

# Specialist Mathematics subject report

2022 cohort

February 2023



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# Introduction

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Throughout 2022, schools and the QCAA worked together to further consolidate the new Queensland Certificate of Education (QCE) system. The familiar challenges of flood disruption and pandemic restrictions were managed, and the system continued to mature regardless.

We have now accumulated three years of assessment information, and our growing experience of the new system is helping us to deliver more authentic learning experiences for students. An independent evaluation will commence in 2023 so that we can better understand how well the system is achieving its goals and, as required, make strategic improvements. The subject reports are a good example of what is available for the evaluators to use in their research.

This report analyses the summative assessment cycle for the past year — from endorsing internal assessment instruments to confirming internal assessment marks, and marking external assessment. It also gives readers information about:

- how schools have applied syllabus objectives in the design and marking of internal assessments
- how syllabus objectives have been applied in the marking of external assessments
- patterns of student achievement.

The report promotes continuous improvement by:

- identifying effective practices in the design and marking of valid, accessible and reliable assessments
- recommending where and how to enhance the design and marking of valid, accessible and reliable assessment instruments
- providing examples, including those that demonstrate best practice.

Schools are encouraged to reflect on the effective practices identified for each assessment, consider the recommendations to strengthen assessment design and explore the authentic student work samples provided.

## Audience and use

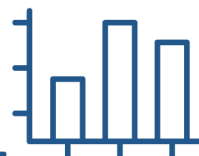
This report should be read by school leaders, subject leaders and teachers to:

- inform teaching and learning and assessment preparation
- assist in assessment design practice
- assist in making assessment decisions
- help prepare students for external assessment.

The report is publicly available to promote transparency and accountability. Students, parents, community members and other education stakeholders can use it to learn about the assessment practices and outcomes for General subjects (including alternative sequences (AS) and Senior External Examination (SEE) subjects, where relevant) and General (Extension) subjects.

## Report preparation

The report includes analyses of data and other information from endorsement, confirmation and external assessment processes. It also includes advice from the chief confirmer, chief endorser and chief marker, developed in consultation with and support from QCAA subject matter experts.



## Subject completion

The following data includes students who completed the General subject or AS.

**Note:** All data is correct as at 31 January 2023. Where percentages are provided, these are rounded to two decimal places and, therefore, may not add up to 100%.

Number of schools that offered the subject: 304.

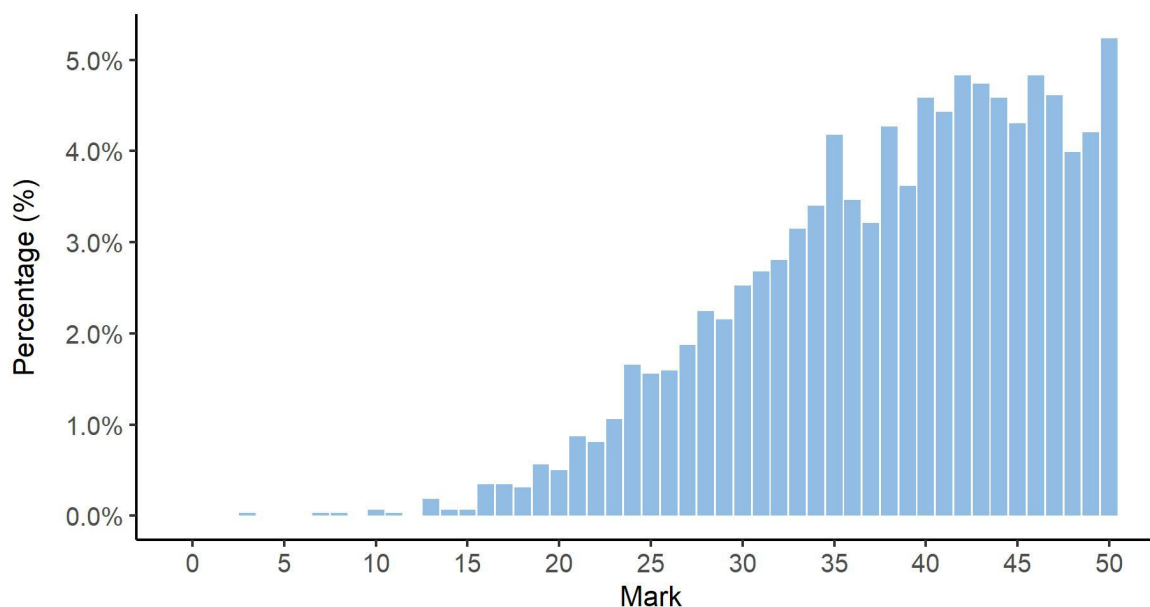
Completion of units	Unit 1	Unit 2	Units 3 and 4
Number of students completed	3736	3519	3192

## Units 1 and 2 results

Number of students	Satisfactory	Unsatisfactory
Unit 1	3511	225
Unit 2	3244	275

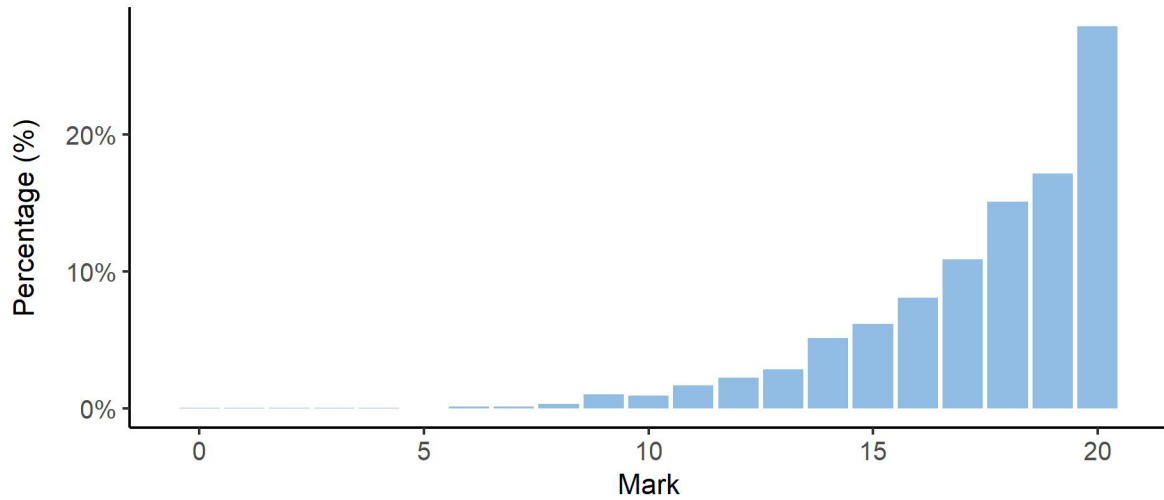
## Units 3 and 4 internal assessment (IA) results

