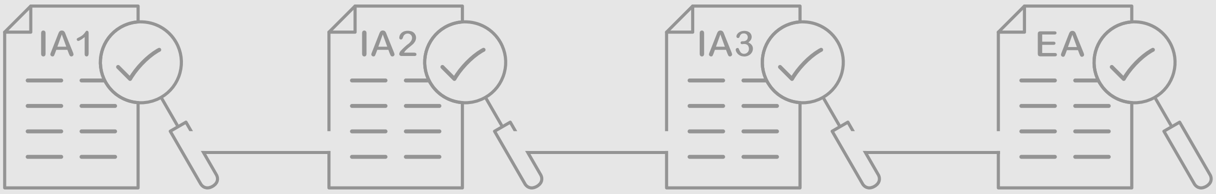
Specialist Mathematics subject report

2023 cohort

January 2024



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Introduction

Throughout 2023, schools and the Queensland Curriculum and Assessment Authority (QCAA) continued to improve outcomes for students in the Queensland Certificate of Education (QCE) system. These efforts were consolidated by the cumulative experience in teaching, learning and assessment of the current General and General (Extension) senior syllabuses, and school engagement in QCAA endorsement and confirmation processes and external assessment marking. The current evaluation of the QCE system will further enhance understanding of the summative assessment cycle and will inform future QCAA subject reports.

The annual subject reports seek to identify strengths and opportunities for improvement of internal and external assessment processes for all Queensland schools. The 2023 subject report is the culmination of the partnership between schools and the QCAA. It addresses school-based assessment design and judgments, and student responses to external assessment for this subject. In acknowledging effective practices and areas for refinement, it offers schools timely and evidence-based guidance to further develop student learning and assessment experiences for 2024.

The report also includes information about:

- how schools have applied syllabus objectives in the design and marking of internal assessments
- how syllabus objectives have been applied in the marking of external assessments
- patterns of student achievement.

The report promotes continuous improvement by:

- identifying effective practices in the design and marking of valid, accessible and reliable assessments
- recommending where and how to enhance the design and marking of valid, accessible and reliable assessment instruments
- providing examples that demonstrate best practice.

Schools are encouraged to reflect on the effective practices identified for each assessment, consider the recommendations to strengthen assessment design and explore the authentic student work samples provided.

Audience and use

This report should be read by school leaders, subject leaders and teachers to:

- inform teaching and learning and assessment preparation
- assist in assessment design practice
- assist in making assessment decisions
- help prepare students for internal and external assessment.

The report is publicly available to promote transparency and accountability. Students, parents, community members and other education stakeholders can use it to learn about the assessment practices and outcomes for senior subjects.

Report preparation

The report includes analyses of data and other information from endorsement, confirmation and external assessment processes. It also includes advice from the chief confirmer, chief endorser and chief marker, developed in consultation with and support from QCAA subject matter experts.

Subject highlights

312
schools offered Specialist Mathematics

85.05%
of students completed 4 units

96.48%
of students received a C or higher

Subject data summary

Subject completion

The following data includes students who completed the General subject or Alternative Sequence (AS).

Note: All data is correct as at January 2024. Where percentages are provided, these are rounded to two decimal places and, therefore, may not add up to 100%.

Number of schools that offered Specialist Mathematics: 312.

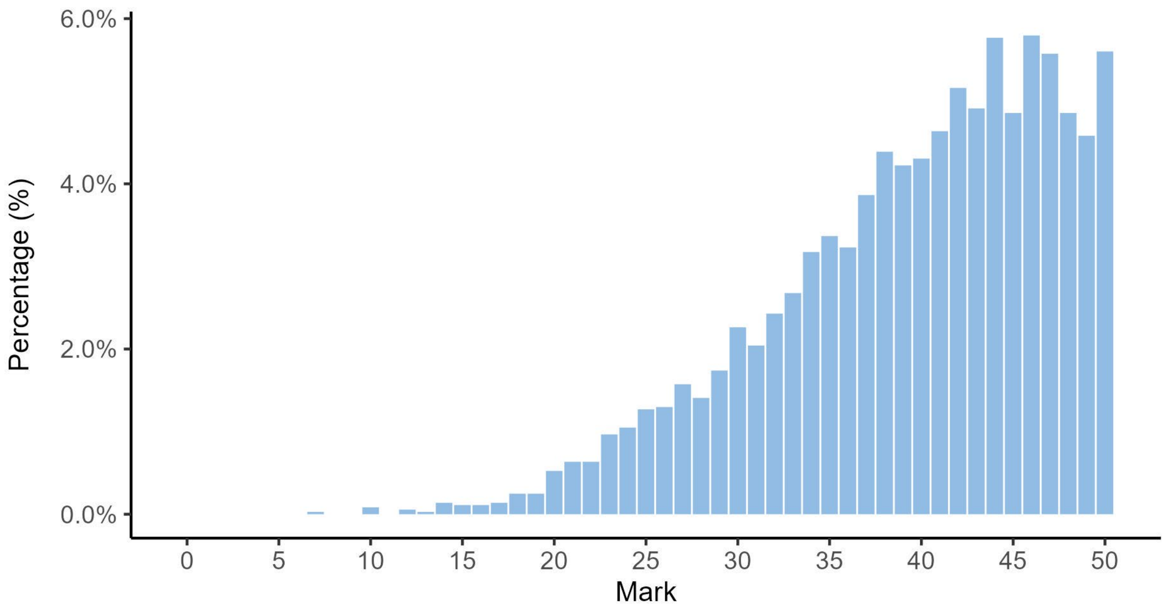
Completion of units	Unit 1	Unit 2	Units 3 and 4
Number of students completed	4,247	3,955	3,612

Units 1 and 2 results

Number of students	Satisfactory	Unsatisfactory
Unit 1	3,980	267
Unit 2	3,622	333

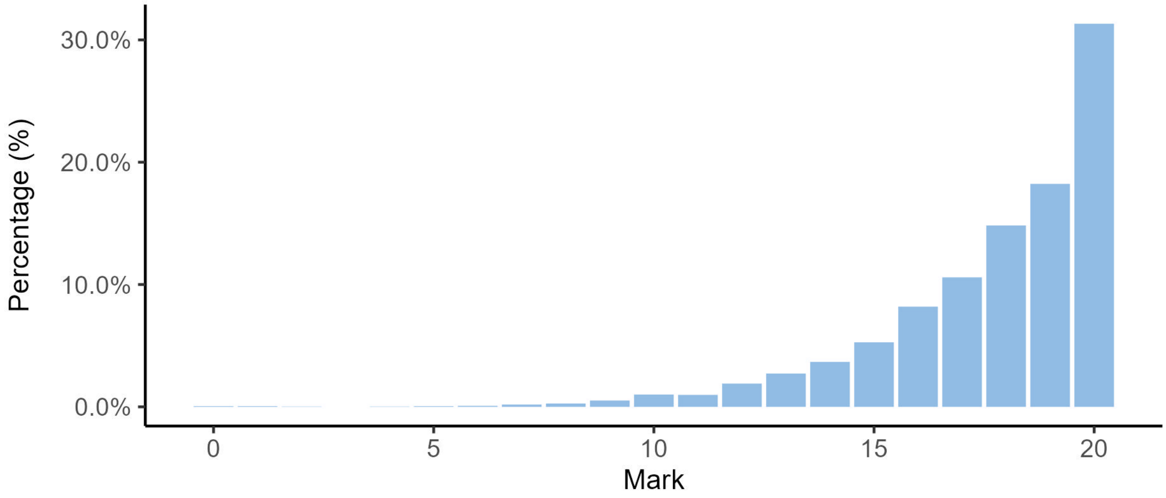
Units 3 and 4 internal assessment (IA) results

