Specialist Mathematics subject report

2021 cohort

February 2022

Queensland Government

QCAA Queensland Curriculum & Assessment Authority

220031

For all Queensland schools

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Contents

Introduction

Despite the challenges brought about by the COVID-19 pandemic, Queensland's education community can look back on 2021 with satisfaction at having implemented the first full assessment cycle in the new Queensland Certificate of Education (QCE) system. That meant delivering three internal assessments and one external assessment in each General subject.

This report analyses that cycle — from endorsing summative internal assessment instruments to confirming internal assessment marks, and designing and marking external assessment. It also gives readers information about:

- applying syllabus objectives in the design and marking of internal and external assessments
- patterns of student achievement.

The report promotes continuous improvement by:

- identifying effective practices in the design and marking of valid, accessible and reliable assessments
- recommending where and how to enhance the design and marking of valid, accessible and reliable assessment instruments
- providing examples of best practice where relevant, possible and appropriate.

Audience and use

This report should be read by school leaders, subject leaders and teachers to:

- inform teaching and learning and assessment preparation
- assist in assessment design practice
- assist in making assessment decisions
- help prepare students for external assessment.

The report is publicly available to promote transparency and accountability. Students, parents, community members and other education stakeholders can learn about the assessment practices and outcomes for General subjects (including alternative sequences (AS) and Senior External Examination (SEE) subjects, where relevant) and General (Extension) subjects.

Report preparation

The report includes analyses of data and other information from endorsement, confirmation and external assessment processes. It also includes advice from the chief confirmer, chief endorser and chief marker, developed in consultation with and support from QCAA subject matter experts.

Subject data summary

Subject completion

The following data includes students who completed the General subject or AS.

For the purposes of this report, while the 2021 summative units for the AS are AS units 1 and 2, this information will be included with the General summative Units 3 and 4.

Note: All data is correct as at 17 December 2021. Where percentages are provided, these are rounded to two decimal places and, therefore, may not add up to 100%.

Number of schools that offered the subject: 313.

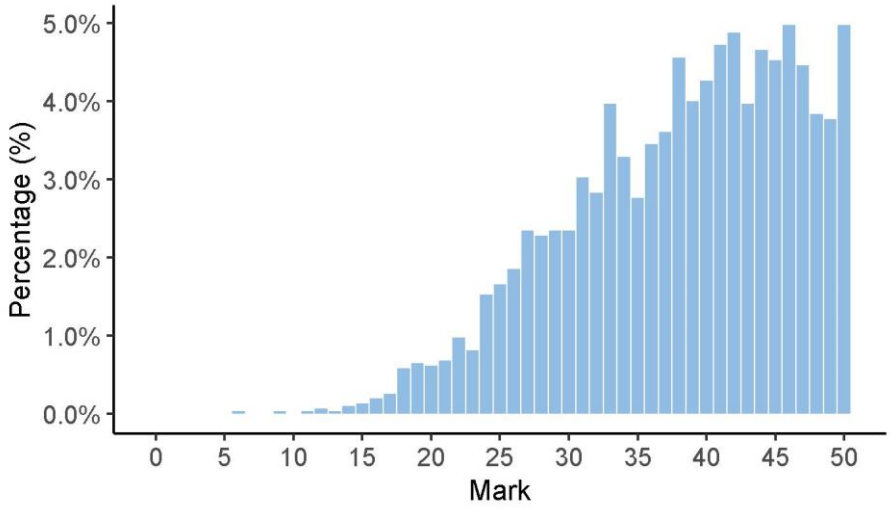
Completion of units	Unit 1	Unit 2	Units 3 and 4
Number of students completed	3577	3336	3061

Units 1 and 2 results

Number of students	Satisfactory	Unsatisfactory
Unit 1	3407	170
Unit 2	3052	284

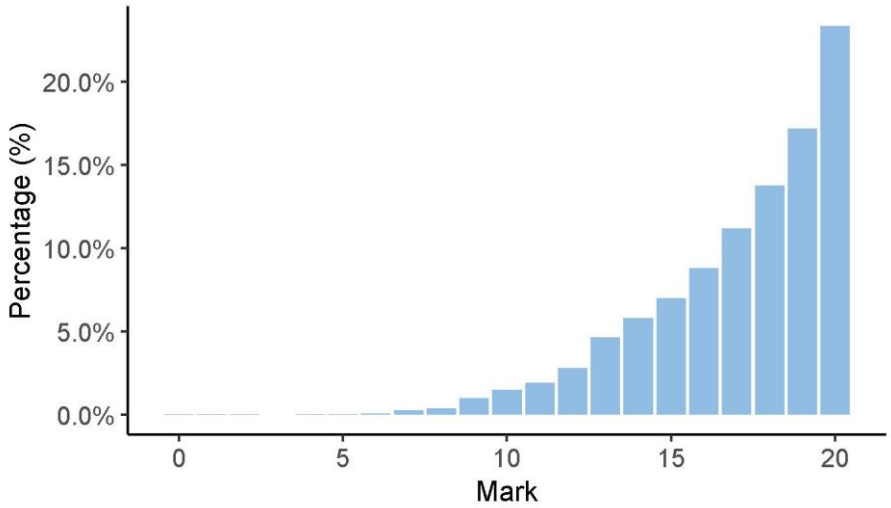
Units 3 and 4 internal assessment (IA) results