### Total marks for IA

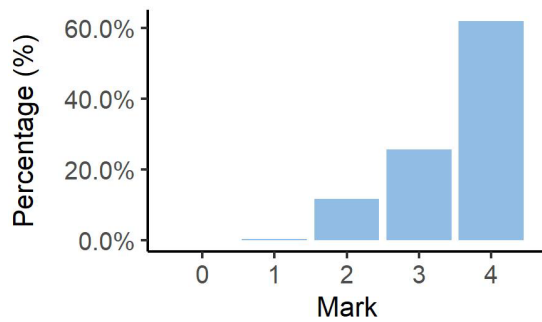


## IA1 marks

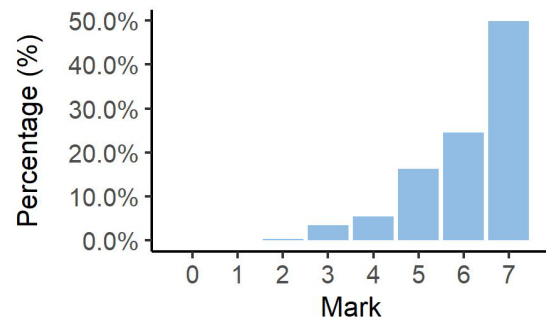
### IA1 total



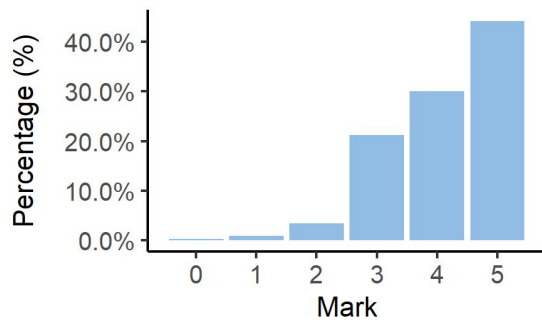
### IA1 Criterion: Formulate



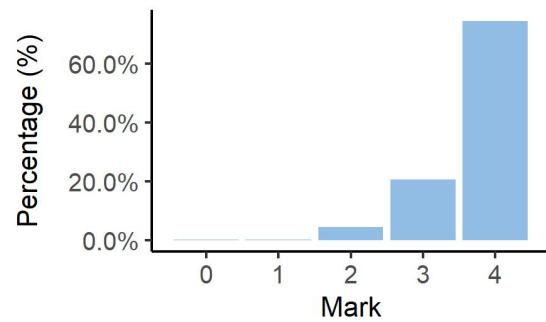
### IA1 Criterion: Solve



### IA1 Criterion: Evaluate and verify

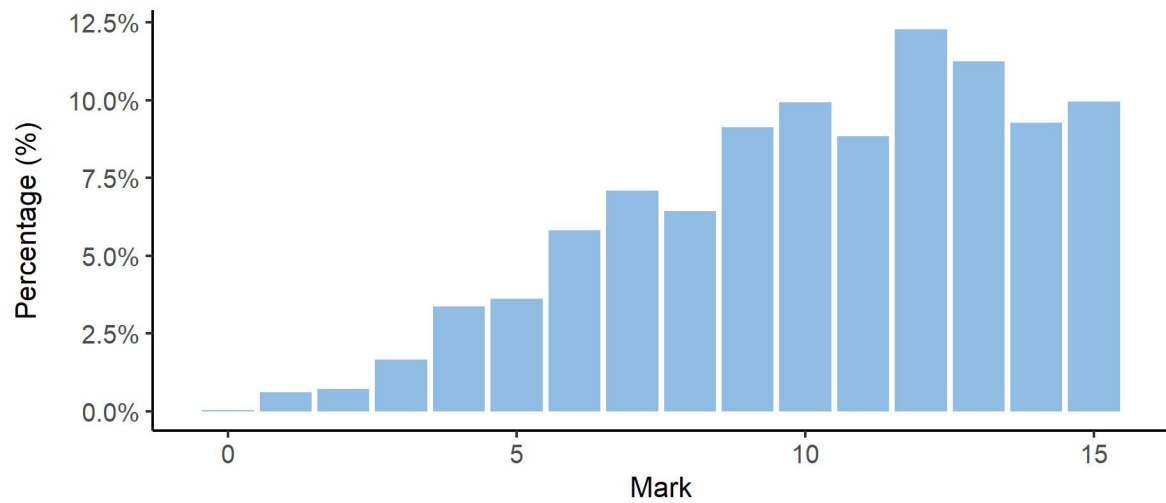


### IA1 Criterion: Communicate

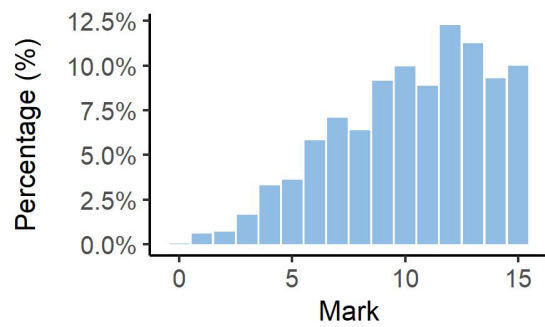


## IA2 marks

### IA2 total

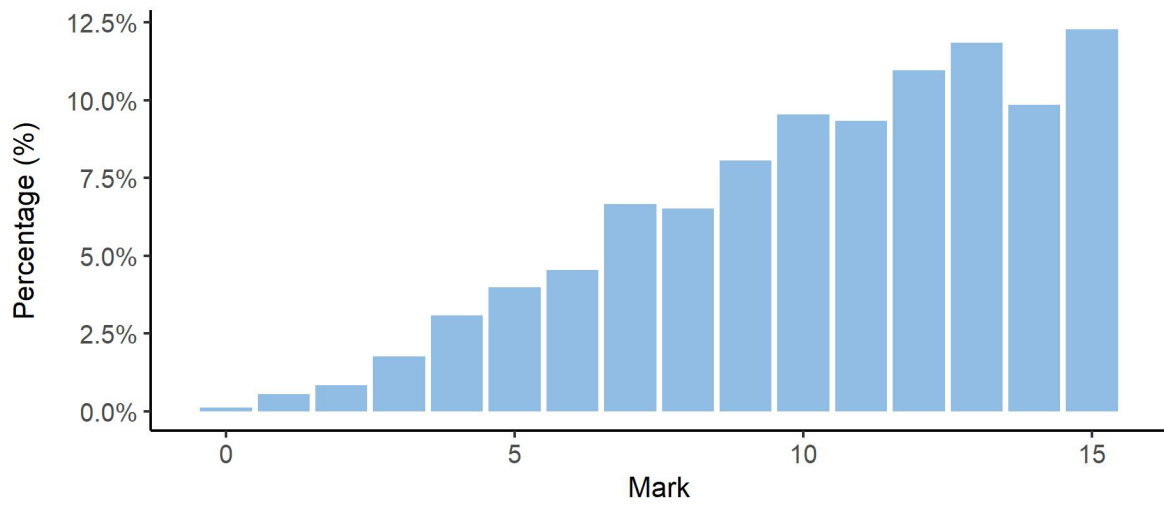


### IA2 Criterion: Foundational knowledge and problem-solving

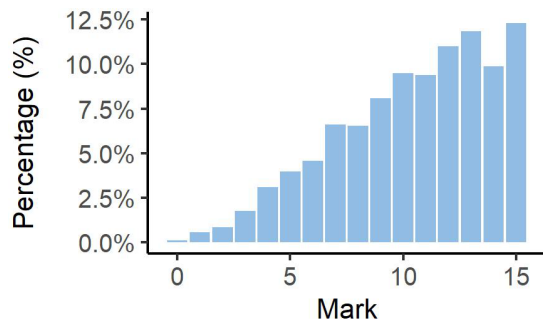


## IA3 marks

### IA3 total

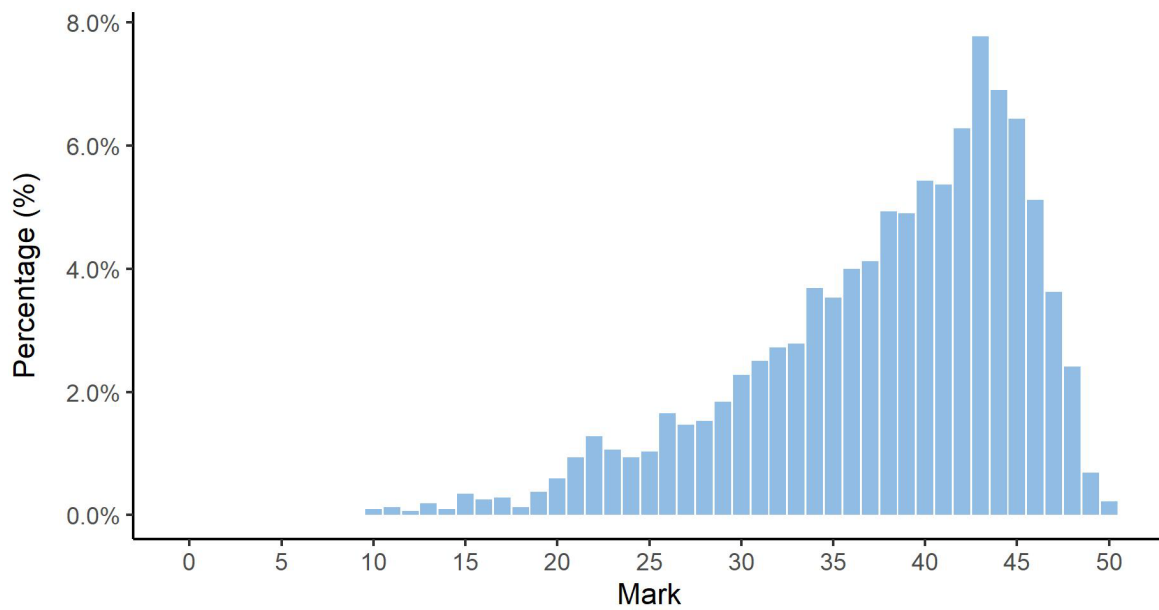


### IA3 Criterion: Foundational knowledge and problem-solving



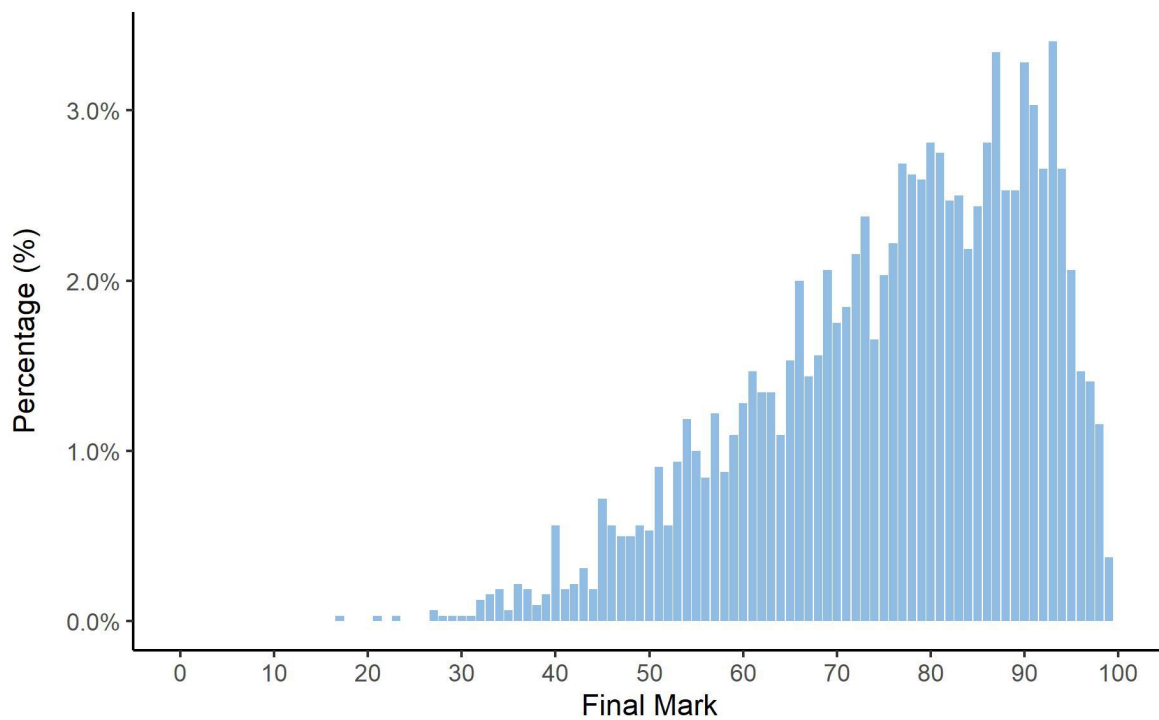


## External assessment (EA) marks



## Final subject results

### Final marks for IA and EA



## Grade boundaries

The grade boundaries are determined using a process to compare results on a numeric scale to the reporting standards.

Standard	A	B	C	D	E
Marks achieved	100–84	83–67	66–47	46–21	20–0

## Distribution of standards

The number of students who achieved each standard across the state is as follows.

Standard	A	B	C	D	E
Number of students	1198	1199	660	134	1

# Internal assessment



The following information and advice relate to the assessment design and assessment decisions for each IA in Units 3 and 4. These instruments have undergone quality assurance processes informed by the attributes of quality assessment (validity, accessibility and reliability).

## Endorsement

Endorsement is the quality assurance process based on the attributes of validity and accessibility. These attributes are categorised further as priorities for assessment, and each priority can be further broken down into assessment practices.

Data presented in the Assessment design section identifies the reasons why IA instruments were not endorsed at Application 1, by the priority for assessments. An IA may have been identified more than once for a priority for assessment, e.g. it may have demonstrated a misalignment to both the subject matter and the assessment objective/s.

Refer to *QCE and QCIA policy and procedures handbook v4.0*, Section 9.5.

### Percentage of instruments endorsed in Application 1

Number of instruments submitted	IA1	IA2	IA3
Total number of instruments	307	307	303
Percentage endorsed in Application 1	56%	27%	36%

## Confirmation

Confirmation is the quality assurance process based on the attribute of reliability. The QCAA uses provisional criterion marks determined by teachers to identify the samples of student responses that schools are required to submit for confirmation.

Confirmation samples are representative of the school's decisions about the quality of student work in relation to the instrument-specific marking guide (ISMG), and are used to make decisions about the cohort's results.

Refer to *QCE and QCIA policy and procedures handbook v4.0*, Section 9.6.

The following table includes the percentage agreement between the provisional marks and confirmed marks by assessment instrument. The Assessment decisions section of this report for each assessment instrument identifies the agreement trends between provisional and confirmed marks by criterion.

### Number of samples reviewed and percentage agreement

IA	Number of schools	Number of samples requested	Number of additional samples requested	Percentage agreement with provisional marks
1	299	1560	25	88.29%
2	296	1335	0	99.32%
3	295	1329	0	98.64%



## Problem-solving and modelling task (20%)

This assessment focuses on the interpretation, analysis and evaluation of ideas and information. It is an independent task responding to a particular situation or stimuli. While students may undertake some research in the writing of the problem-solving and modelling task, it is not the focus of this technique. This assessment occurs over an extended and defined period. Students will use class time and their own time to develop a response.

The problem-solving and modelling task must use subject matter from one or both of the following topics in Unit 3:

- Topic 2: Vectors and matrices
- Topic 3: Complex numbers 2.

## Assessment design

### Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

### Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	58
Authentication	19
Authenticity	25
Item construction	30
Scope and scale	43

\*Each priority might contain up to four assessment practices.

Total number of submissions: 307.

### Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- avoided scaffolding or instructions that directed students to use a particular mathematical method
- provided datasets, or used an open-ended task design, which allowed students to develop a unique response
- included relevant stimulus material where appropriate.

## Practices to strengthen

It is recommended that assessment instruments:

- avoid identifying concepts or techniques that lead students to a particular solution.

## Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

### Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	4
Language	13
Layout	4
Transparency	16

\*Each priority might contain up to four assessment practices.

Total number of submissions: 307.

## Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- had clear instructions to students about the requirements of the task, including identification of the topics being assessed
- were free from spelling and punctuation errors
- included a reference to the approach to problem-solving and mathematical modelling flowchart from Syllabus section 1.2.4 (Syllabus section 4.6.1).

## Practices to strengthen

There were no significant issues identified for improvement.

## Additional advice

- Use checkpoints to indicate in when feedback is to be provided on one complete or near-complete draft.

## Assessment decisions

### Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

### Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Formulate	91.64%	7.69%	0.33%	0.33%
2	Solve	98.33%	1%	0.67%	0%
3	Evaluate and verify	95.32%	4.01%	0.67%	0%
4	Communicate	99.67%	0%	0.33%	0%

### Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- in the Formulate criterion, judgments about the translation of the problem were supported by identifying the breadth and depth of the student's comprehension of how concepts are applied to develop the mathematical model or representation
- in the Solve criterion, judgments about the use of procedures were supported by identifying the 'simple' or 'complex' procedures and whether the solution was 'valid', 'reasonable', or incomplete (refer to the syllabus glossary definitions for *valid* and *reasonable* and see the QCAA high-level sample for IA1)
- in the Evaluate and verify criterion, judgments about the student's decisions were supported by identifying 'justification' or 'statements' (refer to the syllabus glossary definitions for *justify/justified* and *statement* and see the QCAA high-level sample for IA1)
- in the Communicate criterion, judgments about the use of language were supported with examples of appropriately used technical and procedural vocabulary.

### Samples of effective practices

The following excerpts illustrate:

- documentation of both observations and assumptions (Excerpts 1 and 2)
- a variety of appropriate and accurate uses of technology (Excerpts 3–6).

**Note:** The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

**Excerpt 1****Observations**

1. It was observed that the male-to-female ratio in Western Australia is 102 males for every 100 females (McCrindle, 2014). This observation directly impacts the mathematical model as the initial total population must divide by this ratio only to consider the female Western Australian population. This is relevant as males do not reproduce any offspring and cannot be factored into the Leslie matrix.
2. It was observed that Western Australia takes 30% of immigrants into Australia each year (Australian Bureau of Statistic, 2021). Further, 12,706 to 18,200 immigrants settled in Australia during 2018 and 2020. (Lawrence 2018). These observations impacted the mathematical model as an increase in immigrants will impact the projected populations, causing an increase or decrease in total population growth rate.
3. Humans live to approximately 100 years (Vaupel 2010); therefore, a 21 x 21 matrix is reasonable. From this, the age classes for the Leslie matrix were split into 5-year categories. Hence, each new generation produced by the Leslie matrix represents a 5-year gap. This observation is relevant as it impacts the scope mathematical model by investigating only eight new generations.

**Assumptions**

1. It was assumed that birth rates would only impact women aged 15-44 as in Australia. The average woman's reproductive years are between ages 15 and 44 (Watson, 2018). This assumption restricts birth rates to only six of twenty-one age classes. The assumption was made to reduce the anomalies to develop clean data.
2. It was assumed that the male-to-female ratio, survival, and birth rate were the same and remained constant for the Western Australian and immigrant population. This was required to restrict data from different years from impacting the trend and the investigation parameters in order to obtain relevant projections. As only a small number of immigrants are introduced into Western Australia each year, they will have minimal impact on the survival, and birth rate (Australian Bureau of Statistics, 2021).
3. It was assumed that the new immigrant population introduced into Western Australia was equally divided into the age classes from 18 to 34. Most migrants to Australia are young adults, with 61.2% aged between 18 and 34 years (Abs.gov.au, 2018). This assumption was made to create a realistic data spread that included the possible impact immigrants' survival and or birth rate would have on the total population.
4. It was assumed that initially, no one left or entered Western Australia prior to immigration. The developed Leslie matrix does not initially account for interstate and international traveling; therefore, this assumption must be made to prevent skewing the data, creating an unrealistic model.
5. It was assumed that the investigation started during 2016 as the female population data collected was from 2016 (Abs.gov.au, 2016). This assumption impacts the investigation as the potential increasing or decreasing growth rate and total population can be compared to secondary data to determine the model's validity.