Total marks for IA

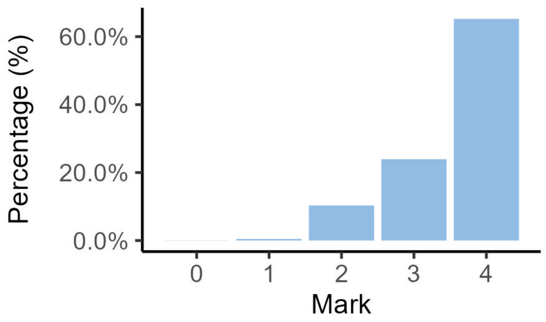


IA1 marks

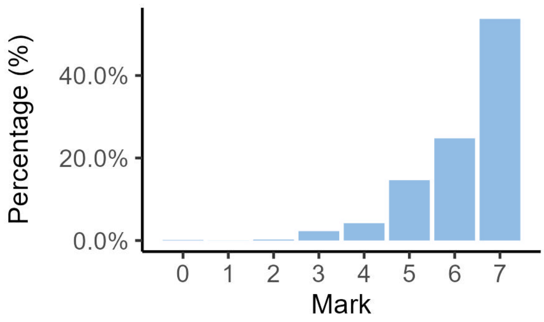
IA1 total



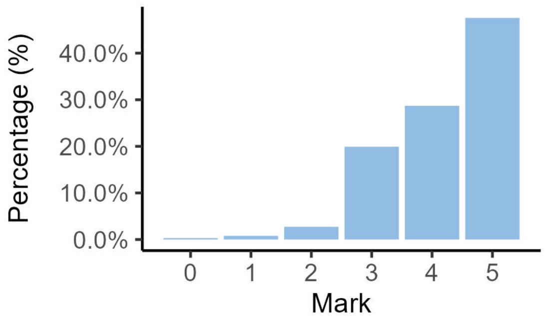
IA1 Criterion: Formulate



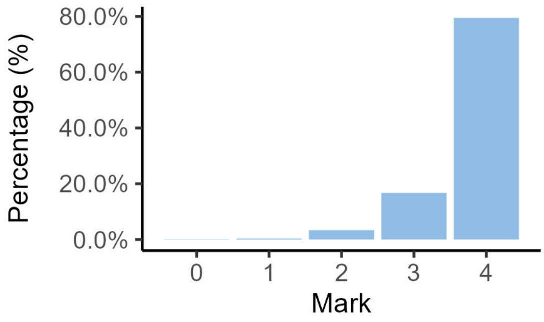
IA1 Criterion: Solve



IA1 Criterion: Evaluate and verify

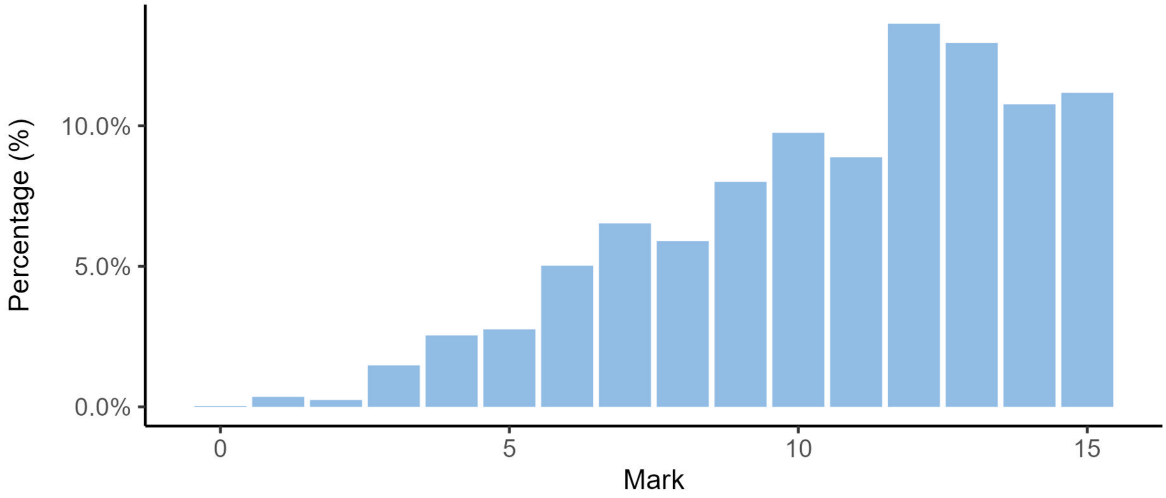


IA1 Criterion: Communicate

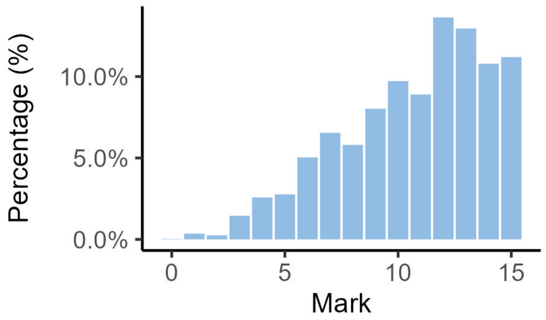


IA2 marks

IA2 total

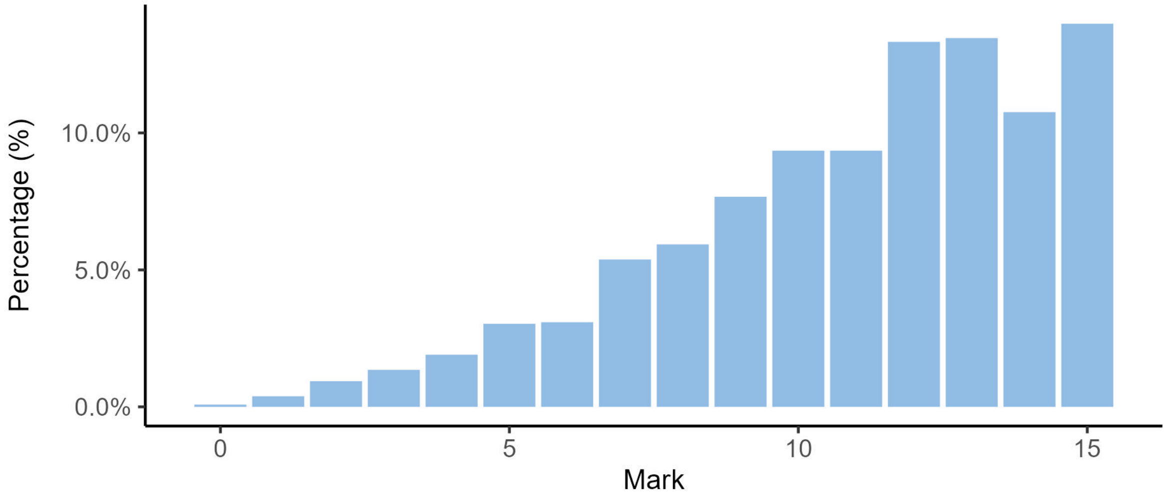


IA2 Criterion: Foundational knowledge and problem-solving

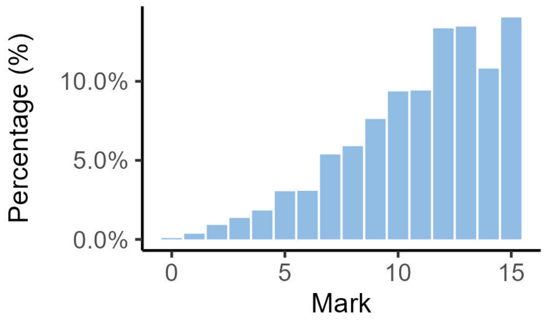


IA3 marks

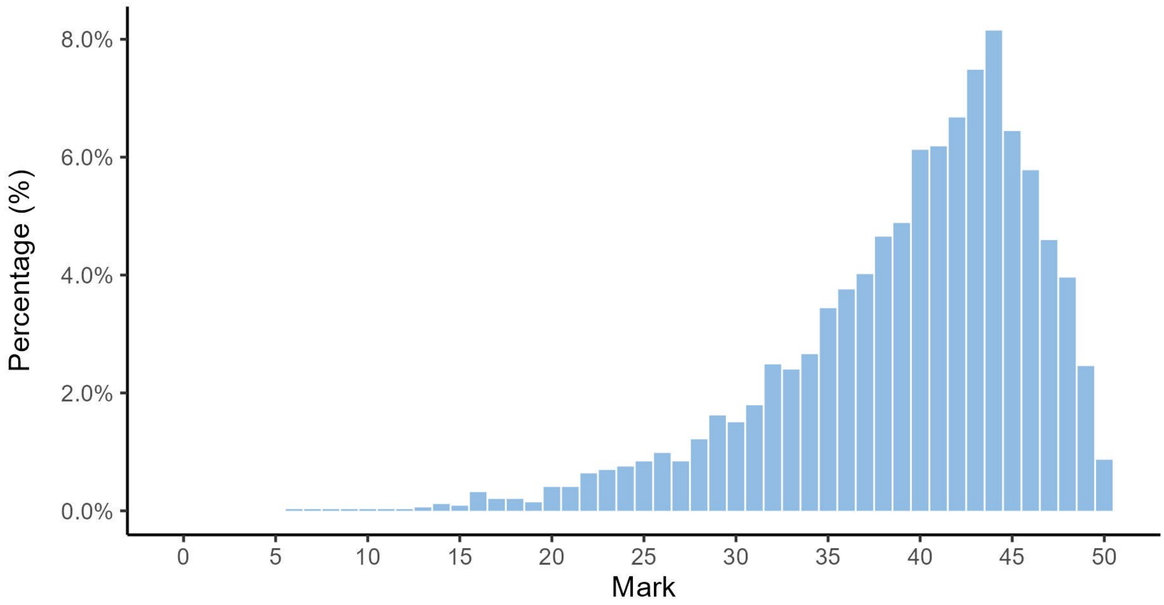
IA3 total



IA3 Criterion: Foundational knowledge and problem-solving

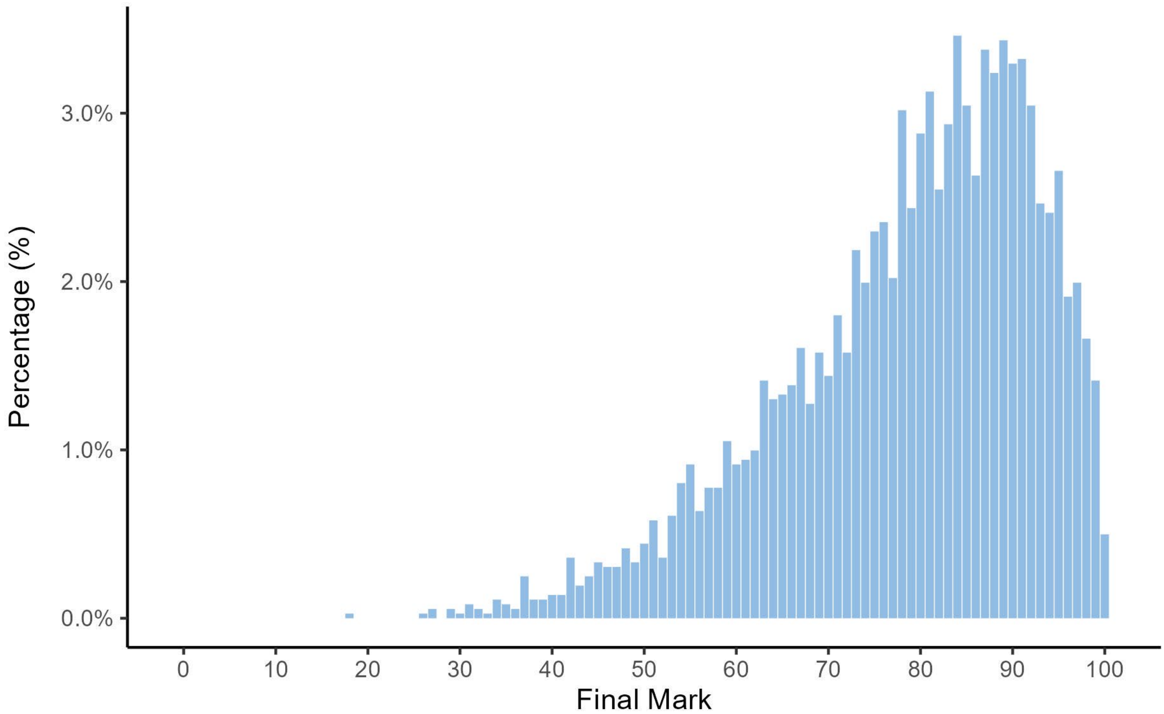


External assessment (EA) marks



Final subject results

Final marks for IA and EA



Grade boundaries

The grade boundaries are determined using a process to compare results on a numeric scale to the reporting standards.

Standard	A	B	C	D	E
Marks achieved	100–85	84–69	68–49	48–22	21–0

Distribution of standards

The number of students who achieved each standard across the state is as follows.

Standard	A	B	C	D	E
Number of students	1,459	1,360	666	126	1

Internal assessment

The following information and advice relate to the assessment design and assessment decisions for each IA in Units 3 and 4. These instruments have undergone quality assurance processes informed by the attributes of quality assessment (validity, accessibility and reliability).

Endorsement

Endorsement is the quality assurance process based on the attributes of validity and accessibility. These attributes are categorised further as priorities for assessment, and each priority can be further broken down into assessment practices.

Data presented in the Assessment design section identifies the reasons why IA instruments were not endorsed at Application 1, by the priority for assessments. An IA may have been identified more than once for a priority for assessment, e.g. it may have demonstrated a misalignment to both the subject matter and the assessment objective/s.

Refer to *QCE and QCIA policy and procedures handbook v5.0*, Section 9.6.

Percentage of instruments endorsed in Application 1

Number of instruments submitted	IA1	IA2	IA3
Total number of instruments	314	334	310
Percentage endorsed in Application 1	72%	43%	47%

Confirmation

Confirmation is the quality assurance process based on the attribute of reliability. The QCAA uses provisional criterion marks determined by teachers to identify the samples of student responses that schools are required to submit for confirmation.

Confirmation samples are representative of the school's decisions about the quality of student work in relation to the instrument-specific marking guide (ISMG), and are used to make decisions about the cohort's results.

Refer to *QCE and QCIA policy and procedures handbook v5.0*, Section 9.7.

The following table includes the percentage agreement between the provisional marks and confirmed marks by assessment instrument. The Assessment decisions section of this report for each assessment instrument identifies the agreement trends between provisional and confirmed marks by criterion.

Number of samples reviewed and percentage agreement

IA	Number of schools	Number of samples requested	Number of additional samples requested	Percentage agreement with provisional marks
1	305	1,783	17	92.46%
2	303	1,415	0	99.01%
3	303	1,407	0	99.67%

Internal assessment 1 (IA1)

Problem-solving and modelling task — extended response (20%)

This assessment focuses on the interpretation, analysis and evaluation of ideas and information. It is an independent task responding to a particular situation or stimuli. While students may undertake some research in the writing of the problem-solving and modelling task, it is not the focus of this technique. This assessment occurs over an extended and defined period. Students will use class time and their own time to develop a response.