Total marks for IA

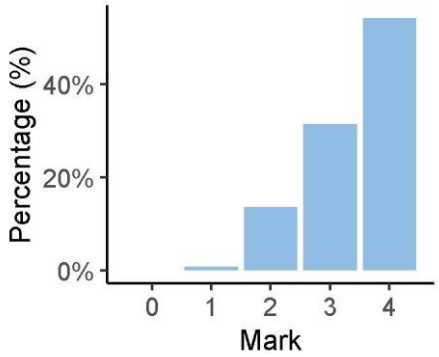


IA1 marks

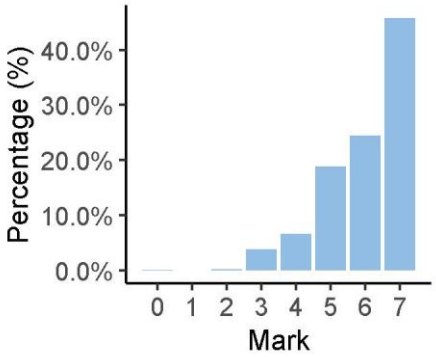
IA1 total



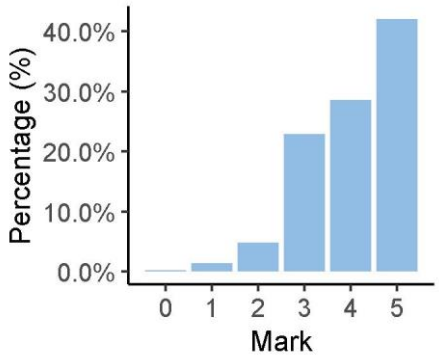
IA1 Criterion: Formulate



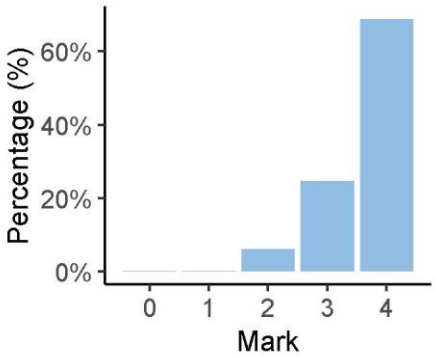
IA1 Criterion: Solve



IA1 Criterion: Evaluate and verify

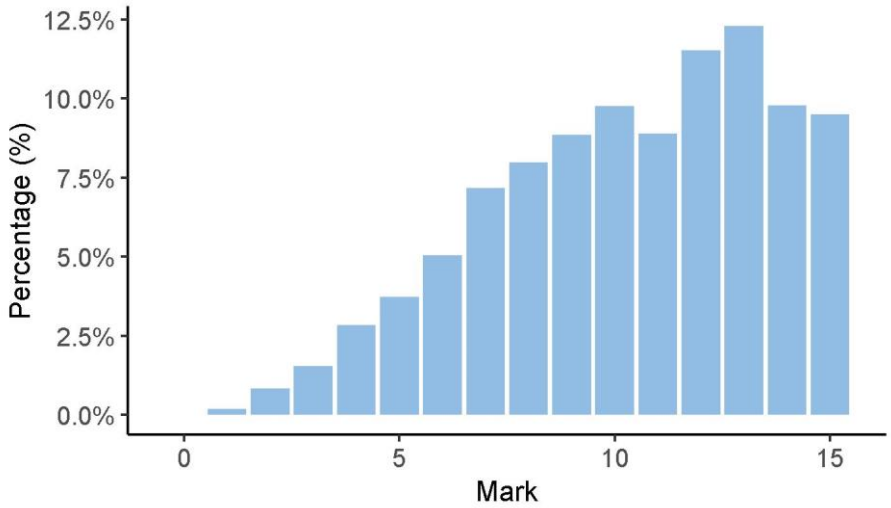


IA1 Criterion: Communicate

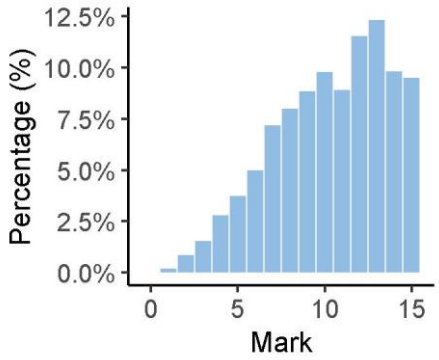


IA2 marks

IA2 total

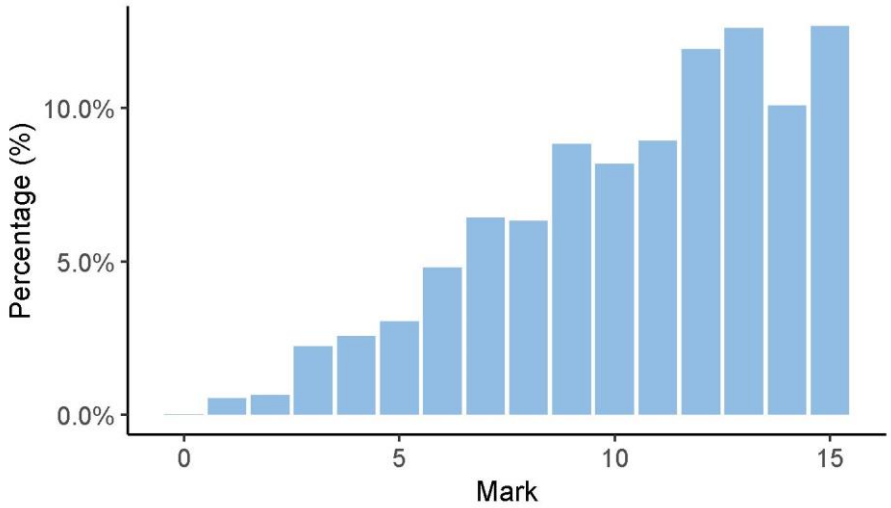


IA2 Criterion: Foundational knowledge and problem-solving

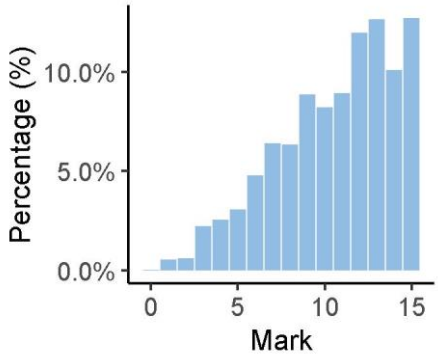


IA3 marks

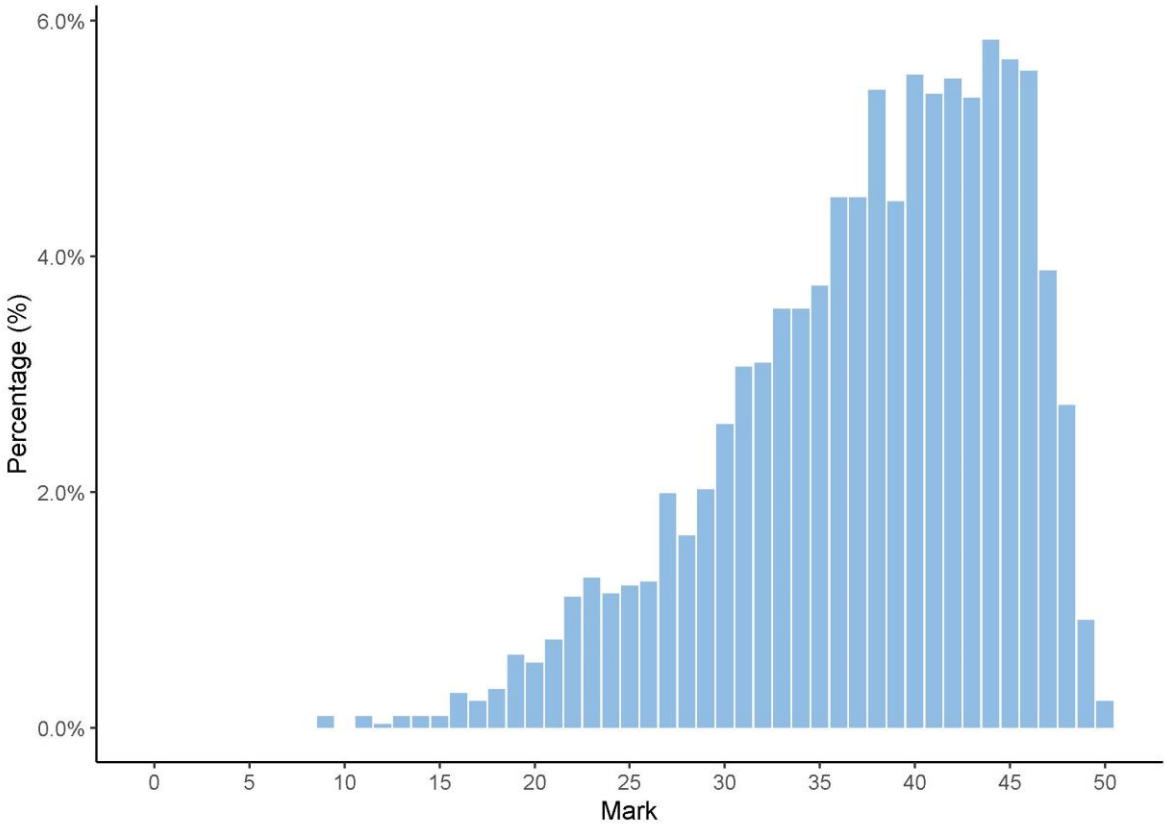
IA3 total



IA3 Criterion: Foundational knowledge and problem-solving

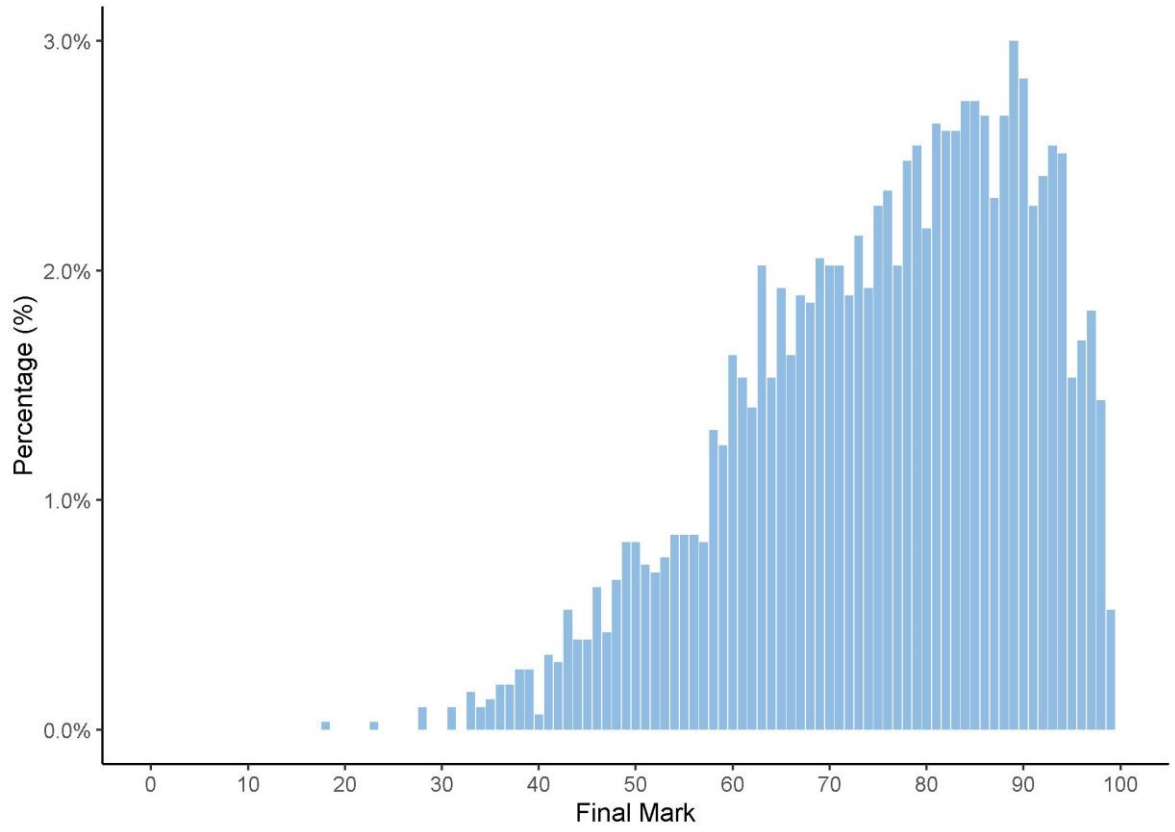


External assessment (EA) marks



Final subject results

Final marks for IA and EA



Grade boundaries

The grade boundaries are determined using a process to compare results on a numeric scale to the reporting standards.

