LUI | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your
barcode ID label here

Book of books used

External assessment 2022

Question and response book

Specialist Mathematics

Paper 1 — Technology-free

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are **not** permitted.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	⬤	◯	◯	◯

	A	B	C	D
1.	◯	◯	◯	◯
2.	◯	◯	◯	◯
3.	◯	◯	◯	◯
4.	◯	◯	◯	◯
5.	◯	◯	◯	◯
6.	◯	◯	◯	◯
7.	◯	◯	◯	◯
8.	◯	◯	◯	◯
9.	◯	◯	◯	◯
10.	◯	◯	◯	◯

Do not write outside this box.

Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (6 marks)

The position vector of a particle, $\boldsymbol{r}_1(\text{cm})$, over time, $t(\text{s})$, is given by

$$\boldsymbol{r}_1(t) = (2t+1)\hat{\boldsymbol{i}} + (t+3)\hat{\boldsymbol{j}} - (2t-3)\hat{\boldsymbol{k}}$$

a) Determine the velocity vector of the particle. *[1 mark]*

b) Determine the time when the position vector of the particle is perpendicular to its velocity vector. *[2 marks]*

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The position vector of a second particle, $\boldsymbol{r}_2(\text{cm})$, over time, $t(\text{s})$, is given by

$$\boldsymbol{r}_2(t) = (16 - 4t)\hat{\boldsymbol{i}} - (3t - 13)\hat{\boldsymbol{j}} + 2\hat{\boldsymbol{k}}$$

c) Determine whether the two particles collide. *[3 marks]*

Do not write outside this box.

QUESTION 12 (6 marks)

Given $z_1 = a + bi,\ z_2 = c + di\ \forall a,\ b,\ c,\ d \in R$, and $z_2 \neq 0$, prove the identity

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

Do not write outside this box.

Do not write outside this box.

QUESTION 13 (6 marks)

a) Use partial fractions to determine $\int \frac{22}{(2x-3)(x+4)}\,dx$ *[4 marks]*

Do not write outside this box.

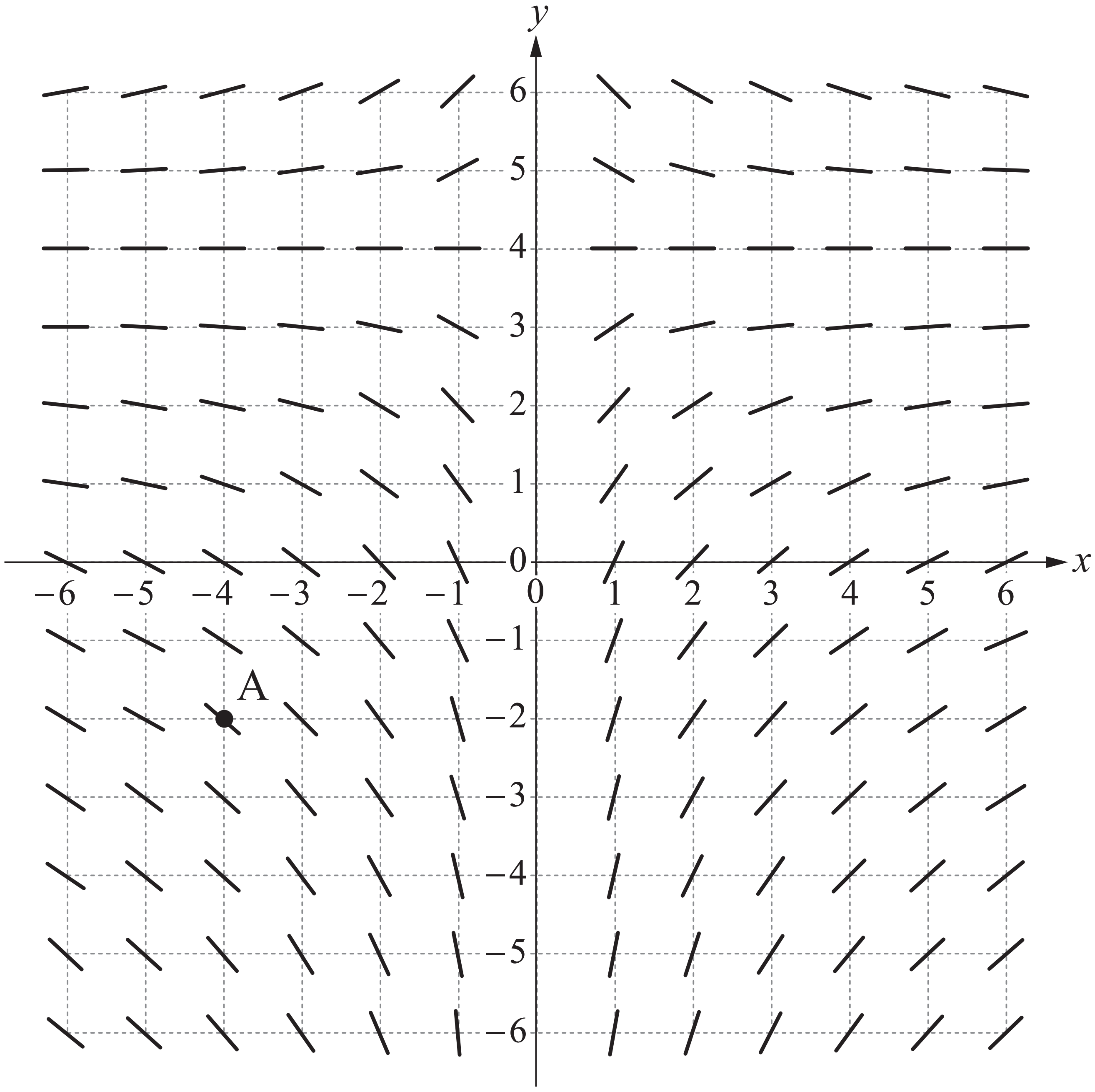
b) Use the result from Question 13a) to determine $\int_{-3}^{0} \frac{22}{(2x-3)(x+4)}\,dx$