The problem-solving and modelling task must use subject matter from one or both of the following topics in Unit 3:

- Topic 2: Vectors and matrices
- Topic 3: Complex numbers 2.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	36
Authentication	5
Authenticity	15
Item construction	15
Scope and scale	29

*Each priority might contain up to four assessment practices.

Total number of submissions: 314.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- included clear instructions to students about the requirements of the task, including identification of the topics being assessed
- provided opportunity to develop a unique response, e.g. providing an open-ended task such that students chose how to use the data and what concepts and techniques were relevant to develop the model and solve the problem
- had authentic real-life contexts that were accessible to students
- included relevant stimulus where appropriate.

Practices to strengthen

It is recommended that assessment instruments:

- are sufficiently different from textbook practice assessments and QCAA sample assessment instruments to ensure responses are not rehearsed and that work submitted is the student's own
- avoid scaffolding or task instructions that indicate to students how to solve the problem as this interferes with students' ability to demonstrate their knowledge and understanding of the relevant criteria and to provide a unique, authentic response.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	3
Language	8
Layout	2
Transparency	7

*Each priority might contain up to four assessment practices.

Total number of submissions: 314.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- focused on interpretation, analysis and evaluation of ideas
- included a reference to the approach to problem-solving and modelling flow chart from Syllabus section 1.2.4 (and Syllabus section 4.6.1)
- provided sample data so the appropriateness of the data could be assessed.

Practices to strengthen

It is recommended that assessment instruments:

- use clear, concise language and avoid jargon that does not contribute to the understanding of the task
- avoid requiring students to do research to be able to use a particular technique
- clearly articulate the task requirements.

Additional advice

- Use checkpoints to indicate that feedback is to be provided on one complete or near-complete draft.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Formulate	95.08%	2.62%	1.97%	0.33%
2	Solve	98.69%	0.66%	0.66%	0%
3	Evaluate and verify	96.72%	2.95%	0.33%	0%
4	Communicate	99.67%	0%	0.33%	0%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- for the Solve criterion
  - judgments about discerning application were supported by identifying where the student demonstrated discriminating intellectual perception or made thoughtful and astute choices in synthesising their approach to solve the problem
  - the application of relevant mathematical concepts and techniques were deemed discerning when the student accurately assessed possible approaches (e.g. refining models and testing hypotheses) and made good judgments about the choices. Evidence of discerning application of concepts and techniques was distinct from the initial choices made in selecting concepts and techniques, and was identified in a student's explanation of their choices made throughout the solving of the problem
- for the Communicate criterion, teachers identified that for appropriate vocabulary to develop the response, it added detail and clarity to the information being communicated. For correct use of mathematical conventions, the student presented their findings in a way that was easily understood. This included
  - labelling tables, figures, graphs (and their axes)
  - aligning equations to their equals signs in the centre of the page
  - using symbols and notation as demonstrated in the syllabus, e.g. matrices, trigonometric functions or summation notation.