## Excerpt 2

### Observations

1. In 2020, Australia's population was 50.2% female and 49.8% male (ABS, 2020), which may be relevant as Leslie matrices only consider females.
2. In 2017, people aged 85 and over made up 1.76% of Queensland's population (ABS, 2018). Queenslanders between 85-100 years of age show almost identical fertility rates, although their death rates vary ('Centenarians now Australia's fastest-growing demographic', 2021).
3. There was a less than 1% chance of pregnancy for women over 50 years of age (Pesce, 2016) and birth rates for girls under the age of 15 years were "unstable" due to low numbers (Marino, 2016). Birth rates of women aged outside of 15-49 years old were thus low and inconsistent.
4. As shown in [Appendix C](#), the death rates of Australians have mostly stabilised over time, due to improved healthcare and technology, with only a minor decrease in the number of yearly female deaths year from 1997-2017 (ABS, 2021).
5. The birth rates of recent immigrants vary due to their different cultural and geographical backgrounds, while the mortality rates of immigrants should be similar to that of long-term residents as they have the same healthcare access (Peri, 2020).
6. As shown in [Appendix D](#), the immigration rate fluctuates from 2003-2017 before stabilising into a linear trend with a gradient of 160.22 (calculated using excel), meaning the net immigration of Queensland changes over time.

### Assumptions

1. The initial population and birth rates will be multiplied by 0.502 before being used in the calculations (Observation 1). Before graphing, projected populations must then be divided by 0.502 to show the entire projected population, rather than that of females only. Finding the overall gender distribution instead of that of each age group could impact the results' accuracy; however, since the gender distributions are almost identical for each group, this impact should be insignificant.
2. As people aged 85 and over show similar characteristics (Observation 2), it was decided that the final age bracket would be 85+. This may negatively impact the results due to variation in death rates within the age bracket. However, since this age group is small and is the final age group, the assumption will not impact the other age brackets and should have a minimal effect on the model's reasonableness.
3. The fertility rates were only considered for women between the ages of 15-49 years old ([Appendix B](#)) (Queensland Government Statistician's Office, 2020). This may negatively impact the results; however, there is too much inconsistency (Observation 3) and not enough information about birth rates outside of this age group to incorporate them effectively.

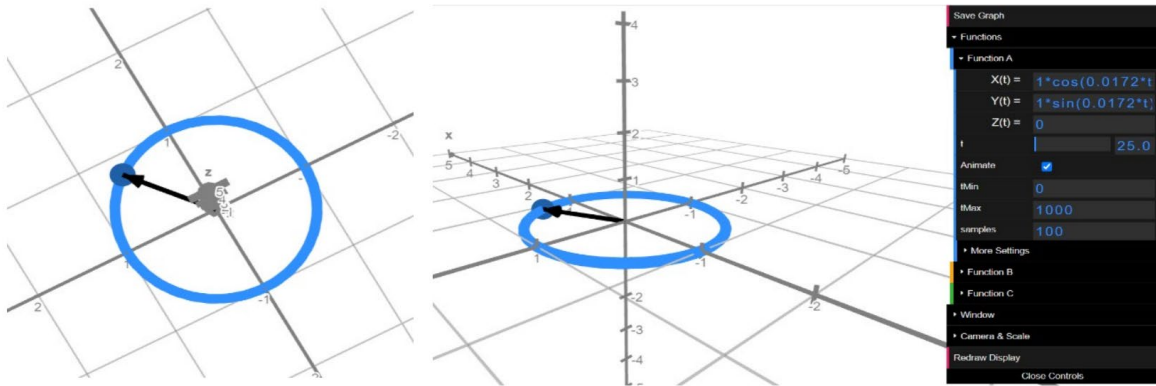


**Excerpt 2 (continued)**

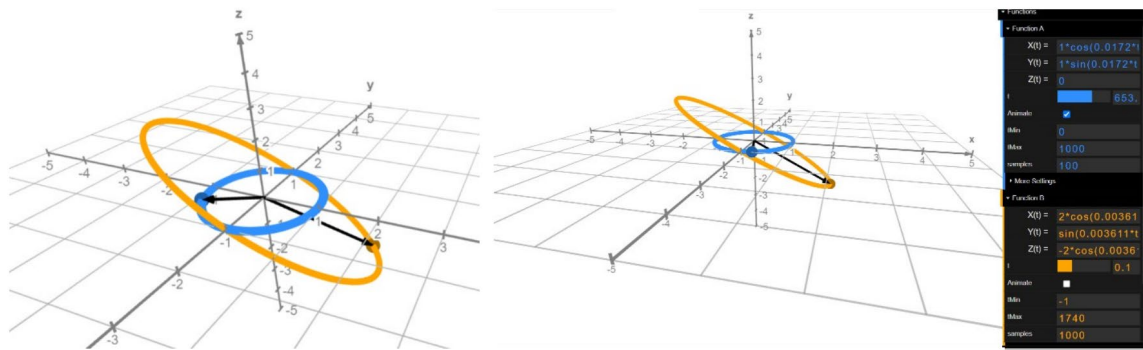
4. It was assumed that the survival rate and fertility rates were constant. This may impact the accuracy of the results since, in a real-life situation, the death and fertility rates change yearly. Since Queensland's death rates are becoming more consistent (Observation 4), the impact of a constant death rate should be minimal. However, the birth rates over time were inconsistent, with high variation, and thus assuming a constant birth rate may impact the projections' reasonableness.
5. It was assumed that overall immigration would increase by 160.22 immigrants every year (Observation 6). This assumption should improve the results' precision, as it means the model will better represent the growth rate of immigration and hence the overall population projections.
6. While was assumed that overall immigration would increase by 160.22 immigrants every year (Observation 6). This assumption should improve the results' precision, as it means the model will better represent the growth rate of immigration and hence the overall population projections.
7. Covid-19 severely impacted immigration worldwide; however, with Queensland's borders opening soon, immigration rates should soon be normalised (Charumilind et al., 2021). Hence, this assignment was based on pre-pandemic data and disregarded the effects of Covid-19. While this assumption will negatively impact the short-term projections' accuracy, it should improve the long-term projections' reasonableness.
8. As well as base (0) immigration, the three levels of initial net immigration investigated were low (29 050 per year), medium (37 350 per year) and high (46 650 per year). These were multiplied by 0.59, the percentage of female Queensland immigrants in 2017 (ABS, 2019), giving low (17 139.5), medium (22 036.5) and high (27 523.5) initial immigration values.
9. In a Leslie matrix, all individuals within an age bracket are assumed to have the same characteristics. The effect of this assumption on the model's reasonableness was minimised by using small five-year age brackets; however, this could be improved by further reducing the size of each age bracket.

**Excerpt 3**

This was then graphed on the parametric curve simulator:



Figures 1 & 2: Earth Model

**Excerpt 4**

Figures 3 & 4: Earth & Asteroid Models

## Excerpt 5

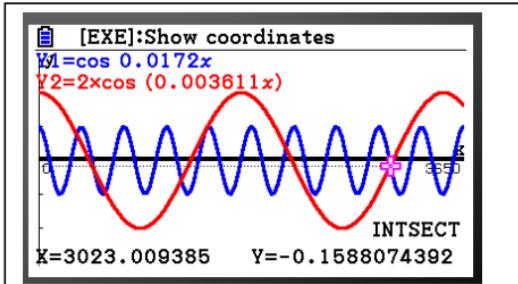


Figure 9:

$$x: \cos(0.0172t) = 2 \times \cos(0.003611t)$$

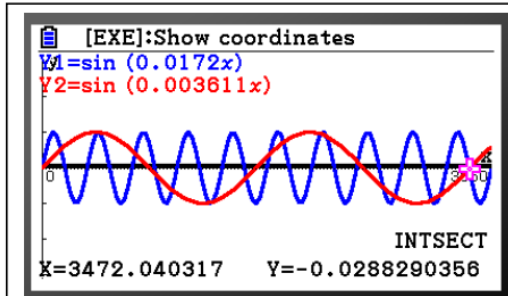


Figure 10:

$$y: \sin(0.0172t) = \sin(0.003611t)$$

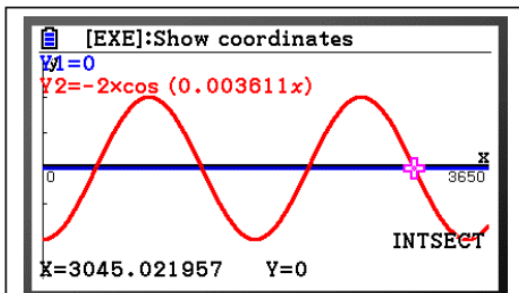


Figure 11:

$$z: 0 = -2 \times \cos(0.003611t)$$

These observed points of intersection were then entered into Excel, where the filter function was used to highlight those values with the smallest differences, shown below.

x	y	z
472.5358	462.3729	435.0031
2122.817	1962.458	2175.016
3023.009	3170.124	3045.022

## Excerpt 6

To verify this a larger Excel spreadsheet was created to solve the parametric equations for a variety of  $t$  values (also extending to ten years):

t	Xleft	Xright	Difference	Intersect?	Yleft	Yright	Difference	Intersect?	Zleft	Zright	Difference	Intersect?
0.5	0.999963	1.999997	1.0000337	FALSE	0.0086	0.001805	0.0067944	FALSE	0	-2	1.999997	FALSE
1	0.999852	1.999987	1.0001349	FALSE	0.017199	0.003611	0.0135882	FALSE	0	-1.999999	1.999987	FALSE