Express your answer in simplest form. *[2 marks]*

Do not write outside this box.

QUESTION 14 (4 marks)

The slope field for the differential equation $\frac{dy}{dx} = \frac{-0.5(y-4)}{x}$, $x \neq 0$ using $-6 \leq x \leq 6$ and $-6 \leq y \leq 6$ is shown.



a) Determine the value of the slope at point A. *[2 marks]*

b) Use the slope field to sketch the solution curve for $\frac{dy}{dx} = \frac{-0.5(y-4)}{x}$ given that when $x = -6,\ y = 3.5$ *[2 marks]*

Note: If you make a mistake in the slope field, cancel it by ruling a single diagonal line through your work and use the additional response space on page 21 of this question and response book.

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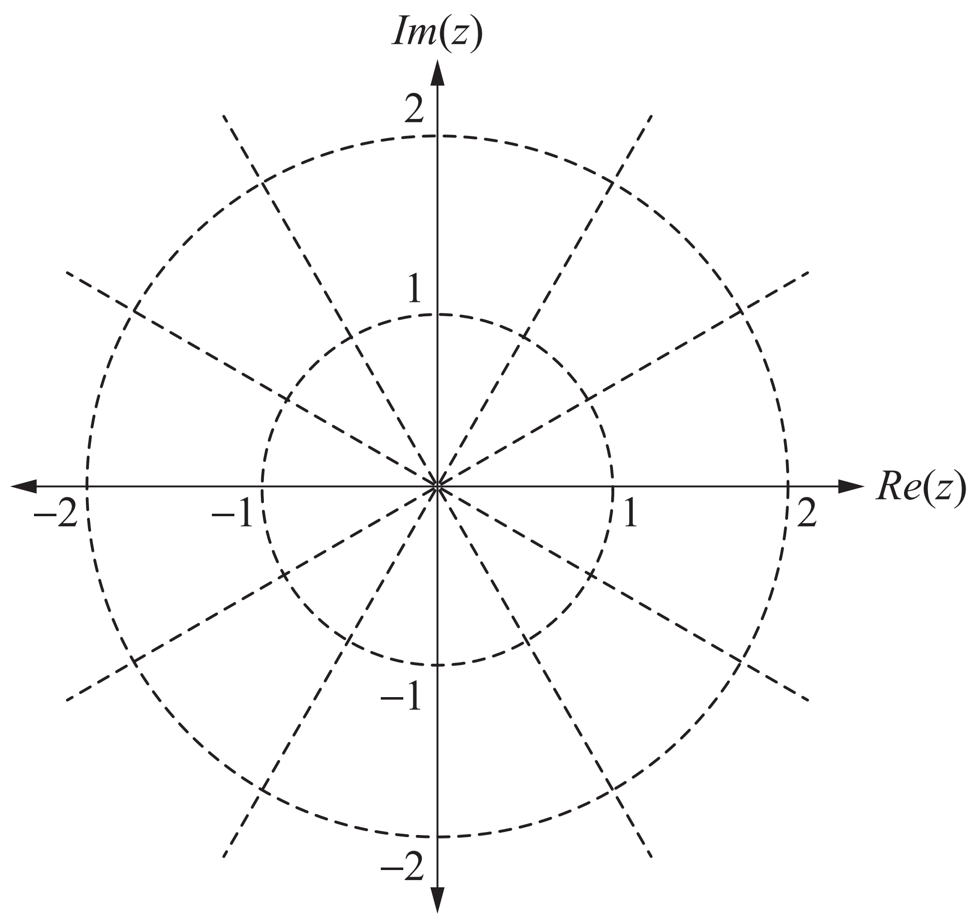
QUESTION 15 (4 marks)

Consider the equation $z^3 = 1$ where $z \in C$.

a) Sketch the solutions to $z^3 = 1$ on the Argand diagram. *[2 marks]*



Note: If you make a mistake in the Argand diagram, cancel it by ruling a single diagonal line through your work and use the additional response space on page 22 of this question and response book.

