LUI | | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your
barcode ID label here

Book of books used

External assessment 2023

Question and response book

Specialist Mathematics

Paper 1 — Technology-free

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are **not** permitted.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

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Section 1

Instructions

- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- Choose the best answer for Questions 1–10.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Ensure you have filled an answer bubble for each question.

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Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 50 marks.

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QUESTION 11 (5 marks)

Determine the following definite integrals.

a) $\int_0^1 \frac{1}{1+x^2}\,dx$ *[2 marks]*

b) $\int_0^{\frac{\pi}{4}} 2\sin^2(x)\,dx$ *[3 marks]*

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QUESTION 12 (5 marks)

Given $\boldsymbol{A}=\begin{pmatrix}1 & -2\\ 1 & 2\end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix}0 & 2\\ 1 & 3\end{pmatrix}$ and $\boldsymbol{C}=\begin{pmatrix}-1 & -1\\ 0 & 3\end{pmatrix}$, determine $\boldsymbol{X}$ in the matrix equation $\boldsymbol{XA}-\boldsymbol{XC}=\boldsymbol{B}$.

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QUESTION 13 (5 marks)

Given $z \in C$, where $z \neq 0$, prove $\dfrac{|z|}{z\overline{z}} = \left| z^{-1} \right|$.

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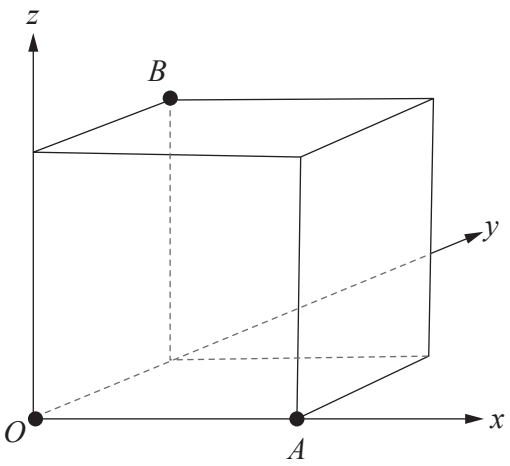
QUESTION 14 (6 marks)

Consider a cube with three edges positioned along the x-, y- and z-axes on the Cartesian plane as shown. Points O, A and B are vertices of the cube.

Not to scale



a) Given $\overrightarrow{OA} = 2\hat{\boldsymbol{i}}$, determine $\overrightarrow{OB}$. Express your answer in terms of $\hat{\boldsymbol{j}}$ and $\hat{\boldsymbol{k}}$. *[1 mark]*

b) Calculate $\overrightarrow{OA} \times \overrightarrow{OB}$. *[1 mark]*

Consider the triangle formed by joining points O, A and B.

c) Use the result from Question 14b) to determine the area of the triangle. *[2 marks]*

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Let points M and N be the midpoints of the triangle's sides OA and OB respectively.

d) Determine $\overrightarrow{MN}$. *[1 mark]*

e) Use the result from Question 14d) to show that the length of AB is twice the length of MN. *[1 mark]*

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QUESTION 15 (5 marks)

The sum of a geometric progression with n terms, where the first term is 1 and the common ratio is r, is given by

$$1 + r + r^2 + r^3 + \ldots + r^{n-1} = \frac{r^n - 1}{r - 1} \quad (\text{for } r \neq 1).$$

Prove that this rule is true $\forall n \in Z^+$ using mathematical induction by completing the steps of the proof as indicated.

a) Initial statement: *[1 mark]*

Assuming the rule is true for $n = k$,

$$1 + r + r^2 + r^3 + \ldots + r^{k-1} = \frac{r^k - 1}{r - 1} \quad (r \neq 1).$$

b) Inductive step: *[3 marks]*

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c) Conclusion: *[1 mark]*

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QUESTION 16 (5 marks)

A curve is defined by the parametric equations $x = 2\tan(\theta)$ and $y = 3\sin(2\theta)$, where $0 \leq \theta < \frac{\pi}{2}$.

Given that $\frac{dy}{dx}$ can be expressed in the form $a\cos^4(\theta) + b\cos^2(\theta)$, where $a, b \in R$, determine the values of a and b.

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QUESTION 17 (7 marks)

An object of mass 2 kg is moving with a constant velocity (m s^{-1}) of $\boldsymbol{v} = 3\hat{\boldsymbol{i}} + \hat{\boldsymbol{k}}$.

At an instant, two forces (N), $\boldsymbol{F}_1 = 5t\hat{\boldsymbol{j}} - 3\hat{\boldsymbol{k}}$ and $\boldsymbol{F}_2 = -t\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$, act simultaneously on the object for t seconds, where $0 \le t \le 2$.

Determine the magnitude of the momentum of the object when $t = 1$.

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QUESTION 18 (6 marks)

A particular solution to the differential equation $\frac{dy}{dx} = \frac{x}{\left(x^2+1\right)\tan\left(y\right)}$, where $x \geq 0$ and $-\frac{\pi}{2} < y \leq 0$, passes through the origin.

Determine this solution in the form $x = f(y)$. Leave your answer in simplified form.

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QUESTION 19 (6 marks)

Object A is released from the origin with constant velocity, $\boldsymbol{v}_A$, such that its position after t seconds is given by

$$\boldsymbol{r}_A = 2\sqrt{3}\,t\,\hat{\boldsymbol{i}} + 3t\hat{\boldsymbol{j}} + 2t\,\hat{\boldsymbol{k}},\ t \geq 0.$$

At a later time, object B is released from point $P\left(3\sqrt{3},\ 6,\ 0\right)$ and travels towards point $Q\left(5\sqrt{3},\ 8,\ 4\right)$ with constant velocity, $\boldsymbol{v}_B$, such that $\left|\boldsymbol{v}_B\right| = \sqrt{2}\left|\boldsymbol{v}_A\right|$.

Given that objects A and B collide, determine the time between the release of the two objects.
Assume all positions are given in metres and all velocities are given in metres per second.

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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