Standard	A	B	C	D	E
Marks achieved	100–84	83–66	65–47	46–21	20–0

Distribution of standards

The number of students who achieved each standard across the state is as follows.

Standard	A	B	C	D	E
Number of students	1100	1199	636	125	1

Internal assessment

The following information and advice pertain to the assessment design and assessment decisions for each IA in Units 3 and 4. These instruments have undergone quality assurance processes informed by the attributes of quality assessment (validity, accessibility and reliability).

Endorsement

Endorsement is the quality assurance process based on the attributes of validity and accessibility. These attributes are categorised further as priorities for assessment, and each priority can be further broken down into assessment practices.

Data presented in the Assessment design section identifies the reasons why IA instruments were not endorsed at Application 1, by the priority for assessments. An IA may have been identified more than once for a priority for assessment, e.g. it may have demonstrated a misalignment to both the subject matter and the assessment objective/s.

Refer to the quality assurance tools for detailed information about the assessment practices for each assessment instrument.

Percentage of instruments endorsed in Application 1

Number of instruments submitted	IA1	IA2	IA3
Total number of instruments	314	314	314
Percentage endorsed in Application 1	48%	26%	39%

Confirmation

Confirmation is the quality assurance process based on the attribute of reliability. The QCAA uses provisional criterion marks determined by teachers to identify the samples of student responses that schools are required to submit for confirmation.

Confirmation samples are representative of the school's decisions about the quality of student work in relation to the ISMG and are used to make decisions about the cohort's results. If further information is required about the school's application of the ISMG to finalise a confirmation decision, the QCAA requests additional samples.

Schools may request a review where an individual student's confirmed result is different from the school's provisional mark in one or more criteria and the school considers this result to be an anomaly or exception.

The following table includes the percentage agreement between the provisional marks and confirmed marks by assessment instrument. The Assessment decisions section of this report for each assessment instrument identifies the agreement trends between provisional and confirmed marks by criterion.

Number of samples reviewed and percentage agreement

IA	Number of schools	Number of samples requested	Number of additional samples requested	Percentage agreement with provisional marks
1	306	1384	156	85.29%
2	306	1370	0	99.02%
3	306	1359	0	100%

Internal assessment 1 (IA1)

Problem-solving and modelling task (20%)

The problem-solving and modelling task must use subject matter from one or both of the following topics in Unit 3:

- Topic 2: Vectors and matrices
- Topic 3: Complex numbers 2.

For the Alternative Sequence, this problem-solving and modelling task must use subject matter from one or more of the following topics in AS unit 1:

- Topic 1: Combinatorics
- Topic 2: Vectors in the plane
- Topic 4: Vector applications in geometry.

The problem-solving and modelling task is an assessment instrument developed in response to a mathematical investigative scenario or context. It requires students to respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Students must provide a response to a specific task or issue that is set in a context that highlights a real-life application of mathematics. The task requires students to use relevant stimulus material involving the selected subject matter and must have sufficient scope to allow students to address all the stages of the problem-solving and modelling approach. Technology must be used.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	83
Authentication	28
Authenticity	36
Item construction	10
Scope and scale	66

*Each priority might contain up to four assessment practices.

Total number of submissions: 314.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided clear instructions for students about the requirements of the task
- identified the topics being assessed on the task sheet, but did not identify the actual method to be used
- had authentic real-life contexts that were accessible to students
- gave opportunities for students to provide a unique response. Examples included
  - open-ended tasks where a variety of approaches were expected
  - individual datasets
- included relevant stimulus where appropriate.

Practices to strengthen

It is recommended that assessment instruments:

- avoid scaffolding or instructions that direct students on how to formulate and solve the problem, e.g. develop a model to do … then use the model to …allow students to develop solutions at the complex level
- provide a clear indication of when and how teachers provide feedback on one draft. Students can be provided with a number of checkpoints for progress checking, but only one draft is submitted as per Section 8.2.5 of the *QCE and QCIA policy and procedures handbook*.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	8
Language	26
Layout	4
Transparency	16

*Each priority might contain up to four assessment practices.

Total number of submissions: 314.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- provided a context that was concise and was accessible to students.

Practices to strengthen

It is recommended that assessment instruments:

- phrase questions in context, using clear, concise language and avoiding jargon that did not contribute to the understanding of the subject matter
- are free from spelling and punctuation errors
- include a reference to the approach to problem-solving and mathematical modelling flowchart from the syllabus (Section 4.6.1).

Additional advice

- It is recommended that schools develop instruments that allow to students to authentically follow the problem-solving and modelling flowchart (Syllabus section 4.6.1) to
  - translate all aspects of the problem themselves
  - be discerning in their application of mathematical concepts and techniques
  - make their own judgments about ideas and consider the strengths and limitations of their responses
  - organise and 'develop' the response independently.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Formulate	89.22%	9.15%	0.65%	0.98%
2	Solve	95.1%	3.92%	0.98%	0%
3	Evaluate and verify	93.46%	5.56%	0.33%	0.65%
4	Communicate	96.08%	2.29%	1.63%	0%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- in the Solve criterion, making judgments about the proficiency level for the use of technology. ‘Accurate and appropriate use of technology’ was well identified from ‘use of technology’ and ‘superficial use of technology’
- in the Evaluate and verify criterion, making judgments about the proficiency level for decision-making. Giving reasons or evidence as ‘justification of decisions’ made using mathematical reasoning was well identified from making ‘statements about decisions’

- in the Communicate criterion, making judgments about the correct use of technical vocabulary, procedural vocabulary and conventions that add detail and fullness to the student's response, as well as making judgments about the organisation and structure of the response with respect to its genre.

Samples of effective practices

The following are excerpts from responses that illustrate the characteristics for the criterion at the performance level indicated. The excerpts may provide evidence of more than one criterion. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

These student response excerpts have been included:

- to demonstrate 'documentation' of assumptions and observations.

Formulate (3–4 marks)
- documentation of appropriate assumptions

Excerpt 1

2.2 Assumptions

Documentation of appropriate assumptions ;
Accurate documentation of relevant observations.

Throughout this report, it has been assumed that:

- That females make up 50% (half) of the total population. This assumes that there are equal numbers of female and male sharks (Evens, 2019). Without this assumption, the report would be invalid because the final population would not be accurate and can impact the amount that the model would need a change to decrease and increase the population. It would also affect the proportions of the age groups.
- That all females can reproduce. Not all females can reproduce, this is due to a series of different possibilities. Female sharks are also able to reproduce without a male (Little, 2017). This means that fertility rates can increase more. However, for this report, it has been assumed the breeding rates are steady and that there is no variation over the different age groups throughout the modelled time period. Without this assumption, the solution would be invalid because the population would grow and decrease at different rates if there was a variation within the breeding rates.
- That a mercury spill would affect the species the way the model has shown. Mercury (Hg) has serious impacts on marine animals' health, in particular, it affects the shark's muscle and nervous systems. For the Australian Sharp-nose shark, high levels of mercury within the water and the shark's body can lead to oxidative stress in the shark's central nervous system, which affects the shark's muscles and brain (Gelsleichter, 2019). It can be assumed that this can then lead to a decrease in survival rates due to the information on how a shark's nervous system slowly starts to shut down from an increased level of Mercury. From this, it was further assumed that it will also have an impact on their breeding rates due to the assumption that sick sharks will be significantly less likely to reproduce, without this assumption the solution would be invalid as the catastrophe would have no major effect on the population.

Formulate (3–4 marks)
- accurate documentation of relevant observations

Excerpt 2

2.0 Considerations

2.1 Observations

To sufficiently solve the task, the following observations were made.

