LUI | | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your barcode ID label here

Book of books used

External assessment 2022

Question and response book

Specialist Mathematics

Paper 2 — Technology-active

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator **permitted.**
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 50 marks.

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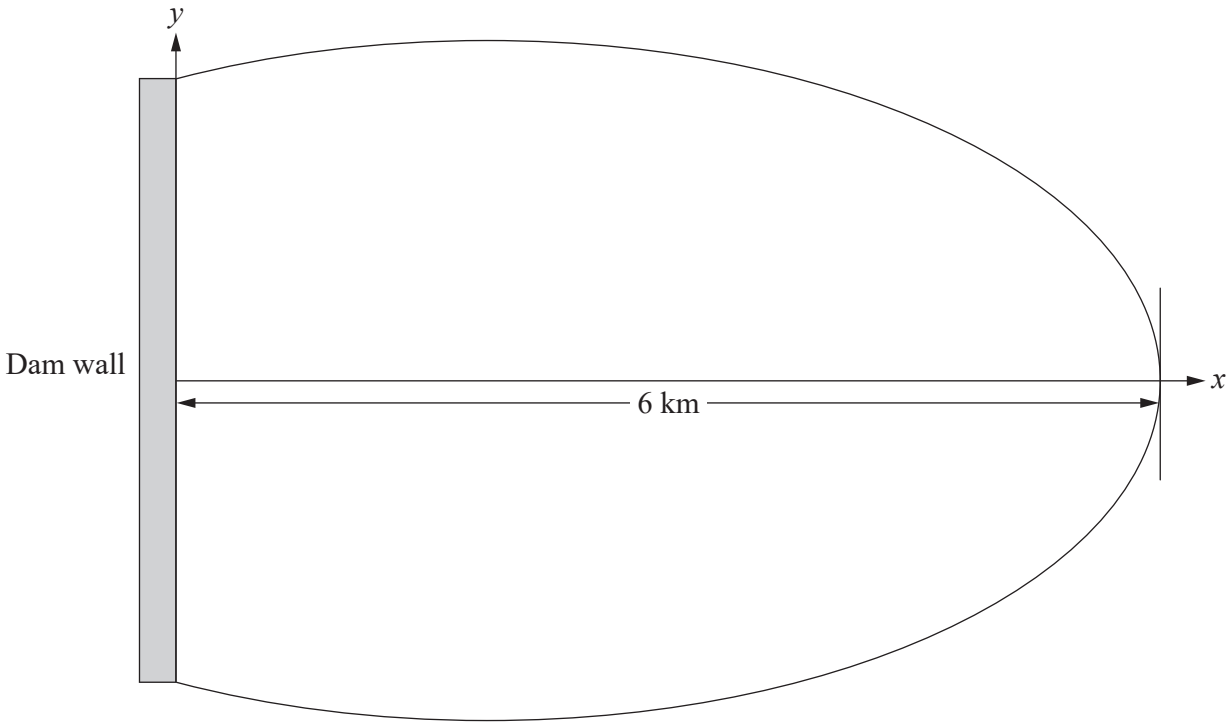
QUESTION 11 (6 marks)

An aerial view of the surface of a dam, 6 km in length, is symmetrically positioned on a Cartesian plane as shown. A dam wall is located along the y-axis.

The surrounding edge of the dam can be modelled by the ellipse $\frac{(x-2)^2}{16}+\frac{y^2}{9}=1$, for $0 \le x \le 6$.



Not to scale

a) Use Simpson's rule with four strips to determine an approximate area of the surface of the dam. *[4 marks]*

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b) Evaluate the reasonableness of this approximation. *[2 marks]*

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QUESTION 12 (5 marks)

A scientist collects data for a species of tree frog in a protected area. Details for the female tree frog population are shown in the table.

Age (years)	0–1	1–2	2–3	3–4
Population in Year 1	150	101	84	62
Birth (breeding) rate	0.4	0.7	0.5	0.1
Survival rate	0.6	0.3	0.2	0

The scientist uses a Leslie matrix model to make predictions about the female tree frog population.

a) State the initial population matrix. *[1 mark]*

b) Determine the Leslie matrix. *[1 mark]*

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A species is considered to be endangered if the female population in a restricted area is predicted to fall to less than 125 in the next 20 years.

c) Determine whether this species of tree frog is considered to be endangered. *[3 marks]*

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QUESTION 13 (5 marks)

An article claims that the mean starting salary of graduates in Australia is currently $64 800 with a standard deviation of $4500.