The solutions to $z^3 = 1$ can be expressed in the form $z = a + bi$, where $a,\ b \in R$.

b) Determine the largest possible positive value of ab. *[2 marks]*

Do not write outside this box.

QUESTION 16 (7 marks)

Consider this system of equations that corresponds to three planes.

$$x + 5y = 1 + 2z$$
$$x + z = 3y + 3$$
$$8y - \lambda = 3z$$

a) Use a Gaussian technique to determine the value of λ for which this system of equations has infinitely many solutions. *[4 marks]*

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b) Use the result from Question 16a) to determine the infinitely many solutions.
Express your answer in the form of a vector equation of a line. *[3 marks]*

Do not write outside this box.

QUESTION 17 (5 marks)

The region between the x-axis and the curve of the function $y = 1 + \sin(2x)$ for $0 \le x \le \frac{\pi}{2}$ is rotated about the x-axis to form a solid of revolution.

Determine the volume of this solid. Express your answer in simplest form.

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Do not write outside this box.

QUESTION 18 (5 marks)

It is proposed that the following expression is divisible by $\left(1+\operatorname{cis}(\theta)\right)$ for $n \in Z^{+}$, $\left(1+\operatorname{cis}(\theta)\right) \neq 0$.

$$\sum_{r=0}^{2n+1} \operatorname{cis}(r\theta)$$

Evaluate the reasonableness of the proposition.

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Do not write outside this box.

QUESTION 19 (7 marks)

The function $f(x)$ passes through the origin.

The gradient function of $f(x)$ is defined as $g(x) = e^x \sin^{-1}(e^x)$.

Determine $f(x)$.

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END OF PAPER

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Do not write outside this box.

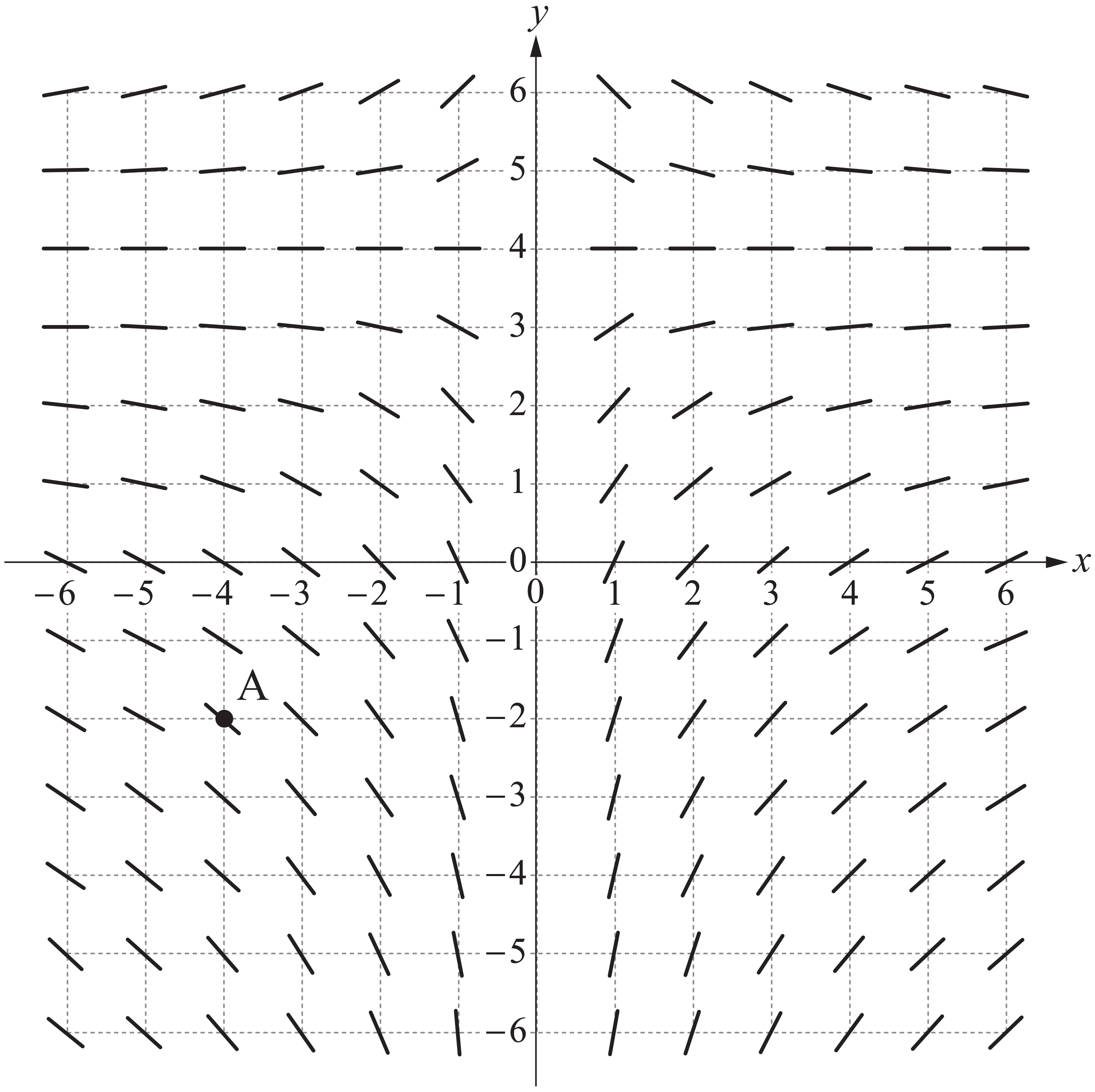
ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Do not write outside this box.