- The model for population growth will be constructed using Leslie matrices. Leslie matrices are an age-structured modelling method that utilises birth and survival rates (Verma & Singh, 2017). This is advantageous as the data given contains the birth and death rates for structured age groups, allowing for a Leslie matrix to project population growth more accurately than other methods. Moreover, they will also be used as they can also effectively project age distribution (Smith, Trout, 1994).
- As Leslie matrices model long term growth, the short-term populations predicted by them fluctuate largely. This is relevant to the task as it means that population growth cannot be determined until a stable long-term growth rate is reached (Verma & Singh, 2017).
- The given data given contains the initial female population, birth rate and death rate.
- As survival rates are used in Leslie matrices, the death rates given in the data will need to be converted to survival rates for their usage (Smith, Trout, 1994).
- Due to the population being sorted in four-month age groups, the Leslie matrices will determine the population growth per 4-months instead of annually. This is relevant to the task as it impacts the growth rate and number of rabbits that will need to be culled.
- Initially, the majority of rabbits are in the young population. This is relevant to the task as the size and growth of the young population significantly influences the total population growth.
- The age group of 0-4 months do not produce any offspring due to their birth rate of 0.
- The age group of 4-8 months have the highest birth rates of 1.5.

These student response excerpts have been included:

- to demonstrate ‘evaluation of reasonableness’.

Evaluate and verify (4–5 marks)

- evaluation of the reasonableness of solutions by considering the results, assumptions and observations

Excerpt 1

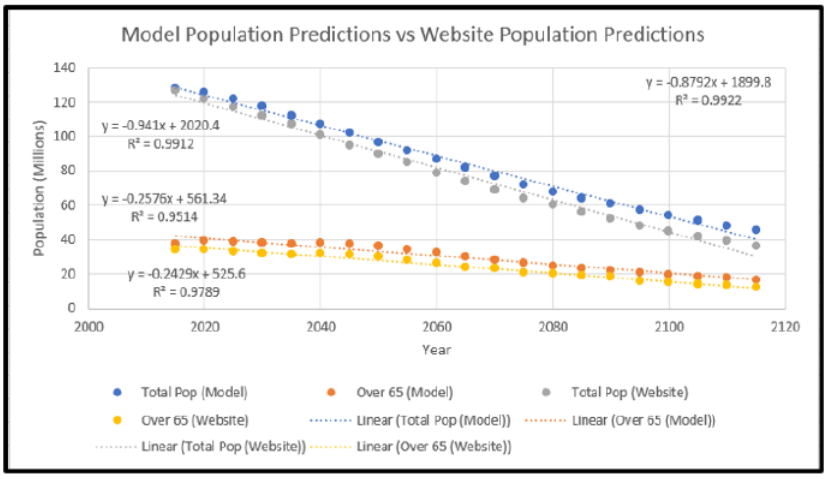
EVALUATION:

The following data, produced by the model created by ‘The Habitable Planet’, has been collated to compare to the above model and be compared for accuracy:

Table 7: Simulation total population, over 65-year-old population, and percentage of population over the age of 65 in Japan for the next 100 years (in 5-year increments). Populations are listed in millions of citizens.

	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060	2065	2070	2075	2080	2085	2090	2095	2100	2105	2110	2115
Total Pop	127	122	117	112	107	101	95	90	85	79	74	69	64	60	56	52	48	45	42	39	36
Over 65	34	34	33	32	31	32	31	30	28	26	24	23	21	20	19	18	16	15	14	13	12
Percentage	26.772%	27.869%	28.205%	28.571%	28.972%	31.683%	32.632%	33.333%	32.941%	32.911%	32.432%	33.333%	32.813%	33.333%	33.929%	34.615%	33.333%	33.333%	33.333%	33.333%	33.333%

Graph 3 displays a comparison between the two models:



Graph 3: Comparison between the above model's predictions and the given simulations predictions for total population and over 65 population in Japan until the year 2115.

Both total population predictions follow a linear trend, both with R^2 values greater than 0.99. The linear nature of the models reinforces their validity as it is evident there are no drastic or unrealistic changes in population over the 100-year period, as would be expected. However, it is evident this model would eventually fail as the total population would soon dip below zero:

$$y = -0.8792x + 1899.8$$

$$Substitute\ y = 0$$

$$0 = -0.8792x + 1899.8$$

$$-1899.8 = -0.8792x$$

$$x = \frac{-1899.8}{-0.8792}$$

$$x = 2160.8$$

It can be seen in *Graph 4* that the patterns followed by the two sets of data are very similar, both increasing until the year 2055 before dropping off slightly. While the model then plateaus at around 36.3% after the year 2075, the website simulation predicts a small spike around the year 2090 before plateauing again at approximately 33.3%. The final difference in percentages is just 3.05%, or a 9.15% error, again very similar results given the large time-period. There is, however, several reasons for the variation between models:

Leslie matrices are one of the best methods for predicting population change amongst a certain population, however they require a variety of assumptions to become viable (Jones, 2008). Firstly, it was observed the model did not account for immigration and emigration; it was therefore assumed this did not impact the population in any way. However, this is an unreasonable assumption to make, as emigration and immigration are inevitable in countries such as Japan and would influence their populations, hence decreasing the reasonableness of the model.

The assumption that the birth and survival rates did not change over the 100-year period was again unrealistic because factors such as medicine and fertility rates amongst populations are constantly changing. Additionally, it was assumed the rounded values from the online simulation would have no impact on calculations and results, however, after having the numbers manipulated and the small uncertainties in rounding compounded over a 100-year period, disparities occurred, reducing the reasonableness of the model.

When calculating the total populations from the female populations the proportion used (51.17% of population is female) was found through research. However, the original data presented by the online simulation suggested the proportion was 51.51% female and hence the female values should have been multiplied by $\frac{1}{0.5151} = 1.9411$ rather than the original 1.95427. This would have had minimal impact on the conclusions drawn, however, could have some impact on the small discrepancies between the two models.

A paper published by the JIPSSR estimates the Japanese population will be approximately 40 million in the year 2115, an over 65 population of roughly 14 million, and hence a percentage of the population over the age of 65 at about 35% (IPSS, 2002). The devised model suggests the population in the year 2115 will be approximately 45.5 million, an over 65 population of roughly 16.5 million, and hence a percentage of the population over the age of 65 at about 36%. This 1% discrepancy between the estimates of the JIPSSR and the Leslie matrix model after 100 years indicates a high degree of reasonableness and reliability.

Strengths	Limitations
The model was tested up to 100 years into the future and remained similar to 'The Habitable Planet' prediction (within 9% error in percentage of population over 65) over the 100-year time-period. Additionally, the model was corroborated by the JIPSSR, with just a 1% difference in prediction of percentage of population over the age of 65.	The model relies on the rounded data provided by 'The Habitable Planet'. These rounded values will have had implications on the conclusions drawn after 20 time-steps of matrix multiplication.
The model uses a 5-year time step to accurately identify the trend that best fits the data and make the most reliable predictions on future populations.	As discussed above, many unreasonable assumptions, such the birth and survival rates will not change over time, immigration and emigration have no impact, and exactly 50% of births are female, are unrealistic and limit the reasonableness of the model.
The linear trends identified verified there were no drastic changes to the course of the total population as would be expected in a large population.	The linear trend identified is only accurate for the given time-period, because as seen above, the population will theoretically reach negatives. If the matrix were to be continued further into the future, a more accurate trend would have been identified.
The required data for the Leslie matrix was successfully extracted from the simulation, such as female birth rate, and the survival rates of the individual age demographics.	

Excerpt 2

The average correlation coefficient for model 1 (0.9048) indicates that the rankings found for each of the three years reflects the win/loss results of the teams and therefore the Official NRL Ladder (observation 2), despite (assumption 1). Whilst model 1 is an improvement from the NRL rank as it considers the number of times each team plays each other, it still contradicts assumption 5, and does not account for other impacting factors. To improve the model, more complex weighting vectors (b) could be used, such as logarithmic and exponential, or could be altered to incorporate the time or location of game play (Sakai, n.d.). Evaluation using assumptions

Although refined model 2 produced the lowest average correlation coefficient (0.7540), it demonstrated a strong correlation with the 2020 data (0.9286). This suggests that top-scoring players, injuries, home/away advantage, and score difference greatly influenced the win/loss results of teams in 2020 (evident as it closely correlated with the official NRL rank which only uses win/loss results), however not as much in 2014 and 2017. Ultimately, the model found that these factors do not have a significant impact on the success of teams during a "normal" season (observation 1). To improve the model, more factors, such as turn-around time, coach changes, time of game and environment of game play could also be considered. Furthermore, examining the weightings added to the R vector could also produce a more accurate rank, i.e., putting more weighting on second dominance, rather than first. Documentation of S & L

The main strength of model 3 is that it satisfied observation 3 and produced ranks for 2017 and 2020 that closely correlated with the Official NRL Ladder (0.9048 and 0.8571 respectively). However, the rank established for 2014 was not closely correlated (0.6194), which indicates that perhaps the established relative importance of the performance factors is not an accurate indication of what is most important for team success. To extend model 3, key players from each team could be examined to develop a more accurate relative importance rank for performance factors, i.e., look at the half-backs from each team, given that receipts and playmaking are the most important factors for these players (Rugby League Base, 2013).