Samples of effective practices

The following excerpts have been included to provide an example of documented assumptions that are appropriate to the task. The assumptions have been documented by including a source, a reason for making the assumption or by explaining how making the assumption will affect the solution.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Excerpt 1

Assumptions

1. It is assumed that female birth, death rate, male to female population ratio and base migration rates stay consistent over time. This means the rates that were gathered from 2021 data are expected to continue. Therefore, the model uses the same rate for every time period.
2. It is assumed that 50% of migrants are female, therefore the Net interstate migration (NIM) and net overseas migration (NOM) numbers were halved. This may not be true to what is experienced in South Australia however data was limited. This impacts the model as only the female migrants are included in calculations and then substituted into the male to female ratio to gather the total population.
3. It was assumed that fecundity rates are only applicable for females aged between 15-49. While there would be outliers that are outside this age bracket, they would be insignificant, and are also not included in the data supplied by the Australian Bureau of Statistics. Therefore, age groups outside of 15-49 are assumed to have a zero-fecundity rate.
4. It is assumed that the percentage of births that are female (48.7% from observation 3) is the same for every age group. While some age groups may be more likely to give birth to males or females, this varies over time, therefore a standard percentage was applied to each age group.
5. It is assumed that migration rates are divided evenly into age groups below 75 years old. It is less likely that people over the age of 75 migrate. This means that the population increase due to migration is distributed amongst the younger age groups and lead to a younger population.

Excerpt 2

Documentation of appropriate assumptions.

2.2 Assumptions

The optimum designs for the escape room maze were influenced by the **assumptions** that:

1. **All persons of a group will be in the same room and move as a group when transitioning between rooms.** As stated in the first observation (refer to Section 2.1), a non-homogenous Markov chain model will be used. This model accounts for the transitioning probability of one group, i.e. the probability of the group as a whole to transition between rooms (Klappencker, 2018). Hence, this model does not support the separation of groups whilst in the maze. Therefore, to fit the non-homogenous Markov chain model, and allow a solution to be calculated, it is assumed groups will not split.
2. As stated in the introduction, this report will calculate the time required for nearly all of both groups to successfully exit the maze, as required by the Escape Room company. 'Nearly all' is not specific and hence licences a default generalization relative to the pragmatic factors of the task. A study by the University of Pennsylvania found that from 85 news articles and reports, 'nearly all' was cited to be 60%-90%, although this reached a cited 99.9% (Liberman, 2010). However, considering the pragmatic factors of the task, all groups must be able to exist the maze. So, it is **assumed 'nearly all' equates to 90%-100% of the groups.**
3. As stated in the introduction, the parent and teen groups house the same probability of success in all rooms, excluding room 3. Said probabilities are used to calculate the transitioning probabilities, allowing an optimal solution to be identified. In many escapes room models utilised by other escape room companies, groups can request a clue which, subsequently, increases their probability of success for a particular room, in turn affecting their transitioning probability and predicted exiting time (Gray, 2022). The Markov Chain model used licenses the assumption that **the conditions of the maze are constant across groups, thus prohibits the groups receival of clues/hints.** Additionally, if a group or group member have previously completed the maze, they would have a competitive advantage over other groups due to their prior knowledge which would hence increase their probability of success. Therefore, it is assumed that, to allow constant conditions, no groups are to receive hints or clues and that **groups are completing the maze for the first time.**
4. In accordance to Observation 3 (refer to Section 2.1), it is assumed that the **cognitive ability of group members is consistent across all teen groups and all parent groups** as to not influence their probabilities of success. Different age groups house different cognitive abilities: groups with higher cognitive ability are more likely to employ more affective problem-solving techniques in addition to prior knowledge which hence increase their likelihood of success (Fredda Blanchard-Fields, 2007) Assuming **all groups comprise members of the same age group** allows for constant conditions for all groups, thus coinciding with the Markov chain model and helping to calculate and identify the optimal solutions.

The following excerpt has been included to provide an example of discerning application of concepts and techniques relevant to the task. The student made thoughtful decisions in determining how to solve the problem.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Since the release angle calculated (37.69°) matches the angle observed earlier (38°), the angle observed can be verified.

Optimising the javelin throw

The goal of a javelin throw is to achieve the maximum range possible. As u must remain constant, the javelin throw can only be improved by changing θ. However, if θ is changed, t and r_x must change (see Appendix 2). Therefore, the expressions for t and r_x, in terms of θ, will now be found. u (30ms^{-1}) will be substituted into (1):

$$r(t) = (30\cos\theta\, t)\vec{\imath} + (-4.905t^2 + 30\sin\theta\, t + 2)\vec{\jmath} \qquad \textbf{(5)}$$

Isolating the $\vec{\jmath}$ components give the vertical displacement of the javelin at time t:

$$r_y(t) = -4.905t^2 + 30\sin\theta t + 2$$

$r_y(t)$ will be solved for 0 to find the value of time when the javelin hits the ground. The quadratic formula will be used to find a solution for time, in terms of θ.

$$0 = -4.905t^2 + 30\sin\theta t + 2$$

$$t(\theta) = \frac{-30\sin\theta \pm \sqrt{(30\sin\theta)^2 - 4(-4.905)(2)}}{-9.81}$$

The discriminant must be added; subtracting it will not provide a positive value for time.

$$\therefore t(\theta) = \frac{30\sin\theta + \sqrt{(30\sin\theta)^2 + 4(9.81)}}{9.81} \qquad \textbf{(6)}$$

Isolating the $\vec{\imath}$ components of (5) gives the horizontal displacement at time t.

$$r_x(t) = 30\cos\theta t$$

$t(\theta)$ will be substituted into $r_x(t)$, creating a function s with input θ.

$$s(\theta) = \frac{30\cos\theta}{9.81}(30\sin\theta + \sqrt{(30\sin\theta)^2 + 4(9.81)}) \qquad \textbf{(7)}$$

$s'(\theta)$ will be found and simplified (see Appendix 3):

$$s'(\theta) = -\frac{30}{9.81}\sin\theta\left(30\sin\theta + \sqrt{30^2\sin^2\theta + 4(9.81)}\right) + \frac{30}{9.81}\cos\theta\left(30\cos\theta + \frac{30^2\sin\theta\cos\theta}{\sqrt{30^2\sin^2\theta + 4(9.81)}}\right)$$

$$= -\frac{30}{9.81}(9.81\cos 2\theta + \sin\theta\,(\frac{-900\cos 2\theta + 39.24}{\sqrt{900\sin^2\theta + 39.24}})$$

$s'(\theta)$ will be solved for zero to find the value of θ at the maximum point of the function. A calculator will be used to find θ:

$$0 = -\frac{30}{9.81}(9.81\cos 2\theta + \sin\theta\,(\frac{-900\cos 2\theta + 39.24}{\sqrt{900\sin^2\theta + 39.24}})$$

$$\theta = 44.39°$$

θ was substituted into (7) to find the maximum range of the javelin throw:

$$s(44.39) = \frac{30\cos(44.39)}{9.81}(30\sin 44.39 + \sqrt{(30\sin 44.39)^2 + 4(9.81)})$$

The following excerpt has been included to provide an example of evaluation of the reasonableness of solutions by considering results, assumptions and observations. The student referenced results, assumptions and observations throughout their evaluation of the solution.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Evaluation of the reasonableness of solutions by considering the results, assumptions and observations.

4.0 Evaluation and Verification

4.1 Reasonableness of the solution

Through the application of Markov chains and related theory, this report completed its task of designing an adjustable escape room that can be transformed into an escape room targeted for either parent of teen groups. Each escape room comprises 6 rooms, 4 two-way doors, and 7 one-way doors as required by the company, further satisfying the aim of this report.

A non-homogenous Markov chain was constructed and calculated to identify the optimal designs (see Observation 1 in Section 2.1). A non-homogenous chain requires constant conditions across all groups. Thus, when calculating the transitioning probabilities of both parent and teen groups, it was assumed all persons of a group will be in the same room (see Assumption 1 in Section 2.2). This created a reasonable solution as it coincided with mathematical theory, allowing for accurate results to be calculated, and is consistent with a normal escape room model.

However, the assumption that groups are completing the maze for the first time and do not receive clues whilst completing the maze (see Assumption 3 in Section 2.2) is not consistent with a normal escape room model nor is it realistic as individuals may wish to complete the maze multiple times. Hence, decreasing the reasonableness of the model.

