LUI | | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your barcode ID label here

Book ☐ of ☐ books used

External assessment 2023

Question and response book

Specialist Mathematics

Paper 2 — Technology-active

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator **permitted**.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

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Section 1

Instructions

- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- Choose the best answer for Questions 1–10.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Ensure you have filled an answer bubble for each question.

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Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
  - If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 50 marks.

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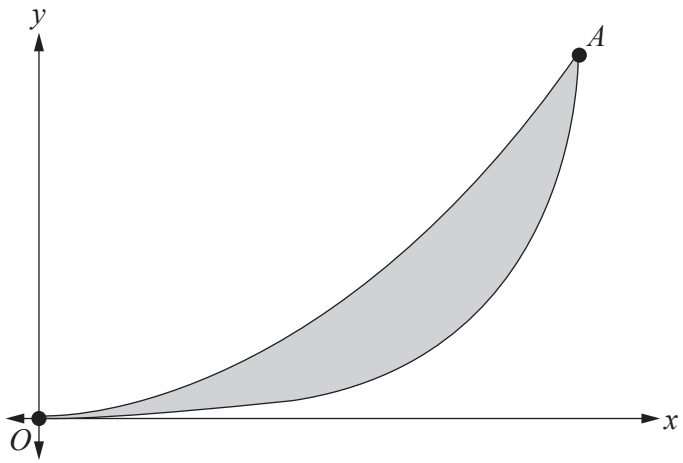
QUESTION 11 (4 marks)

The bounded region between the graphs of the functions $y = -1 + \sec\left(\frac{x}{5}\right)$ and $y = 0.1x^2$ over a certain domain is shaded as shown. The two functions intersect at the origin and point A.



a) Determine the coordinates of point A. *[1 mark]*

b) Calculate the area of the shaded region. *[1 mark]*

The shaded region is rotated about the x-axis to form a solid of revolution.

c) Determine the volume of the solid formed. *[2 marks]*

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QUESTION 12 (7 marks)

Consider the complex number $z = -3 + 2i$.

a) Determine z^3 using the binomial theorem. Leave your answer in the form $a+bi$, where $a, b \in R$. *[2 marks]*

b) Convert z into the form of $r\,\text{cis}(\theta)$, where $-\pi < \theta \leq \pi$. *[1 mark]*

c) Use the result from Question 12b) to determine z^3 using De Moivre's theorem. Leave your answer in the form of $r\,\text{cis}(\theta)$, where $-\pi < \theta \leq \pi$. *[2 marks]*

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d) Evaluate the reasonableness of your results from Questions 12a) and 12c), noting that the two methods to determine z^3 should produce the same result. *[2 marks]*

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QUESTION 13 (4 marks)

The wait time for customers put on hold when calling complaint departments is assumed to be normally distributed. A company claims that the mean wait time for their customers is 7.6 minutes.

The following data represents the wait time (minutes) from a random sample of 12 customers who called the complaint department of this company.

8.3	12.7	9.1	7.3	10.3	5.4	8.5	10.7	6.9	12.5	7.2	11.9

a) Determine the mean of this data. *[1 mark]*

The standard deviation of this data is calculated to be 2.384 minutes.

b) Use an approximate 95% confidence interval for the mean to evaluate the reasonableness of the company's claim. Justify your decision using mathematical reasoning. *[3 marks]*

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QUESTION 14 (4 marks)

At a certain location, a biologist measures the width of a river to be 12 m. She also records the depth of the river at regular 2 m interval widths as shown.

Width (m)	0	2	4	6	8	10	12
Depth (m)	0.52	2.15	3.70	4.27	3.32	1.28	0.59

The biologist estimates the cross-sectional area of the river at this location to be 15 m^2.

Use Simpson's rule to evaluate the reasonableness of this estimation. Justify your area calculation and decision regarding reasonableness using mathematical reasoning.

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QUESTION 15 (7 marks)

The travel time for students attending a certain university is assumed to be normally distributed, with a population mean of 25.2 minutes and standard deviation of 4.7 minutes.

Travel times are collected from a random sample of 120 of these students and used to calculate a sample mean, $\overline{X}_1$, in minutes.

a) Determine $P\left(\overline{X}_1 \leq 25\right)$. *[2 marks]*

b) Given $P\left(\overline{X}_1 > k\right) = 0.9$, determine the value of k. *[1 mark]*

Travel times are collected from a second random sample of the university's students and used to calculate a second sample mean, $\overline{X}_2$, in minutes.

c) Given $P\left(\overline{X}_2 \leq 25\right) \approx 0.4$, determine the number of students in the second sample. *[4 marks]*

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QUESTION 16 (6 marks)

A curve modelled by the relation $xy^2 - y + \cos^{-1}(2x) = 1$, where $-0.35 \leq x \leq 0.27$ and $0 \leq y \leq 1$, intersects the y-axis at point A.

Determine the equation of the tangent to the curve at point A.

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QUESTION 17 (6 marks)

An object is projected upwards from ground level with an initial velocity of 15 m s^{-1} at an angle of 54° to the horizontal.

The object just passes over a drone hovering in the air. An observer is positioned directly below the drone and at a horizontal distance of 20 m from where the object is projected.

The observer commented that:

- it took the object around 2 to 2.5 seconds after its projection to reach the drone
- the object was still moving in an upwards direction as it passed the drone.

Assuming that air resistance is negligible, use a vector calculus approach to evaluate the reasonableness of the observer's comments.

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QUESTION 18 (5 marks)

Consider the complex solutions to the following equation, where $0 < \arg(z) < \pi$.

$$(z+1)\left(z^{14} - z^{13} + z^{12} - z^{11} + \ldots + z^4 - z^3 + z^2 - z\right) = 1 - z$$

Let w_1 be the solution with the maximum possible real part and w_2 be the solution with the maximum possible imaginary part.

Show that $\frac{{w_1}^4}{w_2} \in Z$.

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QUESTION 19 (7 marks)

The height of Year 9 students at a school is assumed to be normally distributed with a population mean height of μ cm.

A teacher at the school measured the height of all the students in her Year 9 class. This data was used to calculate an approximate 95% confidence interval for μ of (163.7, 166.9) cm.

The teacher repeated the procedure using data from another Year 9 class. Although this class had the same number of students, its data produced an approximate 95% confidence interval for μ of (167.8, 172.4) cm.

Using the same data, the teacher recalculated the approximate confidence intervals for μ for each class using a confidence level of x%. She observed that the upper bound of the confidence interval from her Year 9 class now equalled the lower bound of the confidence interval from the other Year 9 class.

Determine the value of x. Give your answer rounded to one decimal place.

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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