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- within both the Formulate criterion and the Evaluate and verify criterion, a clear distinction is made between 'statements' and 'documentation' of observations, assumptions and/or strengths and limitations. In both these criteria
  - responses that clearly demonstrate 'documentation' of assumptions provide evidence of the logic of the mathematisation process, as identified in the flowchart (Syllabus section 4.6.1). While this can often take the form of references or citations, it can also be achieved with coherent reasoning, e.g. I am assuming that the surface is smooth so that friction can be ignored
  - for 'documentation' of observations, supporting evidence could include the source of the observations, the reasoning for the method used to collect the data, or the validity of the observations with respect to the proposed solution and/or model
  - for 'documentation' of strengths and limitations, there needs to be supporting evidence as to why or how the elements identified are strengths or limitations. Where necessary, this will involve refinement of a model and/or solution
- within the Evaluate and verify criterion, a clear distinction is made between 'statements' and 'evaluation of reasonableness'. In this criterion
  - responses that clearly demonstrate an 'evaluation' include consideration of the original real-world problem to be solved, the effects of making the assumptions used to mathematise the problem, as well as both strengths and limitations of the model and/or results
  - an 'evaluation of reasonableness' can take many forms but should involve making an appraisal or judgment supported by reasoning. It could include use of technology, further analysis of data, or algebraic calculations to refine the output of their model and/or check that it provides a valid solution to the problem

- within the Communicate criterion, a 'coherent' response has a logically consistent structure consisting of parts that connect harmoniously. In this criterion
  - responses that clearly demonstrate ‘coherent’ organisation can be read independently of the task sheet and use appropriate structure (e.g. headings and subheadings for a report, labels on diagrams or graphs) and language to make the formulation, solving and evaluation of a solution easy to perceive, understand and interpret. The reader should be free from confusion when following the response.

Additional advice

- It is recommended that, to further ensure accuracy and consistency of the application of the ISMG in this IA, schools continue to deepen their understanding of the syllabus holistically.
  - The PSMT should provide evidence of how well students have achieved the cognitions identified in the objectives of the course (Syllabus section 1.2.1).
  - The Approach to problem-solving and modelling gives context for the proficiency levels and descriptors in the ISMG (Syllabus section 1.2.4).
  - The glossary provides definitions and clarification for specific terminology in the ISMG (Syllabus section 6).
- It is recommended that, when annotating student work, teachers use the terminology from the ISMG in a way that supports the student's understanding of what was required.

Internal assessment 2 (IA2)

Examination (15%)

The examination assesses the application of a range of cognitions to a number of items using a representative sample of subject matter from all Unit 3 (AS unit 1) topics. Where relevant, the focus of this assessment should be on subject matter not assessed in the problem-solving and modelling task. Subject matter from Units 1 and 2 (AS units 3 and 4) is considered assumed knowledge. It is also assumed that work covered in Mathematical Methods will be known before it is required in Specialist Mathematics. Student responses must be completed individually, under supervised conditions, and in a set timeframe (120 minutes plus 5 minutes perusal). The percentage allocation of marks must match the degree of difficulty specifications (~20% complex unfamiliar; ~20% complex familiar; ~60% simple familiar).

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	255
Authentication	0
Authenticity	4
Item construction	22
Scope and scale	76

*Each priority might contain up to four assessment practices.

Total number of submissions: 314.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided questions that assessed a representative sample of Unit 3 topics (AS unit 1) that reflect the intended learning
- explicitly provided opportunities for students to demonstrate all assessment objectives
- had an appropriate number of questions for students to respond to in the time conditions.

Practices to strengthen

It is recommended that assessment instruments:

- provide complex questions where relationships and interactions have a number of elements. These questions should avoid the use of scaffolding (e.g. into Parts a), b), c)) as this reduces the complexity of the question
- provide complex unfamiliar questions that match the syllabus description for both complexity and unfamiliarity
- provide students the opportunity to respond to Assessment objective 4: evaluate the reasonableness of solutions and allocate appropriate marks in the marking scheme accordingly
- provide questions where the focus is on Unit 3 (AS unit 1) subject matter only (standalone questions using only subject matter from Unit 1 or 2 (AS unit 3 or 4) are not suitable)
- include a marking scheme with an indication of how marks will be awarded.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	9
Language	88
Layout	32
Transparency	23

*Each priority might contain up to four assessment practices.

Total number of submissions: 314.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- were free of punctuation, spelling and other errors
- provided simple familiar questions where the required procedure is clear from the way the problem is posed
- used correct mathematical notation, e.g. vectors written in the form $\boldsymbol{a}$, $\underset{\sim}{a}$ or $\overrightarrow{AB}$.

Practices to strengthen

It is recommended that assessment instruments:

- are reviewed using the Print Preview button before submitting, to ensure the layout is appropriate
- used the language of the assessment objectives, e.g. ‘evaluate the reasonableness of solutions’.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	99.02%	0.65%	0.33%	0%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- the marking guide explicitly stated the cognitive processes from the syllabus objectives for which marks were to be awarded, rather than just placing ticks on a sample solution
- annotations on student responses clearly aligned with the allocation of marks on the school's submitted marking guide or, where the student response was different, justification for the mark awarded was provided
- schools clearly identified the total marks awarded for each question on the exam
- schools clearly identified the total marks awarded for the entire instrument and the corresponding percentage on the ISMG
- schools applied the ISMG without rounding the calculated percentage.

Samples of effective practices

The following is an excerpt from a response that illustrates the characteristics for the criterion at the performance level indicated. The excerpt may provide evidence of more than one criterion. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

This excerpt has been included:

- to demonstrate effective annotations that show the total marks awarded to each question on the examination.

Excerpt 1

Examination Marks Summary:

Question	1	2	3	4	5	6	7	8	9	10	
SF	7/7	9/10	10/10	10/10	8/9	8/8	-	-	-	-	52
CF	-	-	-	-	-	-	8/9	9/9	-	-	17
CU	-	-	-	-	-	-	-	-	1/9	3/9	4

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- the same marking guide is used consistently across the cohort when awarding marks
- schools ensure their marking guide accurately matches all questions in the assessment, including any changes from endorsement and updated solutions if errors are identified during marking
- a separate marking guide is provided for comparable assessment
- schools show evidence of the percentage calculation to determine the final mark awarded
- if an error in an endorsed examination was noted post-implementation, evidence that permission has been granted by the QCAA to make changes to the examination has been provided.

Internal assessment 3 (IA3)

Examination — short response (15%)

The examination assesses the application of a range of cognitions to a number of items using a representative sample of subject matter from all Unit 4 (AS unit 2) topics. Subject matter from Units 1, 2 and 3 (AS units 3, 4 and 1) is considered assumed knowledge. It is also assumed that work covered in Mathematical Methods will be known before it is required in Specialist Mathematics. Student responses must be completed individually, under supervised conditions, and in a set timeframe (120 minutes plus 5 minutes perusal). The percentage allocation of marks must match the degree of difficulty specifications (~20% complex unfamiliar; ~20% complex familiar; ~60% simple familiar).

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	213
Authentication	0
Authenticity	0
Item construction	11
Scope and scale	36

*Each priority might contain up to four assessment practices.

Total number of submissions: 314.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided questions that assessed Unit 4 topics (AS unit 2)
- included questions that explicitly provided opportunities to address all assessment objectives
- provided a correct marking scheme with appropriate allocation of marks
- included a balance of technology-free and technology-active questions. For examinations where the students have access to technology for the entire paper, appropriate cues were provided to direct students to use an analytical procedure, e.g. use algebraic techniques to integrate the following.

Practices to strengthen

It is recommended that assessment instruments:

- explicitly assess subject matter from Unit 4 (AS unit 2) with subject matter from Mathematical Methods and previous units as assumed knowledge only
- provide cues for students to use calculus techniques where relevant, rather than using Physics formulas
- provide questions where the focus is on Unit 4 (AS unit 2) subject matter only (standalone questions using only subject matter from Unit 1, 2 or 3 (AS unit 3, 4 or 1) are not suitable)
- provide students the opportunity to respond to Assessment objective 4: evaluate the reasonableness of solutions and allocate appropriate marks in the marking scheme accordingly.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	4
Language	70
Layout	9
Transparency	26

*Each priority might contain up to four assessment practices.

Total number of submissions: 314.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- were free of punctuation, spelling and other errors
- provided simple familiar questions where the required procedure is clear from the way the problem is posed
- for assessment where students have access to technology for the entire paper, it is clearly stated for questions when technology cannot be used, e.g. where the question indicated that an analytical procedure is required
- used correct mathematical notation.