The assumption that all groups comprise members of the same age ranges is additionally not realistic to a real-life scenario as groups consisting of multiple age ranges, like families, may want to complete the maze together (see Assumption 4 in Section 2.2). However, this goes against the observation that age ranges of groups completing the maze influence their probability of success (see Observation 3 in Section 2.1). The incompletion of this assumption in a real-life scenario goes against the non-homogeneous theory of constant conditions across groups, thus rendering the calculations made too simplistic and non-consistent to a real-life business model, hence, further decreasing the reasonableness of the model.

The observation that the spread/compactness of the maze affects a groups success influenced the design of the optimal solutions (see Observation 2 in Section 2.1). The influence of patterns on the design provides confidence as the solution choose is the most optimal solution as it was well considered. As designing and building an escape room requires money and resources, the company request to be confident in the identification of the optimal design is vital, this thus increases the reasonableness of the model as it satisfies the company's needs.

The company's needs are further satisfied as the optimal models allows 52% of teens and parents (2 percentage points above the aim of 50%) out by the end of the timeslot. Thus, providing a reasonable amount of challenge which warrants the assumption participants will find the Escape Room experience relatively enjoyable, in turn foresting a reasonable solution. However, by 70% of the time slot being complete, 36% of teens and 37% of parents had experienced success. Additionally, teen groups (4% of groups) and parent groups (6% of groups) began to experience success after 16% of the time slot (see table 4 and 9). Both factors do not coincide with the company's specification for groups to be actively involved for more than 70% of the time slot, thus inhibiting the reasonableness of the solution. It is noted that equity is experienced over both groups as the same percentage of groups exit the maze after the allocated time slot and by the 70% mark, only 1 percentage point more of parents group experience success compared to the teen groups.

> As stated in Assumption 2, the company's request to find the time of completion for all groups equates to the time of completion for 90%-100% of groups. After three hours, 92% of teen groups and 91% of parent groups experience success whilst 100% of teen groups do not experience success until 6 hours and 5 minutes whilst 100% of parents group do not experience success until 6 and a half hours which is an unrealistic time to be spent completing an escape room (refer to Table 5 and 10).
>
> In summation, the identified optimal models meet the company's specifications, however, the calculations performed to identify the models are not consistent with real life scenarios or business models. Thus, on a surface level, the models are extremely reasonable to the company's specifications, but this reasonableness exponentially decreases if said models where to be employed in a real life business. To increase the overall reasonableness, more complex theory and calculations need to be used.

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- for the Formulate criterion
  - at the high-performance level, observations and assumptions must be relevant and appropriate respectively; they need to directly relate to the purpose of solving the problem, i.e. they influence the choice or application of mathematical principles, concepts, techniques and technology. Their documentation should demonstrate the logic of the student's selected approach to solving the problem
  - judgments about assumptions and observations should be supported by identifying the evidence of their relevance and appropriateness in their documentation
- for an evaluation of reasonableness under the Evaluate and verify criterion, students need to consider more than just the results. They need to consider the relevance of the observations and the implications of the assumptions. Evidence for evaluation of reasonableness should identify discussion of whether the model provides a valid solution to the specific problem identified in the task.

Additional advice

- It is recommended that schools refer to 'Problem-solving and mathematical modelling' in Section 1.2.4 of the syllabus (including the flow chart on page 15), to deepen their understanding of the characteristics described in the performance levels of the ISMG.
- Schools must identify how they applied their school assessment policy to responses exceeding the maximum of 2000 words or the maximum of 10 pages. See the *QCE and QCIA policy and procedures handbook v5.0* (Section 8.2.6) for advice on managing response length. Communication with the QCAA is recommended if further advice is required about managing responses that exceed the allowable length.
- Schools must use the ISMG from the current syllabus to award results (*QCE and QCIA policy and procedures handbook v5.0*, Section 7.3.3 and Section 8.3). Each summative internal assessment instrument should be printed directly from the Endorsement application (app) on the QCAA Portal for use with students. Schools cannot adjust the wording in the descriptors nor create their own task-specific marking guide.
- Schools must apply the best-fit approach to using the ISMG. Marked ISMGs should indicate the characteristics evident in the student response and the mark awarded for each criterion (*QCE and QCIA policy and procedures handbook v5.0*, Section 9.6.1). It is recommended that

teachers and schools refer to *Module 3 — Making reliable judgments* under Resources in the Assessment Literacy app.

- Schools need to check that the result determined by the best-fit approach in each criterion is the same as the result awarded to the student in their data upload for confirmation. Schools are responsible for ensuring that student results are accurately recorded in Student Management (*QCE and QCIA policy and procedures handbook v5.0*, Section 9.7.1 and Section 11.1.2).

Internal assessment 2 (IA2)

Examination — short response (15%)

The examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 3 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	150
Authentication	0
Authenticity	0
Item construction	13
Scope and scale	47

*Each priority might contain up to four assessment practices.

Total number of submissions: 334.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- explicitly provided opportunities for students to demonstrate all assessment objectives
- provided questions that assessed a representative sample of Unit 3 (Unit 1 of alternative sequence (AS)) topics and reflected the intended learning, e.g. vectors in the plane must extend beyond Unit 2 subject matter and extend to three-dimensional space
- provided students the opportunity to respond to Assessment objective 4: evaluate the reasonableness of solutions and allocate appropriate marks in the marking scheme.

Practices to strengthen

It is recommended that assessment instruments:

- provide complex unfamiliar questions that match the syllabus description for both complexity and unfamiliarity
- avoid the use of standalone questions that only assess the review of assumed knowledge from Unit 1 or Unit 2 subject matter
- provide questions where the focus is on Unit 3 (or AS U1) subject matter only (standalone questions using only subject matter from Units 1 or 2 (or AS U3 or U4) are not suitable).

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	5
Language	44
Layout	6
Transparency	3

*Each priority might contain up to four assessment practices.

Total number of submissions: 334.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- where students had access to technology, it was clearly stipulated when technology could not be used, e.g. an algebraic approach was required
- were reviewed using the 'Print preview' button before submitting, to ensure the layout was appropriate
- provided simple familiar questions where the required procedure was clear from the way the problem was posed.

Practices to strengthen

It is recommended that assessment instruments:

- are free from punctuation, spelling and other errors
- maintain a consistent font throughout
- use the language of the assessment objectives, e.g. evaluate the reasonableness of solutions.

Additional advice

- Where Assessment objective 4: evaluate the reasonableness of solutions is assessed in the examination, ensure the marking scheme provides for marks to be awarded for the demonstration of this objective.
- It is recommended that teachers watch the Maths moments videos, accessible under Resources in the Syllabuses app on the QCAA Portal. These videos provide teacher training for writing examinations, including advice on subject matter, syllabus objectives, degree of difficulty, comparable assessment, amending an endorsed instrument and using previously endorsed instruments.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	99.01%	0%	0.99%	0%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- the marking scheme
  - was free from errors and matched the current year's examination
  - clearly described the expected response for each mark allocation
- the teacher accurately determined the student's achievement percentage without significant rounding, identified this on the ISMG, and awarded the mark. When awarding the mark, the use of the 'greater than' symbol was observed, i.e. 80% was awarded 12 marks out of 15.

Samples of effective practices

The following excerpts have been included as illustrations of teacher annotations of student work. In Excerpt 1, the student was awarded 7 out of the possible 9 marks available. The teacher annotations identify where errors occurred and how the marks were awarded accordingly. In Excerpt 2, the student was awarded 3.5 out of the possible 5 marks available. The annotations clearly indicate the marks awarded and the impact of a student omission.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Excerpt 1

collide when $x_A = x_B$ & $y_A = y_B$ & $z_A = z_B$

$5 = 5$ always (x) $\quad -2 = -2$ always (z)

$(t^2 - 7t + 10) = (t - p)$ (y) ✓

for only 1 value of t (collide once)

$t^2 - 8t + 10 + p = 0$ ✓

$$x = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$$

$a = 1$
$b = -8$
$c = 10 + p$

discriminant $= 0$ because 1 solution ✓

$(-8)^2 + 4(1)(10+p) = 0$ mistake in discriminant (two mistakes actually cancel)

$10 + p = 16$

$p = 6$ ✓

$t^2 - 8t + 16 = 0$ ✓

$(t-4) = 0$

$\therefore t = 4$ ✓

(check)

When $t = 4$

$r_A(4) = 5i + (16 - 28 + 10)\hat{j} - 2\hat{k}$

$= 5\hat{i} - 2\hat{j} - 2\hat{k}$

$r_B(4) = 5\hat{i} + (4-6)\hat{k} - 2\hat{k}$

$= 5\hat{i} - 2\hat{j} - 2\hat{k}$

yes, collide

2 seconds after : $t = 6$

but $6^2 - 7\times6 + 10 = 4$

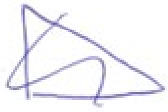
$r_A(6) = 5i + (88)\hat{j} - 2\hat{k}$

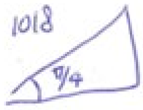
$r_B(6) = 5i + 0\hat{j} - 2\hat{k}$

✓ valid working, error carried.