To check the validity of this claim, an employment agent intends to collect data on the starting salaries of a random sample of 360 graduates.

a) Determine the probability that the sample mean starting salary will be between $64 000 and $65 000. *[2 marks]*

From the data, the agent calculates a confidence interval for the population mean starting salary of ($64 589, $65 811).

b) Determine the sample mean. *[1 mark]*

c) Comment on the reasonableness of the article's claim based on this confidence interval. *[2 marks]*

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QUESTION 14 (5 marks)

An object is moving in a straight line with an acceleration represented by the differential equation $\frac{dv}{dt} = -\left(4 + v^2\right)$, where v is the object's velocity $\left(\text{m s}^{-1}\right)$ over time, $t\left(\text{s}\right)$, where $t \geq 0$, until it comes to rest.

a) Determine the general solution of the differential equation. *[3 marks]*

The initial velocity of the object is 1.5 m s^{-1}.

b) Determine the time when the particle comes to rest. *[2 marks]*

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QUESTION 15 (5 marks)

Consider points $A(3, -1, 3)$ and $B(1, 1, 6)$.

a) Determine $\overrightarrow{AB}$. *[1 mark]*

b) Determine the Cartesian equation of the line that passes through points A and B. *[2 marks]*

Point A lies on the plane, φ, and $\overrightarrow{AB}$ is perpendicular to this plane.

c) Determine the Cartesian equation of the plane. *[2 marks]*

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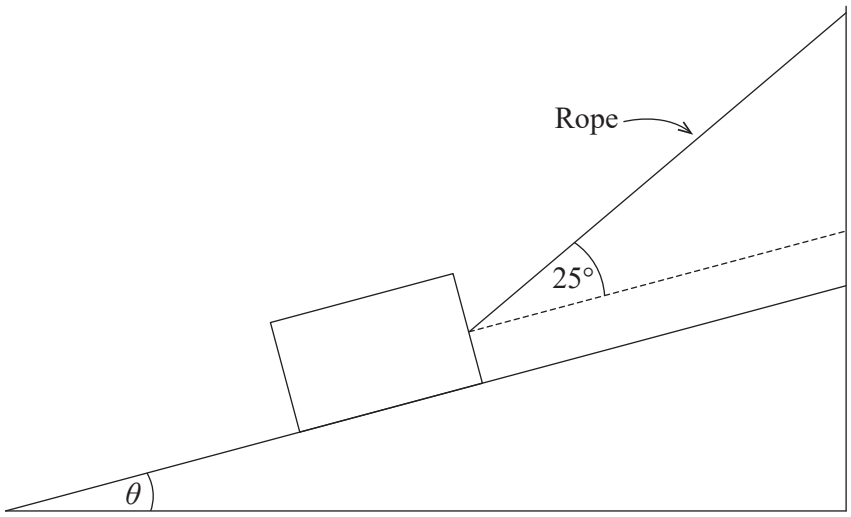
QUESTION 16 (6 marks)

An object with a mass of 12 kg lies on a frictionless inclined plane. A rope is attached to the object at an angle of 25° above the plane, as shown.



Not to scale

The force of the rope, $\boldsymbol{T}$ N, prevents the object from moving. When the rope is detached, the object moves down the plane with an acceleration of 5.6 m s^{-2}.

Determine the magnitude of $\boldsymbol{T}$.

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QUESTION 17 (6 marks)

The mass of a population of elephants is known to be normally distributed.

A biologist randomly selects a number of elephants from this population and measures their masses. The mean mass of the sample is 5206 kg with a standard deviation of 356 kg.

The biologist uses the data to calculate a 90% confidence interval for the population mean mass of (5159.1, 5252.9) kg.

Determine a 99% confidence interval for the population mean mass based on the same data.

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QUESTION 18 (5 marks)

Consider the polynomials $P(z) = z^3 + (i - a)z^2 - 2biz + 3i$ and $Q(z) = z - 2i$, where $a,\ b \in R$.

Given $\frac{P(z)}{Q(z)}$ has a remainder of $a - bi$, evaluate the reasonableness that $(z - (a - bi))$ is a factor of $P(z)$.

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QUESTION 19 (7 marks)

A research organisation plans to use a drone to drop a scientific instrument vertically from a stationary position above the ocean surface. The acceleration $\left(\text{m s}^{-2}\right)$ of the falling instrument can be modelled by $9.8 - 0.1v$, where v is its velocity $\left(\text{m s}^{-1}\right)$.

In order for the instrument sensors to activate, its speed as it hits the ocean surface must reach at least 20 m s^{-1}. However, if it hits with a speed above 50 m s^{-1}, the sensors will be damaged.

Determine the range of the drone's flying height above the ocean surface to ensure that the sensors are activated but not damaged.

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END OF PAPER

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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