Practices to strengthen

It is recommended that assessment instruments:

- are reviewed using the Print Preview button before submitting to ensure the layout is appropriate
- used the language of the assessment objectives, e.g. ‘evaluate the reasonableness of solutions'.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	100%	0%	0%	0%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

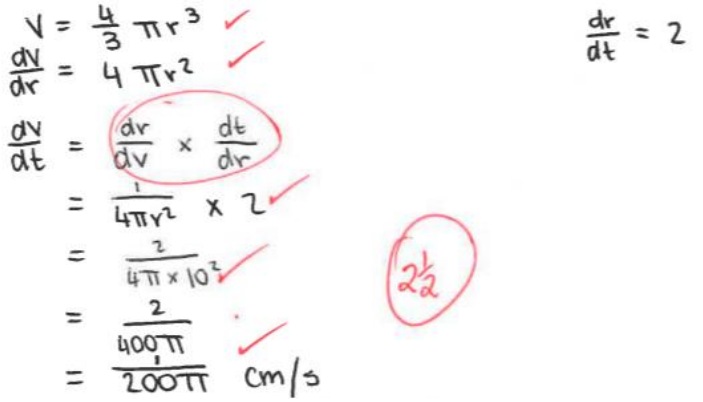
- the marking guide explicitly stated the cognitive processes from the syllabus objectives for which marks were to be awarded, rather than just placing ticks on a sample solution
- annotations on student responses clearly aligned with the allocation of marks on the school's submitted marking guide or, where the student response was different, justification for the mark awarded was provided
- schools clearly identified the total marks awarded for each question on the exam
- schools clearly identified the total marks awarded for entire instrument and the corresponding percentage on the ISMG
- schools applied the ISMG without rounding the calculated percentage.

Samples of effective practices

The following is an excerpt from a response that illustrates the characteristics for the criterion at the performance level indicated. The excerpt may provide evidence of more than one criterion. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

This student response excerpt has been included:

- as the marks awarded for the question are clear and the evidence used to allocate the marks is clearly annotated.

Foundational knowledge and problem-solving (2.5 of a possible 4 marks)	Excerpt 1 

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- the same marking guide is consistently used across the cohort when awarding marks
- schools ensure their marking guide accurately matches all questions in the assessment, including any changes from endorsement and updated solutions if errors are identified during marking
- a separate marking guide is provided for comparable assessment
- schools show evidence of the percentage calculation to determine the final mark awarded
- if an error in an endorsed examination was noted post-implementation, evidence that permission has been granted by the QCAA to make changes to the examination has been provided.

External assessment

External assessment (EA) is developed and marked by the QCAA. The external assessment for a subject is common to all schools and administered under the same conditions, at the same time, on the same day.

Summative external assessment — Examination (50%)

Assessment design

The assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the syllabus. The examination assessed subject matter from Units 3 and 4.

The examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (55 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (55 marks).

The AS assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the AS. The AS examination assessed subject matter from AS units 1 and 2. The AS examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (55 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (55 marks).

Assessment decisions

Assessment decisions are made by markers by matching student responses to the external assessment marking guide (EAMG). The external assessment papers and the EAMG are published in the year after they are administered.

General multiple choice item responses

There were 10 multiple choice items in Paper 1 and 10 multiple choice items in Paper 2.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Paper 1

Question	A	B	C	D
1	14.17	**46.41**	31.86	7.39
2	**63.96**	18.72	13.76	2.77
3	5.92	2.84	**88.71**	2.33
4	8.38	**85.56**	1.1	4.79
5	26.01	27.21	20.09	**26.04**
6	17.49	8.76	**57.46**	15.57
7	0.55	**67.35**	12.63	19.27
8	34.15	4.52	**59.92**	1.27
9	**57.97**	10.03	7.02	24.71
10	19.85	9.45	20.6	**49.76**

Paper 2

Question	A	B	C	D
1	7.4	29.63	**49.67**	12.26
2	5.96	15.24	7.98	**69.89**
3	**59.61**	12.74	11.41	15.66
4	23.91	3.53	**68.07**	4.15
5	7.47	10.59	11.24	**70.09**
6	**51.97**	13.46	27.27	5.86
7	8.7	**68.04**	15.76	6.71
8	15.79	**64.54**	13.6	5.38
9	20.83	5.1	**68.82**	4.69
10	16.55	**44.23**	24.29	14.18

AS multiple choice item responses

There were 10 multiple choice items in Paper 1 and 10 multiple choice items in Paper 2.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

AS Paper 1

Question	A	B	C	D
1	10.57	**64.23**	11.38	13.82
2	**30.08**	36.59	23.58	9.76
3	6.5	1.63	**88.62**	3.25
4	2.44	**73.17**	18.7	5.69
5	17.07	25.2	21.95	**35.77**
6	13.82	10.57	**50.41**	24.39
7	27.64	**29.27**	32.52	10.57
8	18.7	3.25	**75.61**	2.44
9	**57.72**	8.13	23.58	10.57
10	**37.4**	40.65	11.38	9.76

AS Paper 2

Question	A	B	C	D
1	11.48	4.1	18.03	**64.75**
2	8.2	15.57	9.84	**66.39**
3	**50**	13.11	18.85	17.21
4	9.02	**61.48**	20.49	8.2
5	7.38	11.48	16.39	**63.11**
6	**38.52**	10.66	43.44	6.56
7	9.02	**59.84**	22.13	9.02
8	6.56	19.67	**62.3**	10.66
9	5.74	36.07	**48.36**	9.02
10	5.74	32.79	29.51	**31.97**

Effective practices

Overall, students responded well to:

- opportunities to demonstrate knowledge and understanding of the subject matter mathematical induction for proving divisibility results in familiar situations
- opportunities to use technology to justify the reasonableness of solutions, particularly with the use of Simpson's rule
- opportunities to demonstrate knowledge and understanding of Leslie matrices. Students were aware that questions worth more than one mark required working and in general did so.

The following excerpts have been selected to illustrate effective student responses in one or more of the syllabus assessment objectives. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

Samples of effective practices

Short response

Item: Question 12 — Paper 1 (Technology-free)

Assessment objectives: 1, 2 and 3 — simple familiar

This question required students to demonstrate their knowledge and understanding of Vector and Cartesian equations from Unit 3 Topic 2: Vectors and matrices.

Effective student responses:

- identified a vector perpendicular to a given plane
- determined the vector equation of a line perpendicular to a plane and containing a given point
- expressed the vector equation in parametric form
- determined the point of intersection of the line and plane
- demonstrated that two vectors were parallel.

This student response excerpt has been included:

- as it provides evidence that students were aware that questions worth one mark required minimal to no working, but questions worth more than one mark required working or justification
- to demonstrate the use of the scalar multiple to verify that vectors are parallel. Students could also have demonstrated that the cross product of the vectors is zero.

Foundational knowledge and problem-solving (0–8 marks)

Excerpt 1

a) Determine a vector $\boldsymbol{n}$ that is perpendicular to the plane. *[1 mark]*

$n = \hat{i} - \hat{j} - 2\hat{k}$

b) Determine the vector equation of the line l that is perpendicular to the plane and contains the point A$(-2, 1, 3)$. *[1 mark]*

$l = A + kn$

$l = -2\hat{i} + \hat{j} + 3\hat{k} + k(\hat{i} - \hat{j} - 2\hat{k})$

c) Use the result from Question 12b) to express the equation of the line l in parametric form. *[1 mark]*

$x = -2 + k$

$y = 1 - k$

$z = 3 - 2k$

d) Show that the coordinates of S are $(2, -3, -5)$. *[3 marks]*

Let $x = 2$,

$2 = -2 + k$,

$k = 4$, Sub. parametrics into plane,

$(-2 + k) - (1 - k) - 2(3 - 2k) = 15$,

Sub. $k = 4$

$2 - (-3) - 2(-5) = 15$

$2 + 3 + 10 = 15$

$15 = 15$.

Thus, they intersect at $x = 2$.

At which point the line is

$x = 2$, $y = 1 - 4 = -3$, $z = 3 - 2(4) = -5$

$(2, -3, -5)$

e) Determine $\overrightarrow{AS}$. *[1 mark]*

$\overrightarrow{AS} = 2\hat{i} - 3\hat{j} - 5\hat{k} - (-2\hat{i} + \hat{j} + 3\hat{k})$

$= 4\hat{i} - 4\hat{j} - 8\hat{k}$

f) Use a property of parallel vectors to verify that $\overrightarrow{AS}$ and $\boldsymbol{n}$ are parallel. *[1 mark]*

$\overrightarrow{AS} = 4(\hat{i} - \hat{j} - 2\hat{k})$

$n = \hat{i} - \hat{j} - 2\hat{k}$

$n = 4\overrightarrow{AS}$

$\therefore$ They are parallel as one is multiplied by a scalar.

