) Determine the centre of the hyperbolic path of the particle. *[1 mark]*

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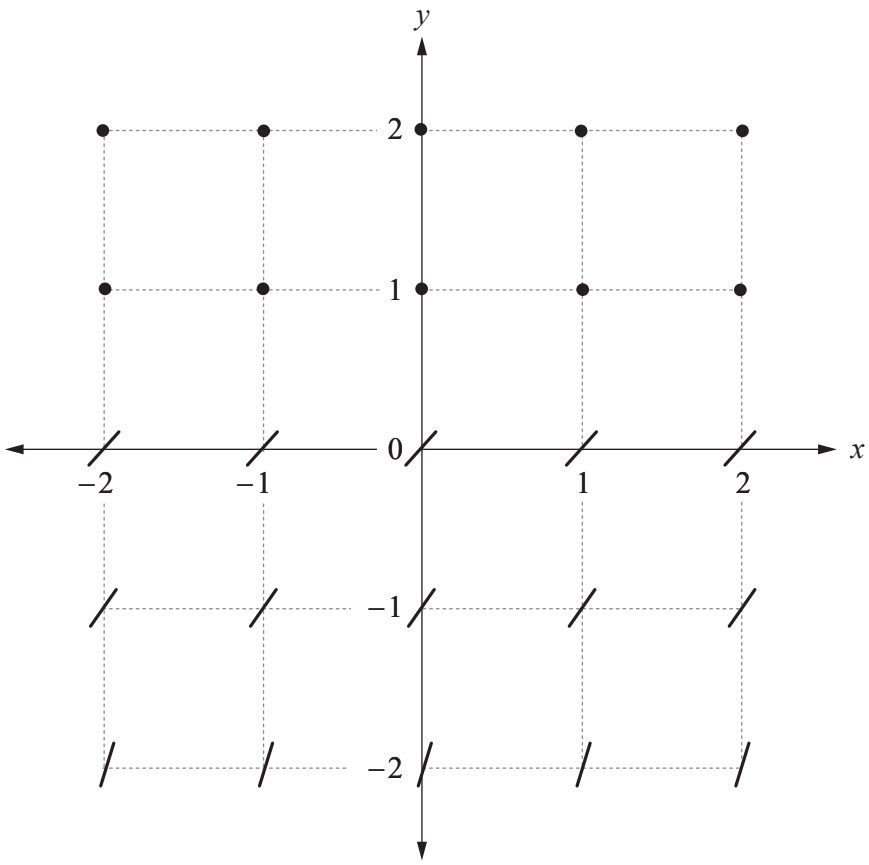
QUESTION 15 (6 marks)

A sketch of a partially completed slope field for the differential equation $\frac{dy}{dx} = 1 - y$ is shown.

a) Complete the slope field by sketching the slopes at the 10 points indicated. *[2 marks]*



b) Use the completed slope field to sketch the solution curve for $\frac{dy}{dx} = 1 - y$, given the point (–1, 0) lies on this curve. *[1 mark]*

Note: If you make a mistake in the diagram, cancel it by ruling a single diagonal line through your work and use the additional response space at the back of this question and response book.

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The differential equation can be solved by rearranging it into the form $\frac{1}{1-y}\frac{dy}{dx}=1$.

c) Determine the equation of the solution curve sketched in Question 15b) by solving the differential equation, given the point (–1, 0) lies on this curve. Leave your answer in the form $y = f(x)$. *[3 marks]*

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QUESTION 16 (6 marks)

Use mathematical induction to prove that $12^n + 2\left(5^{n-1}\right)$ is a multiple of 7 for $n \in Z^+$.

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QUESTION 17 (6 marks)

The acceleration (m s^{-2}) of an object that moves in a straight line in an easterly direction over time t for $0 \le t \le \frac{\pi}{6}$ seconds is given by $a = 2\left(1 + v^2\right)$, where v is its velocity (m s^{-1}).

The object is initially at rest at a position that is $\ln\left(\sqrt{2}\right)$ metres west of the origin.

A student uses this information to calculate that the object is positioned at the origin when $t = \frac{\pi}{6}$ seconds.

Evaluate the reasonableness of the student's calculation.

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QUESTION 18 (6 marks)

A random variable X has a probability density function given by

$$f(x) = \begin{cases} k\sin^{-1}(x), & 0 \le x \le 1 \\ 0 \quad , & \text{otherwise} \end{cases}$$

where k is a positive constant.

Determine the value of k.

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QUESTION 19 (6 marks)

Consider complex numbers of the form $w = x + i$, where x is a positive real number.

If $Re\left(w^7\right) = 0$, determine all possible values of x.

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END OF PAPER

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Do not write outside this box.

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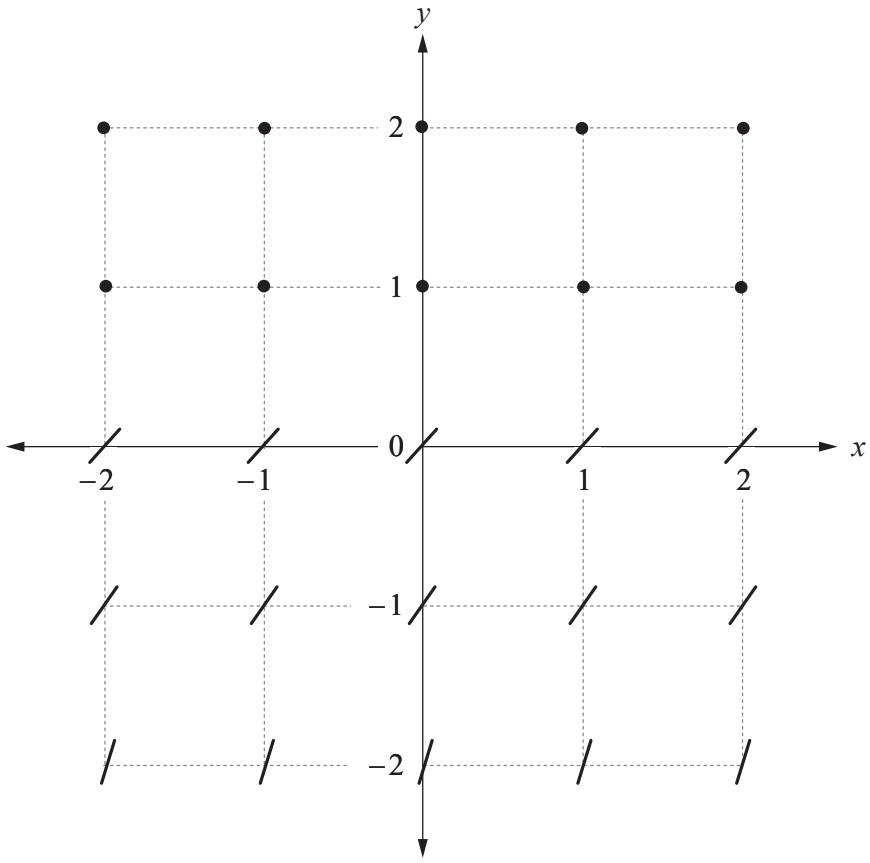
ADDITIONAL RESPONSE SPACE FOR QUESTIONS 15a) AND 15b)

If you want this diagram to be marked, rule a single diagonal line through your original response.



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