$\therefore$ the particles are 88 units apart 2 seconds after collision

Excerpt 2





$v_1(t) = 503\cos 0.46\,\underset{\sim}{i} + 503\sin 0.46\,\underset{\sim}{j}$

$= 449.80\,\underset{\sim}{i} + 224.95\,\underset{\sim}{j}$

$s_1(t) = \int 449.9\,\underset{\sim}{i} + 224.95\,\underset{\sim}{j}\,dt$

$= 449.9t\,\underset{\sim}{i} + 224.95t\,\underset{\sim}{j} + c_1$

as start on ground, $c_1 = 0$

$s_1(t) = 449.4t\,\underset{\sim}{i} + 224.95t\,\underset{\sim}{j}$ ✓

$\underset{\sim}{k}$ this would actually be $\underset{\sim}{k}$ given the orientation of the system!

$v_2(t) = 1018\cos \frac{\pi}{4}\,\underset{\sim}{i} + 1018\sin \frac{\pi}{4}\,\underset{\sim}{j}$

$= 719.83\,\underset{\sim}{i} + 719.83\,\underset{\sim}{j}$

$s_2(t) = \int 719.83\,\underset{\sim}{i} + 719.83\,\underset{\sim}{j}\,dt$

$s_2(t) = 719.83t\,\underset{\sim}{i} + 719.83t\,\underset{\sim}{j} + c_2$ ✓

$c_2 = -1.2\,\underset{\sim}{i} - 3.2\,\underset{\sim}{j} + \underset{\sim}{h}$

$\therefore s_2(t) = (719.83t - 1.2)\,\underset{\sim}{i} + (719.83t - 3.2)\,\underset{\sim}{j} + \underset{\sim}{h}$ ✓

when $s_1(t) = s_2(t)$

Equate components:

$\underset{\sim}{i}$: $449.9t = 719.83t - 1.2$

$1.2 = 269.93t$

$t = 0.0004456\text{hr}$ ✓

$\approx 1.63\text{sec}$ 1.6s.

$\underset{\sim}{j}$: $224.95t = 719.83 - 3.2$

$3.2 = 494.88t$

$t = 0.0006466\text{hr}$

$\approx 2.32\text{sec}$

3rd dimension? They could be in different planes here!

$\therefore$ The two planes will be at the same location at $t = 0.0004\text{hr}$ and $t = 0.0006\text{hr}$

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∴ Considering the slower plane travels at 503km/h,

$\Delta t = 0.00002\,\text{hr} \longrightarrow = 0.07\,\text{sec}$

$\downarrow \times 503$

$= 0.01\,\text{km}$

$\approx 100\,\text{m}$

∴ the planes will be at closest, approx. 100m apart, thus the alarms will go off. Nice reasoning!

However, at the time difference between the planes is only 0.07 sec, (and they are 100m apart); The alarms are ~~an~~ inadequate.

They will go off and only seconds later, will the planes crash meaning no time to fix situation. ✓

1 • Plane (with error

1½ • Correct Plane 2

½ • Equating components not sd.

½ • Some mathematical justification of situation.

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- the marking scheme is applied consistently across all students in the cohort and marking decisions are checked during internal quality assurance processes to ensure that equivalent responses are awarded the same number of marks.

Additional advice

- The marking scheme should provide sufficient information for confirmers to see the match of evidence in student samples.
- It is recommended that the total number of marks awarded for each question is clearly identified on the student response.

- Where a student response differs from the marking guide, schools should clearly identify follow-through marks: where, after an error, subsequent procedures are valid with respect to the error. See Excerpts 1 and 2 as examples.
- Before submitting files for confirmation, schools are responsible for ensuring the quality, accuracy and accessibility of the required files (*QCE and QCIA policy and procedures handbook v5.0*, Section 9.7.3). Schools should refer to the information contained in the *Confirmation submission information* for Specialist Mathematics (available under Resources in the Syllabuses app on the QCAA Portal) to check the submission requirements. Schools are advised to check that all scanning of student work has been completed without error. This includes checking that
  - no pages are missing from the response
  - all pages are visible and easy to read
  - the submitted response matches the sampled student.
- If errors are discovered in a marking guide when marking student work, the corrected marking guide needs to be uploaded. This can be done at any time through the Endorsement app or can be uploaded with the confirmation samples.
- If a comparable assessment instrument is administered to a sampled student, then the school must indicate this in Student Management on the individual student’s learning account and in the Confirmation app. To assist with this, comparable assessments should be developed in the Endorsement app to ensure the correct examination and its matching marking scheme are available for the confirmation review (see the *QCE and QCIA policy and procedures handbook v5.0*, Section 7.4). For further information, see the quick step-guide *Upload samples* for confirmation in the Help section of the Confirmation app.

Internal assessment 3 (IA3)

Examination — short response (15%)

This examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 4 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	136
Authentication	0
Authenticity	0
Item construction	10
Scope and scale	20

*Each priority might contain up to four assessment practices.

Total number of submissions: 310.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided cues for students to use calculus techniques where relevant, rather than using physics formulas.

Practices to strengthen

It is recommended that assessment instruments:

- have questions that focus on Unit 4 (or AS U2) subject matter only. Standalone questions that can only be solved using subject matter from Units 1, 2 or 3 (or AS U3, U4 or U1) or Mathematical Methods subject matter are not suitable
- include complex familiar and unfamiliar questions that match the syllabus description for both complexity and unfamiliarity.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	5
Language	40
Layout	10
Transparency	24

*Each priority might contain up to four assessment practices.

Total number of submissions: 310.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- clearly indicated when technology could not be used, e.g. where the question indicated that an analytical procedure was required
- provided opportunities for students to use technology beyond simple computation.

Practices to strengthen

It is recommended that assessment instruments:

- are reviewed using the ‘Print preview’ button before submitting to ensure the layout is appropriate
- use the language of the Assessment objectives, e.g. evaluate the reasonableness of solutions
- are free from punctuation and spelling errors.

Additional advice

- To support marker reliability, provide a correct marking scheme that indicates clearly how marks will be allocated. This also assists schools to check the scope and scale of the assessment, such as time allocation, adequacy of response space and match to the identified degree of difficulty.
- Where Assessment objective 4: evaluate the reasonableness of solutions is assessed in the examination, ensure that corresponding marks are awarded for the demonstration of this objective in the marking scheme.
- It is recommended that teachers watch the Maths moments videos, accessible under Resources in the Syllabuses app on the QCAA Portal. These videos provide teacher training for writing examinations, including advice on subject matter, syllabus objectives, degree of difficulty, comparable assessment, amending an endorsed instrument and using previously endorsed instruments.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	99.67%	0%	0.33%	0%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- the marking scheme
  - was updated and matched the relevant examination
  - explicitly stated how marks were allocated within each question
- the teacher accurately determined the student's achievement percentage without significant rounding, identified this on the ISMG and awarded the mark, e.g. 46 marks out of a possible 63 marks is 73.02% and corresponds to an ISMG mark of 12 marks out of 15.

Samples of effective practices

The following excerpts have been included to demonstrate possible methods for recording the total number of marks awarded, the percentage calculated and the final ISMG mark awarded.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Excerpt 1

Criterion	Maximum possible marks	Marks allocated	Provisional marks
Foundational knowledge and problem-solving		15	15
Overall	**60.0**	**15**	15

T.F. T.A.

$33 + 24.5 = \frac{57.5}{60} = 95.8\% \rightarrow 15$

Excerpt 2

Criterion	Maximum possible marks	Marks allocated	Provisional marks
Foundational knowledge and problem-solving		15	
Overall	37 / **39.0**	94.87% **15**	15

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- the marking scheme is applied consistently across all students in the cohort and marking decisions are checked during internal quality assurance processes to ensure that equivalent responses are awarded the same number of marks.