Item: Question 13 — Paper 1 (Technology-free)

Assessment objectives: 1, 2 and 3 — simple familiar

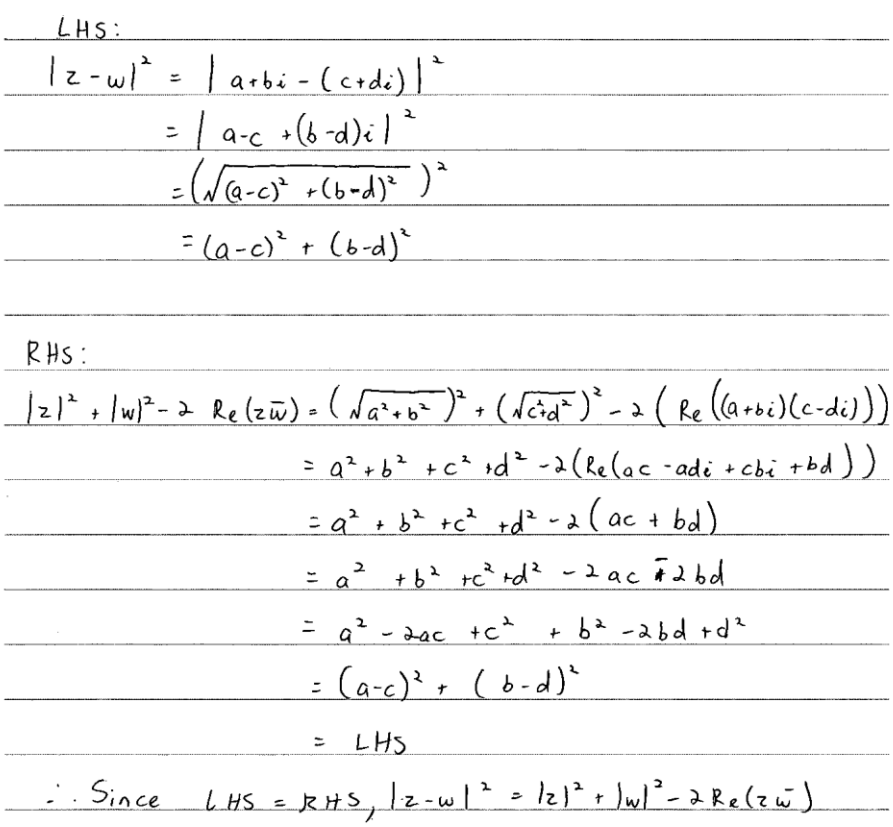
This question required students to construct a proof for a complex number identity.

Effective student responses:

- recognised that it was necessary to express the complex numbers in Cartesian form
- constructed supporting arguments in the form of a proof
- showed logical organisation, communicating key steps.

This student response excerpt has been included:

- as it provides an example of simplifying the left-hand side of the proof and then simplifying the right-hand side of the proof
- because the key steps are clearly communicated, e.g. $z-w$ is written in Cartesian form so it is clear how the modulus is calculated
- as it demonstrates accurate algebraic skills.

Foundational knowledge and problem-solving (0–6 marks)	**Excerpt 1**  LHS: $\|z-w\|^2 = \|a+bi-(c+di)\|^2$ $= \|a-c+(b-d)i\|^2$ $= \left(\sqrt{(a-c)^2+(b-d)^2}\right)^2$ $= (a-c)^2+(b-d)^2$ RHS: $\|z\|^2+\|w\|^2-2\,Re(z\bar{w}) = \left(\sqrt{a^2+b^2}\right)^2+\left(\sqrt{c^2+d^2}\right)^2-2\left(Re\left((a+bi)(c-di)\right)\right)$ $= a^2+b^2+c^2+d^2-2(Re(ac-adi+cbi+bd))$ $= a^2+b^2+c^2+d^2-2(ac+bd)$ $= a^2+b^2+c^2+d^2-2ac-2bd$ $= a^2-2ac+c^2+b^2-2bd+d^2$ $= (a-c)^2+(b-d)^2$ $=$ LHS $\therefore$ Since LHS = RHS, $\|z-w\|^2 = \|z\|^2+\|w\|^2-2Re(z\bar{w})$

Item: Question 17 — Paper 2 (Technology-active)

Assessment objectives: 4, 5 and 6 — complex familiar

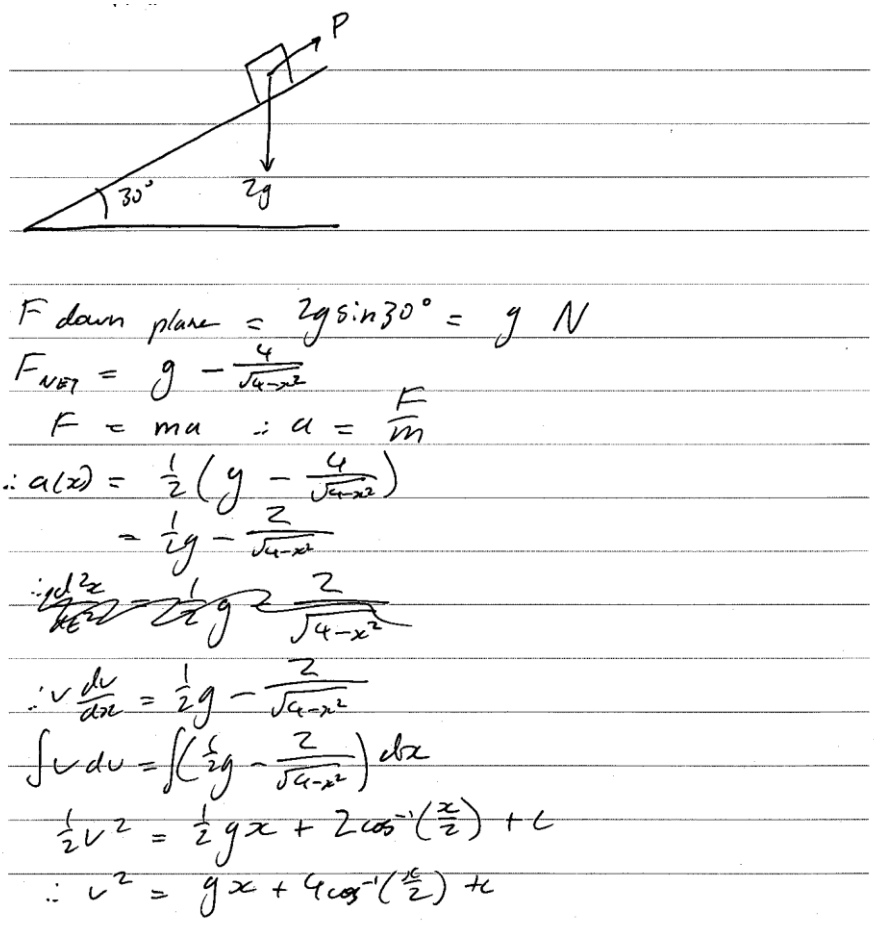
This question required students to determine the position of an object given an equation for force in terms of displacement.

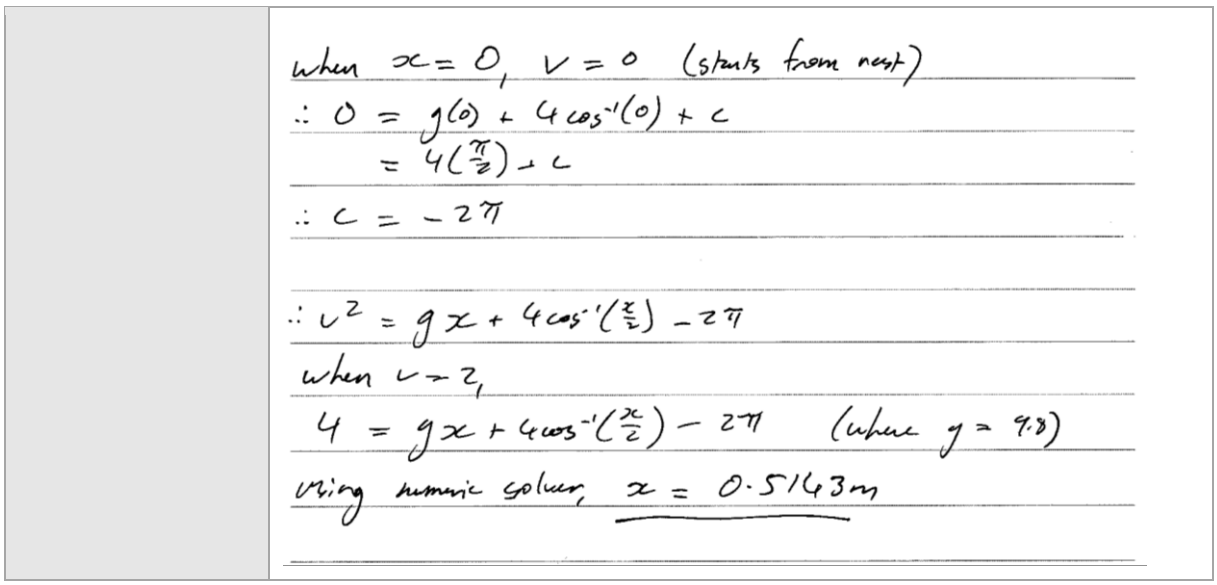
Effective student responses:

- determined the net forces along the plane and used this to obtain an expression for acceleration
- recognised that acceleration was expressed in terms of displacement and that it was necessary to used either $a = v\frac{dv}{dx}$ or $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
- developed and solved a differential equation in terms of velocity and displacement
- determined the position for a given velocity.

