External assessment 2023

Multiple choice question book

Specialist Mathematics

Paper 1 — Technology-free

General instruction

- Work in this book will not be marked.

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

Section 1

Instruction

- Respond to these questions in the question and response book.

QUESTION 1

The position of a particle is given by $\boldsymbol{r} = (t+2)\hat{\boldsymbol{i}} + t^2\hat{\boldsymbol{j}}$ for $t \geq 0$.

Determine the corresponding Cartesian equation.

(A) $y = x^2 - 4$

(B) $y = x^2 + 4$

(C) $y = x^2 - 4x + 4$

(D) $y = x^2 + 4x + 4$

QUESTION 2

Consider the proof of the following proposition using mathematical induction.

$$\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2) \ \forall n \in Z^+$$

An appropriate assumption statement within the proof is

(A) $\sum_{r=1}^{k} k(k+1) = \frac{1}{3}k(k+1)(k+2)$

(B) $\sum_{r=1}^{k} k(k+1) = \frac{1}{3}n(n+1)(n+2)$

(C) $\sum_{r=1}^{k} r(r+1) = \frac{1}{3}k(k+1)(k+2)$

(D) $\sum_{r=1}^{k} r(r+1) = \frac{1}{3}n(n+1)(n+2)$

QUESTION 3

One solution of $z^3 - z^2 - 7z - 2 = 0$ is $z = -2$.

Which equation could be used to determine the remaining solutions?

(A) $z^2 - 3z - 1 = 0$

(B) $z^2 - 3z + 1 = 0$

(C) $z^2 - z - 1 = 0$

(D) $z^2 - z + 1 = 0$

QUESTION 4

The age-specific population distribution of a particular species of animal is shown.

Age (years)	**0–1**	**1–2**	**2–3**	**3–4**
Female population	94	82	37	6
Breeding rate	0	1.3	0.9	0.2
Survival rate	0.6	0.8	0.4	0

The Leslie matrix based on this data is

(A) $\begin{bmatrix} 94 & 82 & 37 & 6 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1.3 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0.6 & 0.8 & 0.4 & 0 \\ 1.3 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1.3 & 0.9 & 0.2 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \end{bmatrix}$

QUESTION 5

A confidence interval for a parameter is a range of values within which the

(A) sample estimate of the parameter always lies.

(B) sample estimate of the parameter never lies.

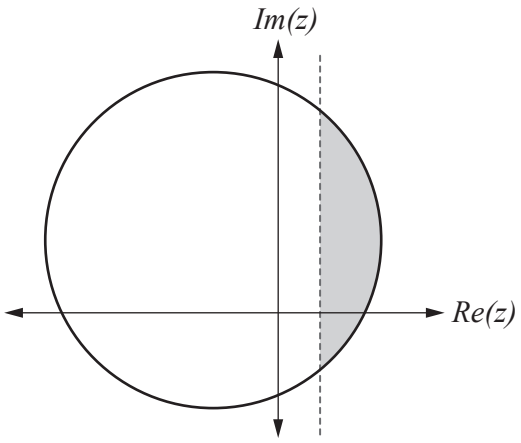
(C) parameter always lies.

(D) parameter never lies.

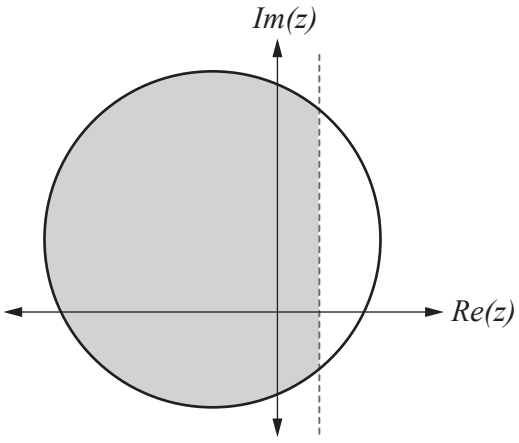
QUESTION 6

The shaded region defined as $\{z : |z+2-i| \leq 5\} \cap \{z : Re(z) < 1\}, z \in C$ is best represented by

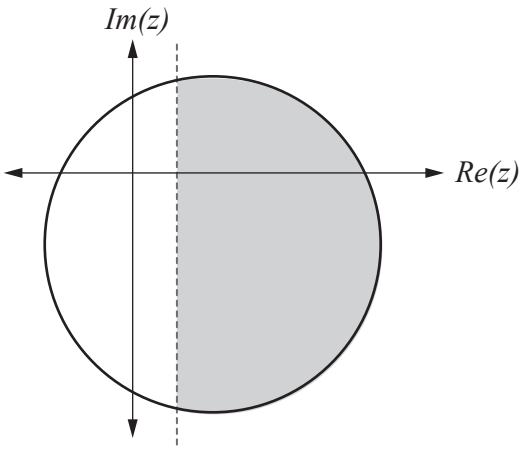
(A)



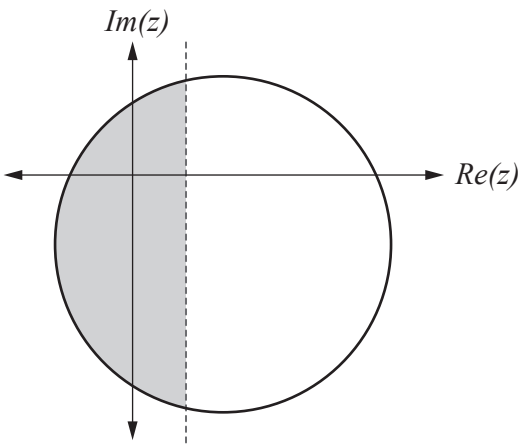
(B)



(C)