Additional advice

- The marking scheme should provide sufficient information for confirmers to see the match of evidence in student samples.
- It is recommended that the total number of marks awarded for each question is clearly identified on the student response.
- Where a student response differs from the marking guide, schools clearly identify follow-through marks: where, after an error, subsequent procedures are valid with respect to the error.
- Before submitting files for confirmation, schools are responsible for ensuring the quality, accuracy and accessibility of the required files (*QCE and QCIA policy and procedures handbook v5.0*, Section 9.7.3). Schools should refer to the information contained in the *Confirmation submission information* for Specialist Mathematics (available under Resources in the Syllabuses app on the QCAA Portal) to check the submission requirements. Schools are advised to check that all scanning of student work has been completed without error. This includes checking that
  - no pages are missing from the response
  - all pages are visible and easy to read
  - the submitted response matches the sampled student.
- If errors are discovered in a marking guide when marking student work, the corrected marking guide needs to be uploaded. This can be done at any time through the Endorsement app.
- If a comparable assessment is administered to a sampled student, the school must indicate this in Student Management on the individual student's learning account and in the Confirmation app. To assist with this, comparable assessments should be developed in the Endorsement app, to ensure the correct examination and its matching marking scheme are available for the confirmation review (see the *QCE and QCIA policy and procedures handbook v5.0*, Section 7.4). For further information see the quick-step guide *Upload samples* for confirmation in the Help section of the Confirmation app.

External assessment

External assessment (EA) is developed and marked by the QCAA. The external assessment for a subject is common to all schools and administered under the same conditions, at the same time, on the same day.

Examination — short response (50%)

Assessment design

The assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the syllabus.
The examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (50 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (50 marks).

The examination assessed subject matter from Units 3 and 4. Questions were derived from the context of Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus.

The assessment required students to respond to multiple choice and short response questions.

The AS assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the AS.
The AS examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (50 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (50 marks).

The AS examination assessed subject matter from AS units 1 and 2. Questions were derived from the context of Vectors and matrices, Real and complex numbers, Trigonometry, Statistics and Calculus.

The AS assessment required students to respond to multiple choice and short response questions.

Assessment decisions

Assessment decisions are made by markers by matching student responses to the external assessment marking guide (EAMG). The external assessment papers and the EAMG are published in the year after they are administered.

Multiple choice question responses

There were 10 multiple choice questions in Paper 1 (General).

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Specialist Mathematics (General): Paper 1

Question	A	B	C	D
1	4.42	8.78	**76.01**	10.29
2	15.79	1.72	**81.71**	0.73
3	**69.76**	16.4	8	5.06
4	9.33	1.02	6.6	**82.84**
5	**74.12**	6.83	16.37	2.38
6	6.92	**71.53**	6.02	15.3
7	33.53	17.65	32.25	**15.5**
8	**42.34**	6.37	33.35	17.53
9	17.51	33.73	19.48	**28.41**
10	33.53	**37.63**	18.67	9.83

There were 10 multiple choice questions in Paper 2 (General).

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Specialist Mathematics (General): Paper 2

Question	A	B	C	D
1	12.03	18.69	12.18	**56.23**
2	1.83	**83.52**	12.64	1.45
3	6.19	10.78	**60.74**	21.21
4	10.72	8.17	13.19	**67.22**
5	**65.79**	6.13	22.38	4.94
6	10.78	12.03	20.17	**56.15**

Question	A	B	C	D
7	**88.14**	6.51	3.69	1.13
8	**64.95**	6.19	3.84	24.44
9	35.37	**53.44**	6.92	3.52
10	11.19	12.9	**57.8**	17.2

There were 10 multiple choice questions in Paper 1 (AS).

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Specialist Mathematics (AS): Paper 1

Question	A	B	C	D
1	5.96	12.58	**64.24**	16.56
2	25.83	1.99	**70.2**	1.99
3	0.66	1.32	2.65	**95.36**
4	12.58	**63.58**	12.58	10.6
5	14.57	**58.94**	5.3	20.53
6	19.21	11.26	**54.3**	13.91
7	3.31	18.54	3.31	**74.83**
8	**29.14**	9.27	39.74	19.87
9	34.44	28.48	**23.84**	11.26
10	**86.09**	4.64	5.3	3.97

There were 10 multiple choice questions in Paper 2 (AS).

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Specialist Mathematics (AS): Paper 2

Question	A	B	C	D
1	9.27	**29.8**	11.92	48.34
2	6.62	5.96	**65.56**	21.85
3	23.84	**56.29**	11.92	6.62
4	15.89	13.25	18.54	**50.33**
5	**47.02**	13.91	28.48	9.27
6	1.99	7.28	6.62	**84.11**
7	0	7.95	**88.74**	3.31
8	**76.16**	9.27	6.62	7.95
9	33.77	**52.32**	8.61	4.64
10	6.62	**23.18**	50.99	18.54

Effective practices

Overall, students responded well to:

- the opportunity to demonstrate knowledge and understanding of matrix algebra within the solution of a matrix equation, although care is needed in recalling the assumed knowledge of calculating the inverse and matrix multiplication of matrices of dimension 2, without technology
- the opportunity to demonstrate knowledge and understanding of Simpson's rule and the justification of the reasonableness of solutions related to its application
- the opportunity to demonstrate the use of technology in determining the coordinates of points of intersection of two functions and the area between two functions
- simple familiar questions on both papers where students could demonstrate their understanding of practised procedures.

Samples of effective practices

Short response

The following excerpt is from Question 15 of Paper 1. It required students to use mathematical induction to prove a given result for the sum of a geometric progression.

Effective student responses:

- understood the nature of inductive proof using the initial statement and inductive step
- constructed supporting arguments in the form of a proof.

This excerpt has been included:

- to show an example of simplifying the left-hand side of the proof to reach the right-hand side of the proof
- to demonstrate a suitable proof of the initial statement and inductive step
- to demonstrate an appropriate conclusion statement.

a) Initial statement: *[1 mark]*

Prove for the base case, $n=1$:

$RHS = \frac{r^1 - 1}{r-1} = 1 \quad LHS = r^{1-1} = r^0 = 1$

$\therefore LHS = RHS$ as required

Assuming the rule is true for $n = k$,

$$1+r+r^2+r^3+...+r^{k-1} = \frac{r^k - 1}{r-1} \ (r \neq 1).$$

b) Inductive step: *[3 marks]*

Prove for $n=k+1$

$RHS = \frac{r^{k+1}-1}{r-1}$

$LHS = 1+r+r^2+r^3+...+r^{k-1} + r^{(k+1)-1}$

$= \frac{r^k - 1}{r-1} + r^k$

$= \frac{r^k-1}{\cancel{r+1}(r-1)} + \frac{r^k(r-1)}{r-1}$

$= \frac{r^k - 1 + r^{k+1} - r^k}{r-1}$

$= \frac{r^{k+1}-1}{r-1}$

$= RHS$

as required

c) Conclusion: *[1 mark]*

$\therefore$ The statement was proven true for $n=1$, assumed true for $n=k$ and proven true for $n=k+1$. $\therefore$ By mathematical induction $1+r+r^2+r^3+...+r^{n-1} = \frac{r^n - 1}{r-1} \ \forall n \in \mathbb{Z}^+$

The following excerpt is from Question 18 of Paper 1. This complex unfamiliar question required students to use a variety of integration techniques to determine a particular solution to a differential equation of the form $\frac{dy}{dx} = f(x)g(y)$.

Effective student responses:

- recognised the need to use separation of variables
- used a combination of integration techniques to determine the general solution of the differential equation
- recalled assumed knowledge related to reciprocal trigonometric functions and identities
- considered the given domain and range to evaluate the reasonableness of the solution.

This excerpt has been included:

- to demonstrate the use of suitable integration techniques
- to demonstrate the use of given information to determine the constant of integration
- to demonstrate logical setting in the sequential development of the response
- to provide evidence of the evaluation of the reasonableness in determining the particular solution to the differential equation.