ADDITIONAL RESPONSE SPACE FOR QUESTION 14b)

If you want this slope field to be marked, rule a single diagonal line through the slope field on page 9.



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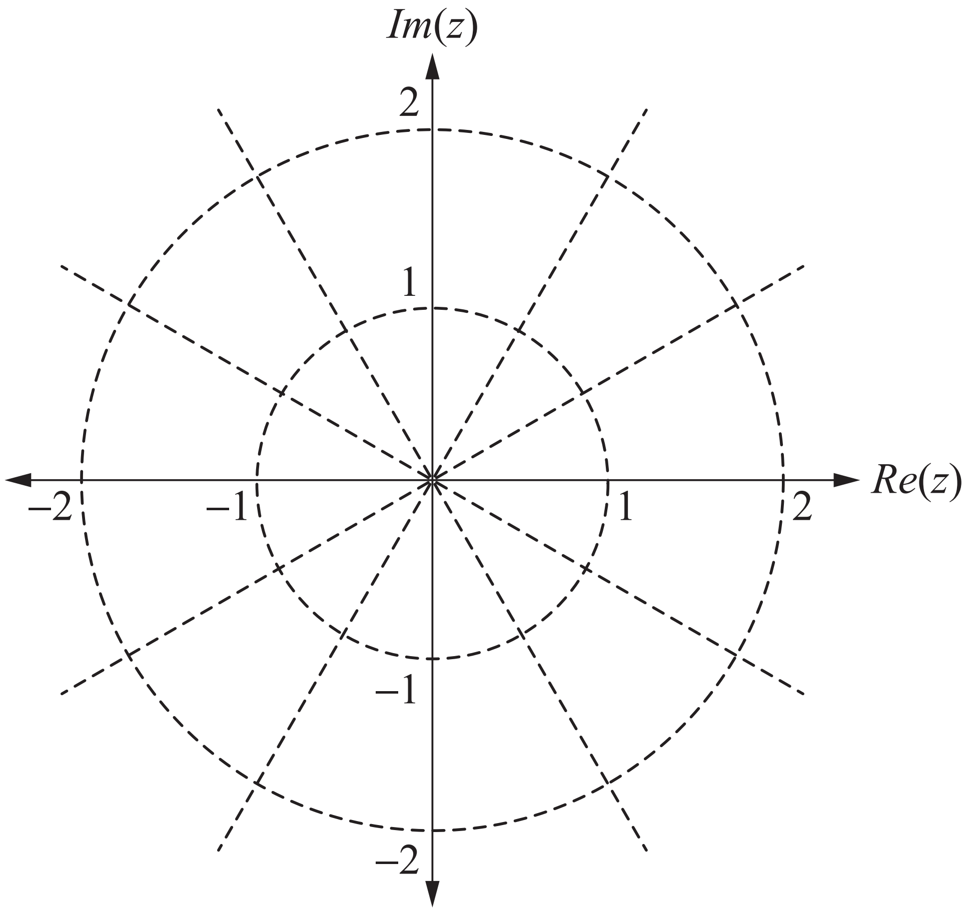
ADDITIONAL RESPONSE SPACE FOR QUESTION 15a)

If you want this Argand diagram to be marked, rule a single diagonal line through the Argand diagram on page 10.



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