=COS(0.0172\*A2)    =2\*COS(0.003611\*A2)    =ABS(B2-C2)

Figure 12: Spreadsheet

The Excel equation inserted in the 'Intersect?' column (Figure 13), calculated if the difference between the two sides of the parametric equation was less than  $4.2587 \times 10^{-5}$ , the conversion of Earth's radius to AU, to ensure constant units.

$$=IF(D2<(0.0000425875),"YES")$$

## Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- in the Formulate criterion, judgments about observations and assumptions are supported by making a clear distinction between 'statements' and 'documentation'. Responses that demonstrate documentation of assumptions provide evidence of the logic of the mathematisation process, as identified in the flowchart (Syllabus section 1.2.4). Well-documented assumptions should form part of the modelling process and support the student's

justification for selecting particular rules and procedures. Responses that demonstrate documentation of observations includes the source of the data, the reasoning for the method used to collect the data, or the validity of the observations with respect to the proposed solution and/or model

- in the Solve criterion, ‘use of technology’ must go beyond simple computation or word processing; it must be a key component in the development of the solution to the problem. Evidence of accuracy and appropriateness should be identified and included in the body of the response, e.g. screenshot of excerpt from spreadsheet showing formulas used
- in the Evaluate and verify criterion, judgments about the evaluation of the reasonableness of solutions requires consideration of all three elements — results, assumptions and observations — in relation to the original task and context. Judgments about the documentation of relevant strengths and limitations must include both strengths and limitations of the solution and/or model. Well-documented strengths and limitations should form part of the evaluation process and support the student’s judgment about the reasonableness of the solution.

### **Additional advice**

- Schools should identify how they have applied their school assessment policy to responses exceeding the maximum of 2000 words or the maximum of 10 pages (see *QCIA and QCE policy and procedures handbook v4.0*, Section 8.2.6: Managing response length).
- As appendixes are not to be marked, students should provide evidence relevant to performance descriptors (e.g. use of complex procedures, use of technology) in the body of the report. Large datasets, evidence of authentication (e.g. creating a piece of code, completing an experiment), or repeated calculations can be put in an appendix.
- Use the ISMG from the current syllabus. Do not use an ISMG from a previous version of the syllabus. Do not create your own ISMG.



## Examination (15%)

The examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 3 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

## Assessment design

### Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

### Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	177
Authentication	0
Authenticity	2
Item construction	22
Scope and scale	46

\*Each priority might contain up to four assessment practices.

Total number of submissions: 307.

### Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided questions that assessed a representative sample of Unit 3 (AS unit 3) topics and reflected the intended learning
- used questions that were relevant to the school context, and sufficiently different from both textbook questions and QCAA sample questions to ensure responses were the student's own and not rehearsed
- had an appropriate number of questions for students to respond to in the time conditions
- assessed Assessment objective 4: Evaluate the reasonableness of solutions.

### Practices to strengthen

It is recommended that assessment instruments:

- provide complex unfamiliar questions that match the syllabus description for both complexity and unfamiliarity

- remove scaffolding on complex familiar questions. Avoid splitting these questions into parts a), b), etc. as this reduces the opportunity for clarification and analysis required to develop responses
- avoid the use of standalone questions that only assess the review of assumed knowledge from Unit 1 or Unit 2 subject matter
- assign appropriate mark allocations to questions, based on the level of complexity and number of steps required to solve.

## Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

### Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions *
Bias avoidance	7
Language	59
Layout	3
Transparency	46

\*Each priority might contain up to four assessment practices.

Total number of submissions: 307.

### Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- avoided unnecessary jargon, specialist language and colloquial language
- clearly stipulated use of technology or an algebraic approach for questions where examinations were not separated into separate technology-free and technology-active papers.

### Practices to strengthen

It is recommended that assessment instruments:

- are free of punctuation, spelling and other errors
- are checked by the school for typographical errors and layout issues, using an appropriate proofing process, before they are submitted
- model correct language conventions, including the use of appropriate complex number and vector notation
- provide suitable cues to ensure that a vector calculus approach is used in complex questions involving motion in a plane, including projectile and circular motion.

## Additional advice

- If assessments are not endorsed at Application 1, an updated marking scheme reflecting any changes made should be provided with any subsequent applications.
- Where Assessment objective 4: Evaluate the reasonableness of solutions is assessed in the examination, ensure the marking scheme provides for marks to be awarded for the demonstration of this objective.

## Assessment decisions

### Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

### Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	99.32%	0.68%	0%	0%

### Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- the marking scheme clearly identified the objective, cognition or procedure for which each mark was awarded
- the marking scheme was used accurately and consistently across a cohort when awarding marks, with the allocation of marks on the student response aligning with those in the marking scheme
- the total number of marks awarded for each question was clearly identifiable, e.g. at the top of the page, at the bottom of the page, or in a table of results
- evidence of the percentage calculation used to determine the awarded mark out of 15 is provided on the ISMG.

### Samples of effective practices

The following excerpts illustrate:

- a school's clear allocation of marks for a response to a question involving a diagram (Excerpt 1)
- how a school has identified the awarding of follow-through marks (Excerpt 2)
- how a school has clearly identified the total marks for an IA2, the corresponding percentage calculation and the awarded mark out of 15 on the ISMG (Excerpt 3).

**Note:** The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.



## Excerpt 1

$$|z - (2 - 3i)| < 3 \quad \therefore \text{let } z = x + yi$$

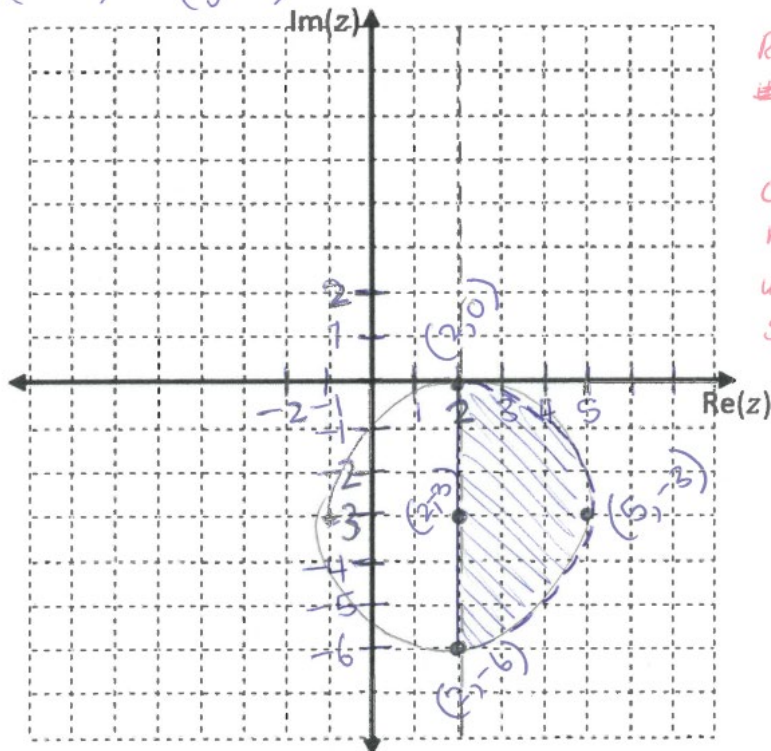
$$|(x + yi) - (2 - 3i)| < 3$$

$$|(x - 2) + (y + 3)i| < 3$$

$$\sqrt{(x - 2)^2 + (y + 3)^2} < 3$$

$$\therefore (x - 2)^2 + (y + 3)^2 < 3^2$$

$\therefore$  circle  
 centre  $(2, -3)$   
 radius = 3



Re line ✓  
~~solid~~ solid ✓ (for subset)  
 intersects 2 ✓  
 circle  $(2, -3)$  ✓  
 radius ✓  
 unbroken ✓ (for subset)  
 shading ✓  
 labels ✓