This student response excerpt has been included:

- to demonstrate how the use of a diagram assisted the student to clarify the information provided in order to accurately determine the net forces parallel to the plane
- as an example of appropriate use of technology to determine the displacement.

Foundational knowledge and problem-solving (0–7 marks)	**Excerpt 1**  $F_{down\ plane} = 2g\sin 30° = g\ N$ $F_{NET} = g - \frac{4}{\sqrt{4-x^2}}$ $F = ma \quad \therefore a = \frac{F}{m}$ $\therefore a(x) = \frac{1}{2}\left(g - \frac{4}{\sqrt{4-x^2}}\right)$ $= \frac{1}{2}g - \frac{2}{\sqrt{4-x^2}}$ $\therefore \frac{d^2x}{dt^2} = \frac{1}{2}g - \frac{2}{\sqrt{4-x^2}}$ $\therefore v\frac{dv}{dx} = \frac{1}{2}g - \frac{2}{\sqrt{4-x^2}}$ $\int v\,dv = \int\left(\frac{1}{2}g - \frac{2}{\sqrt{4-x^2}}\right)dx$ $\frac{1}{2}v^2 = \frac{1}{2}gx + 2\cos^{-1}\left(\frac{x}{2}\right) + c$ $\therefore v^2 = gx + 4\cos^{-1}\left(\frac{x}{2}\right) + c$



Item: Question 19 — Paper 2 (Technology-active)

Assessment objectives: 4, 5 and 6 — complex unfamiliar

This question required students to use a given probability density function to determine the probability that a sample mean is greater than a given value.

Effective student responses:

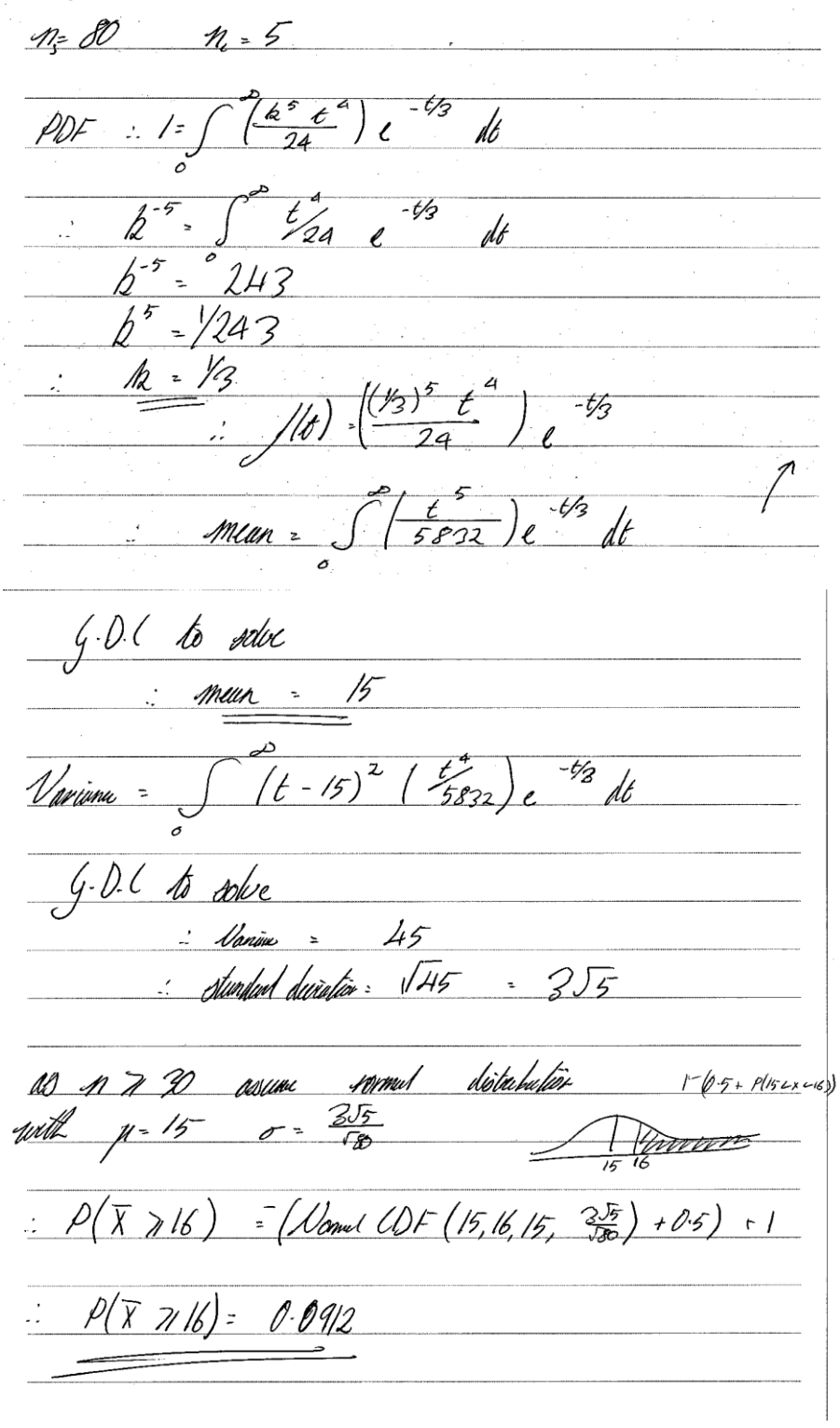
- used the given Mathematical Methods formulas to determine the population mean and standard deviation
- recognised that the distribution of sample means will be approximately normal due to the sample size
- determined the mean and standard deviation of the sample
- used the cumulative normal distribution to determine the probability.

This student response excerpt has been included:

- to demonstrate efficient use of technology
- to show evidence of how the method is justified by observing that the large sample size means that the distribution of sample means is approximately normal.

Foundational knowledge and problem-solving (0–7 marks)

Excerpt 1



Practices to strengthen

It is recommended that when preparing students for external assessment, teachers consider:

- supporting students to recognise cues embedded in the question, e.g.
  - some simple familiar questions are scaffolded to step students through the question. The word ‘show’ is used so that students can make progress even if they are unable to

complete some parts of the question, e.g. in Paper 1 Question 12, students are asked to verify the coordinates of point S in 12d), meaning that they can potentially progress to 12e) and 12f) even if they are unable to complete 12d).

  - when students are asked for an explanation, the response space available is indicative of the required response length. It should not be necessary to exceed this space

- encouraging students to ensure that they have explicitly answered the question, e.g. in Paper 2 Question 12, students were asked if an assumption was needed. Students were able to discuss the link between large sample size and the assumption of normality for the distribution of sample means, but they also needed to conclude whether the assumption was required or not
- supporting students to make connections between the subject matter and the glossary terms so that terminology such as use ‘matrix algebra’ to solve a system of equations is familiar to students
- supporting students to recognise that when variables are defined over a particular domain, they should consider this domain when completing their solution (see Paper 1 Question 11)
- supporting students to use knowledge from Mathematical Methods, Units 1 and 2 and the P–10 Australian curriculum for these reasons
  - Paper 1 Question 15 required students to have knowledge of logarithm laws
  - formulas from Mathematical Methods are given where needed (see Paper 2 Question 19)
  - algebraic skills are required. Students should be encouraged to take care when expanding brackets (see Paper 1 Question 19)
  - where relevant, units should be included
- encouraging students to evaluate the reasonableness of their solutions during the solution process. This may occur by rejecting a negative solution, choosing an appropriate level of accuracy for intermediate results, or considering if an additional solution is possible
- enhancing students’ abilities in their use of a graphics calculator in the technology-active section of the examination, e.g. recognising that often a numerical solver will only provide one solution and students need to consider if a second solution is possible, perhaps using a graphical technique.