(D)



QUESTION 7

The differential equation for which the solution is a logistic equation of the form $y = \dfrac{a}{b + Ce^{-at}}$ where a, b and C are constants is

(A) $\dfrac{dy}{dt} = 0.25(1 - 0.01t)$

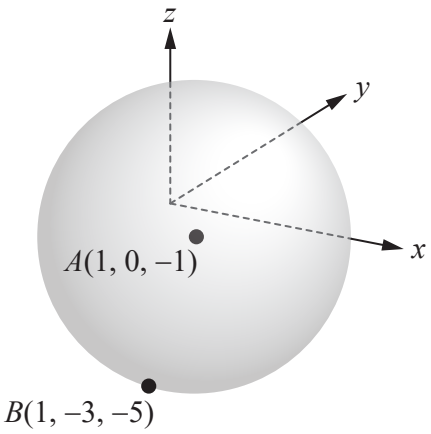
(B) $\dfrac{dy}{dt} = 0.25(1 - 0.01y)$

(C) $\dfrac{dy}{dt} = 0.25t(1 - 0.01t)$

(D) $\dfrac{dy}{dt} = 0.25y(1 - 0.01y)$

QUESTION 8

Point A is the centre of a sphere and point B lies on its surface as shown.



The equation of the sphere is

(A) $x^2 - 2x + y^2 + z^2 + 2z = 23$

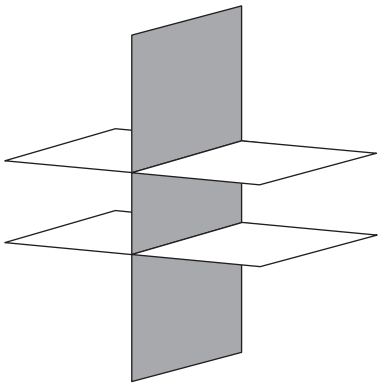
(B) $x^2 + 2x + y^2 + z^2 - 2z = 23$

(C) $x^2 - 2x + y^2 + z^2 + 2z = 25$

(D) $x^2 + 2x + y^2 + z^2 - 2z = 25$

QUESTION 9

The geometric interpretation of a certain system of three equations with no solution is shown.



Given two of the equations are $x + y - z = 0.5$ and $x - y - z = 0.5$, the third equation could be

(A) $2x - 2y - 2z = 1$

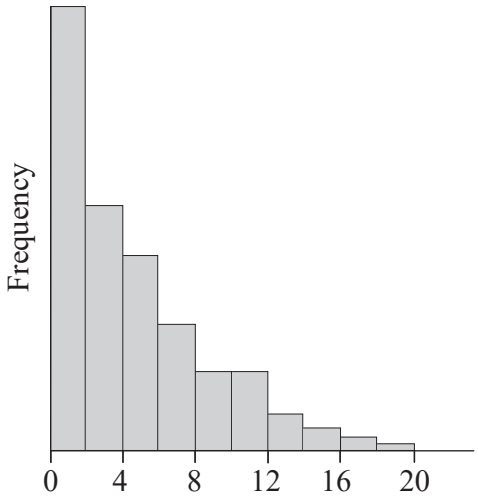
(B) $2x + 2y - 2z = 1$

(C) $2x - 2y + 2z = 3$

(D) $2x + 2y - 2z = 3$

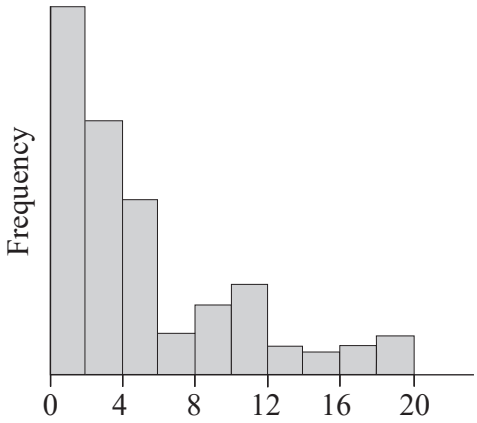
QUESTION 10

A random variable is drawn from a population with the distribution shown in the histogram.

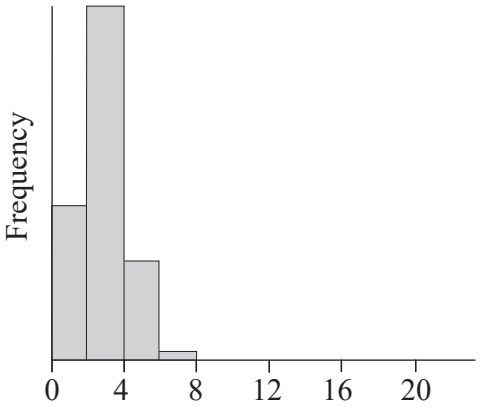


A number of samples of size 10 were randomly selected from this distribution and the sample means, $\overline{x}$, were recorded. The histogram that most likely represents the distribution of the sample means is

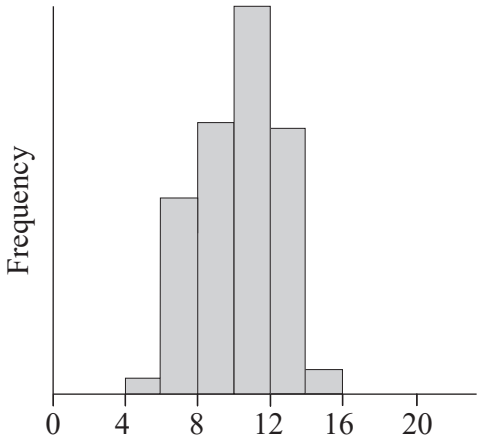
(A)



(B)



(C)



(D)