4



## Excerpt 2

$$|z - i| = k \quad \text{let } z = x + yi$$

$$|x + yi - i| = k$$

$$|x + (y-1)i| = k$$

$$\sqrt{x^2 + (y-1)^2} = k$$

$$x^2 + (y-1)^2 = k^2 \quad \therefore$$

circle  $c(0,1)$

sketch



$$\therefore (z+2)(z^2-6z+13) = 0$$

$$(z+2)(z^2-6+(-3)^2+13+(-3)^2)$$

$$(z+2)[(z-3)^2+4] = 0$$

$$(z+2)[(z-3)^2-4i^2] = 0$$

$$(z+2)(z-3+2i)(z-3-2i) = 0$$

$$z_1 = 2$$

$$z_2 = 3-2i$$

$$z_3 = 3+2i$$

$$z^3 - 4z^2 + z + 26 = 0$$

Try  $z = -2$

$$(2)^3 - 4(-2)^2 + (-2) + 26 = 0$$

$$-8 - 16 - 2 + 26 = 0$$

$$0 = 0$$

$\therefore (z+2)$  is a factor

$$z^2 - 6z + 13$$

$$z+2 \overline{) z^3 - 4z^2 + z + 26}$$

$$-(z^3 + 2z^2) \downarrow$$

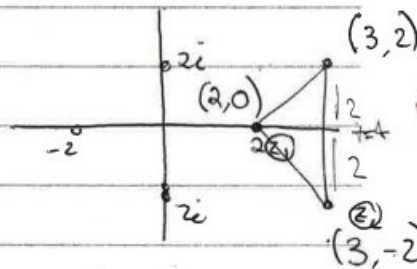
$$-6z^2 + z$$

$$-(-6z^2 - 12z)$$

$$13z + 26$$

$$\underline{(13z + 26)}$$

$$0$$



Area of Triangle

$$= \frac{b \times h}{2}$$

$$= \frac{4 \times 1}{2}$$

$$= \frac{4 \times 1}{2}$$

$$= 2 \text{ units}^2$$

$$= \frac{4}{2}$$

$$= 2 \text{ units}^2$$

**Excerpt 3****Instrument-specific marking guide (IA2): Examination (15%)**

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics

$$\frac{41}{56}$$

$$73.21\%$$

The student work has the following characteristics:	Cut-off	Marks
<ul style="list-style-type: none"> <li>consistently correct selection, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>authoritative</u> and <u>accurate</u> command of mathematical concepts and techniques; <u>astute</u> evaluation of the <u>reasonableness of solutions</u> and <u>use</u> of mathematical reasoning to correctly <u>justify</u> procedures and decisions; and <u>fluent</u> application of mathematical concepts and techniques to <u>solve</u> problems in a <u>comprehensive</u> range of <u>simple familiar</u>, <u>complex familiar</u> and <u>complex unfamiliar</u> situations.</li> </ul>	> 93%	15
	> 87%	14
<ul style="list-style-type: none"> <li>correct selection, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>comprehension</u> and <u>clear</u> communication of mathematical concepts and techniques; <u>considered</u> evaluation of the <u>reasonableness of solutions</u> and <u>use</u> of mathematical reasoning to <u>justify</u> procedures and decisions; and <u>proficient</u> application of mathematical concepts and techniques to <u>solve</u> problems in <u>simple familiar</u>, <u>complex familiar</u> and <u>complex unfamiliar</u> situations.</li> </ul>	> 80%	13
	> 73%	12
<ul style="list-style-type: none"> <li><u>thorough</u> selection, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>comprehension</u> and <u>communication</u> of mathematical concepts and techniques; <u>evaluation</u> of the <u>reasonableness of solutions</u> and <u>use</u> of mathematical reasoning to <u>justify</u> procedures and decisions; and <u>application</u> of mathematical concepts and techniques to <u>solve</u> problems in <u>simple familiar</u> and <u>complex familiar</u> situations.</li> </ul>	> 67%	11
	> 60%	10
<ul style="list-style-type: none"> <li><u>selection</u>, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>comprehension</u> and <u>communication</u> of mathematical concepts and techniques; <u>evaluation</u> of the <u>reasonableness of some solutions</u> using mathematical reasoning; and <u>application</u> of mathematical concepts and techniques to <u>solve</u> problems in <u>simple familiar</u> situations.</li> </ul>	> 53%	9
	> 47%	8
<ul style="list-style-type: none"> <li><u>some selection</u>, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>basic comprehension</u> and <u>communication</u> of <u>rudimentary</u> mathematical concepts and techniques; <u>inconsistent</u> evaluation of the <u>reasonableness of solutions</u> using mathematical reasoning; and <u>inconsistent</u> application of mathematical concepts and techniques.</li> </ul>	> 40%	7
	> 33%	6
<ul style="list-style-type: none"> <li><u>infrequent selection</u>, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>basic comprehension</u> and <u>communication</u> of <u>some mathematical concepts</u> and techniques; <u>some description</u> of the <u>reasonableness of solutions</u>; and <u>infrequent application</u> of mathematical concepts and techniques.</li> </ul>	> 27%	5
	> 20%	4
<ul style="list-style-type: none"> <li><u>isolated selection</u>, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>partial comprehension</u> and <u>communication</u> of <u>rudimentary mathematical concepts</u> and techniques; <u>superficial description</u> of the <u>reasonableness of solutions</u>; and <u>disjointed</u> application of mathematical concepts and techniques.</li> </ul>	> 13%	3
	> 7%	2
<ul style="list-style-type: none"> <li><u>isolated and inaccurate selection</u>, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>disjointed and unclear</u> communication of mathematical concepts and techniques; <u>illogical</u> description of the <u>reasonableness of solutions</u>.</li> </ul>	> 0%	1
<ul style="list-style-type: none"> <li>does not satisfy any of the descriptors above.</li> </ul>		0

**Practices to strengthen**

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- schools check that the correct marking scheme is uploaded, it matches all questions on the endorsed instrument, and the total number of marks for each question matches the endorsed instrument
- the solutions in the marking scheme are checked carefully, with any errors identified during the marking of student work corrected and re-uploaded at confirmation
- a separate marking scheme is provided to match any comparable assessment
- when awarding follow-through marks, teachers explicitly identify the error and note that they are awarding follow-through marks

- follow-through marks are only awarded when, after making an error, a student completes valid subsequent steps of the solution correctly
- schools check the totalling of marks before calculating an overall percentage and do not round the calculated percentage before applying the cut-off on the ISMG.

### **Additional advice**

- If a student does not respond to any questions in the assessment instrument, a not rated (NR) should be awarded. A mark of 0/15 is not appropriate in these circumstances.
- Schools should take care when scanning student responses to ensure that both sides of double-sided pages are scanned correctly, and any additional working pages are also scanned before uploading for confirmation.
- If an error in an endorsed examination was noted post-implementation, schools must provide evidence that permission to make changes to the examination was granted by the QCAA, as outlined in communication with the subject PEO.



## Examination (15%)

This examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 4 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

## Assessment design

### Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

### Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	164
Authentication	0
Authenticity	1
Item construction	32
Scope and scale	20

\*Each priority might contain up to four assessment practices.

Total number of submissions: 303.

### Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided a correct marking scheme with a clear indication of how marks would be awarded in each question
- provided a representative sample across all three topics
- assessed Assessment objective 4: Evaluate the reasonableness of solutions.

### Practices to strengthen

It is recommended that assessment instruments:

- have questions that focus on Unit 4 subject matter only. Standalone questions that can be solved using only subject matter from Units 1, 2 or 3 (AS units 1, 2 or 3) or Mathematical Methods subject matter are not suitable
- provide a representative sample that accurately reflects the intended learning across the topics of Unit 4 (AS unit 4)

- include complex unfamiliar questions that match the syllabus description for both complexity and unfamiliarity
- provide suitable cues to ensure that a calculus approach is used in complex questions involving applications of simple differential equations in context, such as kinematics
- clearly stipulate use of technology or an algebraic approach for questions where examinations are not separated into separate technology-free and technology-active papers.

## Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

### Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	7
Language	45
Layout	9
Transparency	41

\*Each priority might contain up to four assessment practices.

Total number of submissions: 303.

### Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- phrased questions in context, using clear, concise language and avoiding jargon that did not contribute to the understanding of the subject matter
- were free of punctuation, spelling and other errors.

### Practices to strengthen

It is recommended that assessment instruments:

- are reviewed using the 'Print preview' button before they are submitted to ensure their layout is appropriate (see *Developing internal summative assessment instruments: Endorsement user guide*, Section 4, available from the Endorsement application in the QCAA Portal)
- model correct language conventions, including the use of appropriate calculus and statistical notation
- use mathematical terminology that matches syllabus subject matter, including the use of statistical language.

### Additional advice

- Provide a correct marking scheme that indicates clearly how marks will be allocated. This assists schools to check the scope and scale of the assessment, such as time allocation, adequacy of response space and match to the identified degree of difficulty.

- Where Assessment objective 4: Evaluate the reasonableness of solutions is assessed in the examination, ensure that corresponding marks are awarded for the demonstration of this objective in the marking scheme.

## Assessment decisions

### Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

#### Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	98.64%	1.02%	0.34%	0%

### Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- the marking guide clearly identified the objective, cognition or procedure for which each mark was to be awarded
- the marking guide was used accurately and consistently across a cohort when awarding marks, with the allocation of marks on the student response aligning with those on the marking guide
- the total number of marks awarded for each question was clearly identifiable, e.g. at the top of the page, at the bottom of the page, or in a table of results
- evidence of the percentage calculation used to determine the awarded mark out of 15 was provided on the ISMG.

#### Samples of effective practices

The following excerpts illustrate:

- how a school has clearly identified the total marks for the IA3 and the corresponding percentage calculation on a table placed at the front of the instrument (Excerpt 1)
- how a school has identified the awarding of follow-through marks (Excerpt 2)
- how a school has clearly identified the total marks for an IA2, the corresponding percentage calculation and the mark allocation on an ISMG (Excerpt 3)
- how a school has summarised the various mark allocations across Paper 1 and Paper 2, and calculated the total marks for a student, within the instruction section of an IA3 (Excerpt 4).