$\tan(y)\,dy = \dfrac{x}{x^2+1}\,dx$

First $\int \tan(y)\,dy$:

$= \int \dfrac{\sin y}{\cos y}\,dy$

let $\cos y = u$

$\dfrac{du}{dy} = -\sin y \Rightarrow dy = \dfrac{du}{-\sin y}$

$\int \dfrac{x}{x^2+1}\,dx$

let $x^2+1 = u$

$\dfrac{du}{dx} = 2x \qquad dx = \dfrac{du}{2x}$

$\dfrac{1}{2}\int \dfrac{1}{u}\,du = \dfrac{1}{2}\ln|u| + c_1$

$= \dfrac{1}{2}\ln|x^2+1| + c_1$

$= -\int \dfrac{1}{u}\,du = -\ln|u| + c$, Sub $u = \cos y$ $\cancel{\int \tan y\,dy = \ln|\cos y|}$

$\int \tan y\,dy = -\ln|\cos y| + c$

$\therefore -\ln|\cos y| = \dfrac{1}{2}\ln|x^2+1| + c$

Sub $(0,0)$:

$-\ln|\cos 0| = \dfrac{1}{2}\ln|0+1| + c$

$-\ln(1) = \dfrac{1}{2}\ln(1) + c \quad \therefore c = 0$

$\rightarrow -\ln|\cos y| = \dfrac{1}{2}\ln|x^2+1|$

$-2\ln|\cos y| = \ln|x^2+1|$

$\ln|x^2+1| = \ln|\sec^2 y|$

$\therefore x^2+1 = \sec^2 y$

$x^2 = \sec^2 y - 1$

$x^2 = \tan^2 y$

$\rightarrow x = \pm\tan y$

$\boxed{x = -\tan y}$ Answer

~~as when~~ $y \in (-\frac{\pi}{2}, 0)$, $\tan y \in \mathbb{R}$, since $\cancel{x \geq 0}$, ~~add a~~

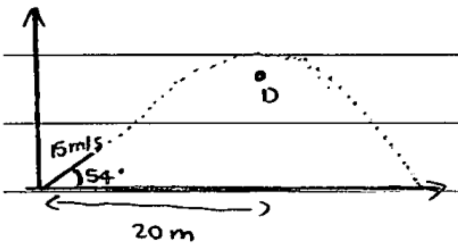
The following excerpt is from Question 17 of Paper 2. This complex familiar question required students to use a vector calculus approach to model the projectile motion of an object in a plane and evaluate the reasonableness of comments from an observer in relation to the object passing a stationary drone.

Effective student responses:

- made suitable assumptions regarding the position of the origin and the direction of the unit vectors used to model the motion of the object
- used a vector calculus approach as required in the problem.

This excerpt has been included to demonstrate:

- the use of a suitable vector calculus approach to model the velocity and displacement vectors of the object
- the usefulness of a diagram to assist in the clarification of the information provided
- well-constructed arguments within the evaluation of the reasonableness of both comments of the observer
- the logical organisation and the communication of key steps, including the use of appropriate vector notation and clear flow of the solution.



$\underset{\sim}{a} = -g\underset{\sim}{j}$ assume $g = 9.8$

$\underset{\sim}{v} = -gt\underset{\sim}{j} + \underset{\sim}{v}_0$

$\underset{\sim}{v} = 15\cos(54°)\underset{\sim}{i} + 15\sin(54°)\underset{\sim}{i} - gt\underset{\sim}{j}$

$= 8.82\underset{\sim}{i} + (12.14 - 9.8t)\underset{\sim}{j}$

$\underset{\sim}{r}(t) = 8.82t\underset{\sim}{i} + (12.14t - 4.9t^2)\underset{\sim}{j} + \underset{\sim}{c}$

$\underset{\sim}{c} = (0,0)$

$\therefore \underset{\sim}{r}(t) = 8.82t\underset{\sim}{i} + (12.14t - 4.9t^2)\underset{\sim}{j}$

when $x = 20$	when $t = 2.27$
$20 = 8.82t$	$y = 12.14 \times 2.27 - 4.9(2.27)^2$
$t \approx$ 2.27 seconds	$y \approx 2.31$ m

$\underset{\sim}{v}$ when $t = 2.27$ seconds

$\underset{\sim}{v}(2.27) = 8.82\underset{\sim}{i} + (12.14 - 9.8 \times 2.27)\underset{\sim}{j}$

$\approx 8.82\underset{\sim}{i} + {-10.106}\underset{\sim}{j}$

the observer's comment that the object took between 2-2.5 seconds to reach the drone is reasonable as it to the object 2.27 seconds to reach the drone, which is in the interval they provided. The comment that the drone was still moving in an upwards direction after passing the drone, however, is unreasonable as the $\hat{j}$ component of the velocity vector at that time was negative, signifying that the object was and would continue to travel in a downwards direction.

The following excerpt is from Question 19 of Paper 2. This complex unfamiliar question required students to compare recalculated approximate confidence intervals for the population mean based on an unknown level of confidence from two different samples of the same size.

Effective student responses:

- understood that an approximate confidence interval for the population mean, based on a particular sample, contained information related to the sample mean and the sample standard deviation of that particular sample
- modelled clear communication of statistical information
- effectively compared relevant aspects of the various confidence intervals.

This excerpt has been included:

- to demonstrate an effective use of the graphics calculator, including the use of relevant statistical functions
- to demonstrate the effective equating of results related to the matching of the upper and lower limits of the two confidence intervals in the second situation
- to demonstrate understanding of the given instructions, illustrated by the provision of an answer rounded correctly to one decimal place
- to model clear communication of statistical information.

$z = 1.96$

$(163.7, 166.9)$, $\bar{x} = 165.3\text{ cm}$

$1.6 = \dfrac{1.96 \times \sigma}{\sqrt{n}}$ $\quad \varepsilon = 1.6\text{cm}$ $\quad \rightarrow \dfrac{\sigma}{\sqrt{n}} = \dfrac{1.6}{1.96}$

$(167.8, 172.4)$

$2.3 = \dfrac{1.96\ \sigma_2}{\sqrt{n}} \rightarrow \dfrac{\sigma_2}{\sqrt{n}} = \dfrac{2.3}{1.96}$

Now $165.3 + \dfrac{z\sigma}{\sqrt{n}} = 170.1 - \dfrac{z\sigma_2}{\sqrt{n}}$

$\dfrac{z\sigma}{\sqrt{n}} + \dfrac{z\sigma_2}{\sqrt{n}} = 4.8$

$z\left(\dfrac{\sigma}{\sqrt{n}} + \dfrac{\sigma_2}{\sqrt{n}}\right) = 4.8$

$z\left(\dfrac{1.6}{1.96} + \dfrac{2.3}{1.96}\right) = 4.8$

$1.9897\ldots z = 4.8$

$z \approx 2.412$ (3dp)

Use NCD, $\mu = 0$, $\sigma = 1$

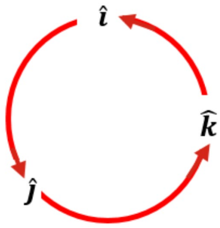
$Pr(-2.412 < X < 2.412) \approx 0.9841\ldots$

$\therefore$ The teacher used a 98.4 % confidence interval

Practices to strengthen

When preparing students for external assessment, it is recommended that teachers consider:

- providing opportunities for students to practise Objective 4: evaluate reasonableness of solutions, e.g.
  - comparing results against given information
  - using given domains to recognise all possible solutions to trigonometric equations
  - recognising that when using a graphics calculator to numerically solve equations and, in particular, quadratic equations, more than the one solution provided may be possible
- providing opportunities for students to apply the definition of the vector (cross) product
  - to unit vectors $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$ and $\hat{\boldsymbol{k}}$ such as $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}} = 0,\ \hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}},\ \hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}} = -\hat{\boldsymbol{k}}$ and other similar results as represented by the cyclic illustration as shown



  - in problems involving area of shapes including triangles and parallelograms
    - area of triangle = $\frac{1}{2}|\boldsymbol{a} \times \boldsymbol{b}|$
    - area of parallelogram = $|\boldsymbol{a} \times \boldsymbol{b}|$
- providing opportunities for students to recognise integration techniques involving the substitution of $u = g(x)$ combined with the use of the formula $\int \frac{1}{x}\,dx = \ln|x| + c,$ for $x \neq 0$, that is, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c,\ f(x) \neq 0,$ e.g. $\int \tan(x)\,dx = -\ln(\cos(x)) + c$
- providing opportunities for students to determine a volume of solid of revolution formed by rotating a bounded region between two functions using an appropriate algorithm such as $\pi\int_a^b \left({y_1}^2 - {y_2}^2 \right) dx$ rather than the flawed $\pi\int_a^b (y_1 - y_2)^2\,dx$.

Additional advice

- Teachers should ensure students are competent in performing simple algebraic procedures, e.g. expanding expressions of the form $(a+b)^2$ and simplifying algebraic fractions.
- Teachers should encourage students to become more familiar with the full capabilities of graphics calculator facilities, including determining the solution of a range of equations using an approximate root (in both calculate and graph mode) and using the range of statistical operations available, including those related to probability and confidence interval calculations associated with the sample mean.
- Teachers should provide opportunities for students to respond to questions using alternative methods of solution.
- Teachers should encourage students to use part of their perusal time to identify the total number of questions in the examination and where the 'END OF PAPER' message is located.