**Note:** The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

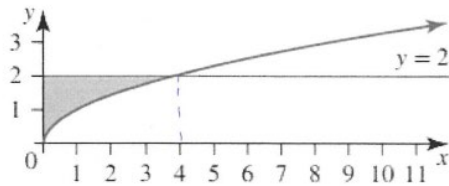
## Excerpt 1

Marking

Paper	Marks
1	23 / 33
2	18½ / 27
Total	41½ / 60
Provisional	11 / 15

69.2%

## Excerpt 2



$y = \sqrt{x}$   
 $y^2 = x$   
 $V = \pi \int_a^b [f(y)]^2 dy$

intersection:  
 sub  $y=2$  into  $y^2=x$   
 $2^2 = x$   
 $4 = x$

*Incorrect terminals.*  
 $V = \pi \int_0^4 (y^2)^2 dy$

*Correct expression.*  
 $V = \pi \int_0^4 y^4 dy$

$= \pi \left[ \frac{y^5}{5} \right]_0^4$

*FI error with terminals.*  
 $= \pi \left[ \left( \frac{4^5}{5} \right) - 0 \right]$

$= \frac{1024\pi}{5} \text{ units}^3$



**Excerpt 3****Instrument-specific marking guide (IA3): Examination (15%)**

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

$$\frac{49}{49.5} = 98.99\%$$

The student work has the following characteristics:	Cut-off	Marks
<ul style="list-style-type: none"> <li>consistently correct selection, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>authoritative</u> and <u>accurate</u> command of mathematical concepts and techniques; astute evaluation of the <u>reasonableness of solutions</u> and use of mathematical reasoning to correctly <u>justify</u> procedures and decisions; and <u>fluent</u> application of mathematical concepts and techniques to <u>solve</u> problems in a <u>comprehensive</u> range of <u>simple familiar</u>, <u>complex familiar</u> and <u>complex unfamiliar</u> situations.</li> </ul>	> 93%	15
	> 87%	14
<ul style="list-style-type: none"> <li>correct selection, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; comprehension and <u>clear</u> communication of mathematical concepts and techniques; <u>considered</u> evaluation of the <u>reasonableness of solutions</u> and <u>use</u> of mathematical reasoning to <u>justify</u> procedures and decisions; and <u>proficient</u> application of mathematical concepts and techniques to <u>solve</u> problems in <u>simple familiar</u>, <u>complex familiar</u> and <u>complex unfamiliar</u> situations.</li> </ul>	> 80%	13
	> 73%	12
<ul style="list-style-type: none"> <li><u>thorough</u> selection, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the <u>reasonableness of solutions</u> and use of mathematical reasoning to <u>justify</u> procedures and decisions; and application of mathematical concepts and techniques to <u>solve</u> problems in <u>simple familiar</u> and <u>complex familiar</u> situations.</li> </ul>	> 67%	11
	> 60%	10
<ul style="list-style-type: none"> <li>selection, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to <u>solve</u> problems in <u>simple familiar</u> situations.</li> </ul>	> 53%	9
	> 47%	8

**Excerpt 4**

- The marks allocation appears in the table below:

Criterion	Marks TA (Part A)	Result	Marks TF (Part B)	Result	Total Marks	Total Result
Simple Familiar	17	14½	19	18	36	32½
Complex Familiar	8	6	4	3	12	9
Complex Unfamiliar	6	6	6	2	12	8
Total	31	26½	29	23	60	49½
Total out of 15						13

**Practices to strengthen**

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- schools check that the correct marking guide is uploaded, that it matches all questions on the endorsed item, and that the total number of marks for each question matches the endorsed item
- the solutions on the marking guide are checked carefully, with any errors identified during the marking of student work corrected and re-uploaded at confirmation
- a separate marking guide is provided for comparable assessment

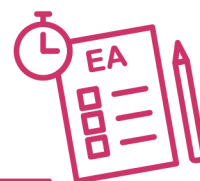


- when awarding follow-through marks, teachers explicitly identify the error and note that they are awarding follow-through marks
- follow-through marks are awarded only when, after making an error, a student completes valid subsequent steps of the solution correctly
- schools check the totalling of marks before calculating an overall percentage and do not round the calculated percentage before applying the cut-off on the ISMG.

### **Additional advice**

- If a student does not respond to any questions in the assessment instrument, then a not rated (NR) is awarded; a mark of 0/15 is not appropriate.
- Follow-through marks are only awarded when, after making an error, a student completes valid subsequent steps of the solution correctly.
- Schools should take care when scanning student assessment, ensuring that both sides of double-sided pages are scanned correctly and that any additional working pages are also scanned before uploading for confirmation.
- If an error in an endorsed examination was noted post-implementation, schools must provide evidence that permission to make changes to the examination was granted by the QCAA, as outlined in communication with the subject PEO.

# External assessment



External assessment (EA) is developed and marked by the QCAA. The external assessment for a subject is common to all schools and administered under the same conditions, at the same time, on the same day.

## Examination (50%)

### Assessment design

The assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the syllabus. The examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (50 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (50 marks).

The examination assessed subject matter from Units 3 and 4.

The assessment required students to respond to multiple choice and short response questions.

The AS assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the AS.

The AS examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (50 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (50 marks).

The AS examination assessed subject matter from AS units 3 and 4.

The AS assessment required students to respond to multiple choice and short response questions.

### Assessment decisions

Assessment decisions are made by markers by matching student responses to the external assessment marking guide (EAMG). The external assessment papers and the EAMG are published in the year after they are administered.

### Multiple choice question responses

There were 10 multiple choice questions in Paper 1 (General).

#### Percentage of student responses to each option

##### Note:

- The correct answer is **bold** and in a **blue** shaded table cell.
- Some students may not have responded to every question.

## Specialist Mathematics General: Paper 1

Question	A	B	C	D
1	1.61	<b>95.52</b>	2.17	0.66
2	10.2	24.19	<b>52.06</b>	13.1
3	25.67	<b>61.53</b>	8.92	3.52
4	13.29	1.02	1.78	<b>83.71</b>
5	<b>76.97</b>	19.61	2.93	0.36
6	1.15	5.43	4.28	<b>88.98</b>
7	<b>40.7</b>	20.93	27.48	10.33
8	5.33	<b>80.78</b>	5.73	7.8
9	3.72	4.48	8.56	<b>82.86</b>
10	5.99	6.75	<b>79.86</b>	7.01

There were 10 multiple choice questions in Paper 2 (General).

## Percentage of student responses to each option

## Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

## Specialist Mathematics General: Paper 2

Question	A	B	C	D
1	5.63	49.93	<b>26.53</b>	17.51
2	<b>78.87</b>	5.46	10.04	5.33
3	12.51	13.4	<b>53.03</b>	20.11
4	1.74	7.14	12.05	<b>78.18</b>
5	3.06	5.92	<b>85.09</b>	5.5
6	9.64	3	<b>85.45</b>	1.58
7	8.95	<b>63.56</b>	11.75	13.79
8	6.29	<b>75.67</b>	14.32	3.13
9	<b>72.19</b>	20.34	3.32	3.52
10	<b>70.14</b>	14.42	10.37	4.25

There were 10 multiple choice questions in Paper 1 (AS).

#### Percentage of student responses to each option

##### Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

#### Specialist Mathematics Alternative Sequence: Paper 1

Question	A	B	C	D
1	3.85	<b>95.38</b>	0	0.77
2	7.69	25.38	<b>47.69</b>	18.46
3	35.38	<b>46.92</b>	12.31	5.38
4	7.69	12.31	8.46	<b>70.77</b>
5	<b>70</b>	21.54	7.69	0.77
6	11.54	4.62	<b>63.08</b>	20
7	<b>39.23</b>	20.77	23.85	15.38
8	8.46	7.69	<b>56.92</b>	26.92
9	11.54	<b>66.15</b>	19.23	3.08
10	6.92	6.15	<b>79.23</b>	7.69

There were 10 multiple choice questions in Paper 2 (AS).

#### Percentage of student responses to each option

##### Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

#### Specialist Mathematics Alternative Sequence: Paper 2

Question	A	B	C	D
1	<b>86.15</b>	0.77	3.08	10
2	<b>79.23</b>	6.15	11.54	3.08
3	12.31	13.85	<b>44.62</b>	26.92
4	38.46	<b>54.62</b>	4.62	1.54
5	2.31	3.08	<b>90</b>	4.62
6	10.77	5.38	<b>81.54</b>	2.31
7	3.08	4.62	3.08	<b>89.23</b>
8	3.85	<b>74.62</b>	19.23	2.31
9	<b>80.77</b>	14.62	2.31	2.31

Question	A	B	C	D
10	66.15	13.85	13.08	5.38

## Effective practices

Overall, students responded well to:

- questions requiring integration by partial fractions, although care was needed when integrating fractions of the form  $\frac{a}{bx+c}$  where  $b \neq 1$
- opportunities to demonstrate knowledge and application of Leslie matrices
- opportunities to demonstrate knowledge and understanding of Simpson's rule and use of technology to justify the reasonableness of their solution.

## Samples of effective practices

### Short response

The following excerpt is from Question 12 from Paper 1. It required students to construct a proof for a complex number identity.

Effective student responses:

- realised the given fraction by multiplying the numerator and denominator by the complex conjugate of  $z_2$
- determined the modulus of an expression
- constructed supporting arguments in the form of a proof.

This excerpt has been included:

- as it provides an example of writing the realised fraction in Cartesian form so that the modulus can be calculated correctly
- to show an example of simplifying the left-hand side of the proof and then simplifying the right-hand side of the proof to reach an intermediate position.

$$\begin{aligned} \left| \frac{z_1}{z_2} \right| &= \frac{|z_1|}{|z_2|} \\ \text{LHS} &= \left| \frac{a+bi}{c+di} \right| \\ &= \left| \frac{a+bi}{c+di} \times \frac{c-di}{c-di} \right| \\ &= \left| \frac{ac-ad^2i+bc^2i+bd}{c^2+d^2} \right| \\ &= \left| \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i \right| \\ &= \sqrt{\left(\frac{ac+bd}{c^2+d^2}\right)^2 + \left(\frac{bc-ad}{c^2+d^2}\right)^2} \\ &= \frac{1}{c^2+d^2} \sqrt{(ac+bd)^2 + (bc-ad)^2} \\ &= \frac{1}{c^2+d^2} \sqrt{a^2c^2 + 2abcd + b^2d^2 + b^2c^2 - 2abcd + a^2d^2} \\ &= \frac{\sqrt{a^2c^2 + b^2d^2 + b^2c^2 + a^2d^2}}{c^2+d^2} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \frac{|a+bi|}{|c+di|} \\ &= \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}} \times \frac{\sqrt{c^2+d^2}}{\sqrt{c^2+d^2}} \\ &= \frac{\sqrt{(a^2+b^2)(c^2+d^2)}}{c^2+d^2} \\ &= \frac{\sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2}}{c^2+d^2} \\ &= \frac{\sqrt{a^2c^2 + b^2d^2 + b^2c^2 + a^2d^2}}{c^2+d^2} \\ &= \text{LHS} \end{aligned}$$

~~$$\therefore \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$$~~

$$\therefore \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$$

### Short response

The following excerpt is from Question 17 from Paper 2. It required students to use a 90% confidence interval for the population mean to determine a 99% confidence interval based on the same data.

Effective student responses:

- used technology appropriately to determine the value of  $z$  for a 90% confidence interval
- recognised that the sample size needed to be an integer
- determined a 99% confidence interval.

This excerpt has been included:

- as it provides an example of using the formula book to obtain a suitable formula for upper value of the confidence interval
- as it correctly identifies the  $z$ -value for the 90% confidence interval
- to model clear communication of statistical information.

Given  $\bar{x} = 5206$  kg and  $s = 356$  kg

Consider the top interval 5252.9

$$P_{\text{upper}} = \bar{x} + z \frac{s}{\sqrt{n}}$$

For 90% confidence,  $z = 1.6449$

$$\text{Thus, } 5252.9 = 5206 + 1.6449 \times \frac{356}{\sqrt{n}}$$

From GDC nSolve,  $n = 155.887$

Thus,  $n = 156$  elephants ( $n \in \mathbb{Z}^+$ )

Thus, for 99% confidence,  $z = 2.5758$

$$CI = \left( \bar{x} \pm z \frac{s}{\sqrt{n}} \right) = \left( 5206 \pm 2.5758 \times \frac{356}{\sqrt{156}} \right)$$

$$CI = (5132.58 \text{ kg}, 5279.42 \text{ kg})$$

## Short response

The following excerpt is from Question 19 from Paper 1. It required students to determine a function given information about a point and the derivative of the function.

Effective student responses:

- successfully applied the integration by parts rule
- determined a suitable substitution variable and used the substitution method of integration
- determined a general solution for  $f(x)$
- determined a value of the constant of integration to obtain  $f(x)$ .

This excerpt has been included:

- as it clearly identifies  $u$  and  $\frac{dv}{dx}$  in preparation for their use in integration by parts
- as it defines a different variable in preparation for use in integration by substitution
- as it demonstrates the logical communication of key steps.



$$g(x) = f'(x) = e^x \sin^{-1}(e^x)$$

$$f(x) = \int e^x \sin^{-1}(e^x) dx$$

$$\text{Let } u = \sin^{-1}(e^x) \quad v' = e^x$$

$$u' = \frac{e^x}{\sqrt{1-e^{2x}}} \quad v = e^x$$

$$f(x) = uv - \int u'v dx$$

$$= e^x \sin^{-1}(e^x) - \int \frac{e^x x e^x}{\sqrt{1-e^{2x}}} dx$$

$$= e^x \sin^{-1}(e^x) - \int \frac{e^{2x}}{\sqrt{1-e^{2x}}} dx$$

$$\text{For } \int \frac{e^{2x}}{\sqrt{1-e^{2x}}} dx$$

$$\text{Let } m = 1 - e^{2x}$$

$$\frac{dm}{dx} = -2e^{2x}$$

$$dx = \frac{dm}{-2e^{2x}}$$

$$\therefore \int \frac{e^{2x}}{\sqrt{m}} \cdot \frac{dm}{-2e^{2x}}$$

$$= -\frac{1}{2} \int \frac{1}{\sqrt{m}} dm$$

$$= -\frac{1}{2} \int m^{-1/2} dm$$

$$= -\frac{1}{2} \times 2\sqrt{m} + C$$

$$= -\sqrt{m} + C$$

$$= -\sqrt{1-e^{2x}} + C$$

$$\therefore f(x) = e^x \sin^{-1}(e^x) - (-\sqrt{1-e^{2x}}) + C$$

$$f(x) = e^x \sin^{-1}(e^x) + \sqrt{1-e^{2x}} + C$$

The function passes through (0,0)

$$0 = e^0 \sin^{-1}(e^0) + \sqrt{1-e^0} + C$$

$$0 = 1 \sin^{-1}(1) + \sqrt{1-1} + C$$

$$0 = \frac{\pi}{2} + C$$

$$C = -\frac{\pi}{2}$$

$$\therefore f(x) = e^x \sin^{-1}(e^x) + \sqrt{1-e^{2x}} - \frac{\pi}{2}$$

## Short response

The following excerpt is from Question 18 from Paper 2. It required students to use information about polynomials  $P(z)$ ,  $Q(z)$  and their quotient to evaluate the reasonableness of a statement about a factor of  $P(z)$ .

Effective student responses:

- used the remainder theorem to determine an expression for  $P(2i)$
- used substitution to determine an expression for  $P(2i)$
- determined an expression for  $P(z)$
- used mathematical reasoning to determine the reasonableness of the statement.

This excerpt has been included:

- as it provides an example of using the remainder theorem rather than polynomial division
- as it provides an example where technology is used to show  $P(a - bi) \neq 0$
- as the mathematical reasoning is clearly communicated.

$$P(2i) = a - bi$$

$$(2i)^3 + (i-a)(2i)^2 - 2bi(2i) + 3i = a - bi$$

$$-8i - 4(i-a) + 4b + 3i = a - bi$$

$$-8i - 4i + 4a + 4b + 3i = a - bi$$

$$(4a + 4b) + (-8 - 4 + 3)i = a - bi$$

$$(4a + 4b) - 9i = a - bi$$

Equate coefficients:

$$\text{Im}(z): -9 = -b \quad \therefore b = 9$$

$$\text{Re}(z): 4a + 4b = a$$

$$4a + 4 \times 9 = a$$

$$3a = -36$$

$$a = -12$$

$$\therefore P(z) = z^3 + (i+12)z^2 - 18iz + 3i$$

If  $(z - (a - bi))$  is a factor then  $P(a - bi) = 0$

$$a - bi = -12 - 9i \quad \therefore P(-12 - 9i) = 0 \text{ if claim is true}$$

$$(-12 - 9i)^3 + (i+12)(-12 - 9i)^2 - 18i(-12 - 9i) + 3i = 1566 - 285i \neq 0$$

$$\therefore \text{Not reasonable that } (z - (a - bi)) \text{ is a factor of } P(z)$$

## Practices to strengthen

It is recommended that when preparing students for external assessment, teachers consider:

- providing opportunities to practise sketching complex numbers using the polar grid
- supporting students to make connections between the subject matter and the glossary terms so that terminology used to scaffold questions is familiar and students can provide clear and accurate explanations about their thinking
- encouraging students to become familiar with technology in order to recognise when it can be used effectively in Paper 2
- encouraging students to recognise the difference between the distribution of  $X$  compared to the distribution of  $\bar{X}$ .

## Additional advice

- If students require additional pages to complete a response, they should continue their solution on the additional pages at the back of the book. On the additional pages, they should clearly indicate the question they are responding to. If a second solution is completed, students must rule a diagonal line through the work that is incorrect.
- While not all questions will award marks for logical communication of key steps, students should be encouraged to logically communicate key steps, as this helps students in developing their solutions and assists markers when looking for evidence of specific concepts or techniques.
- Teachers should remind students to take care when simplifying algebraic terms, especially when expanding brackets with negative signs.
- Teachers should encourage students to become familiar with the Specialist Mathematics formula book.
- Where relevant, units should be included in the response.