Specialist Mathematics General Senior Syllabus 2019 v1.2

Subject report 2020

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Queensland Curriculum & Assessment Authority

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For all Queensland schools

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Introduction

The first summative year for the new Queensland Certificate of Education (QCE) system was unexpectedly challenging. The demands of delivering new assessment requirements and processes were amplified by disruptions to senior schooling arising from the COVID-19 pandemic. This meant the new system was forced to adapt before it had been introduced — the number of summative internal assessments was reduced from three to two in all General subjects. Schools and the QCAA worked together to implement the new assessment processes and the 2020 Year 12 cohort received accurate and reliable subject results.

Queensland's innovative new senior assessment system combines the flexibility and authenticity of school-based assessment, developed and marked by classroom teachers, with the rigour and consistency of external assessment set and marked by QCAA-trained assessment writers and markers. The system does not privilege one form of assessment over another, and both teachers and QCAA assessors share the role of making high-stakes judgments about the achievement of students. Our commitment to rigorous external quality assurance guarantees the reliability of both internal and external assessment outcomes.

Using evidence of student learning to make judgments on student achievement is just one purpose of assessment. In a sophisticated assessment system, it is also used by teachers to inform pedagogy and by students to monitor and reflect on their progress.

This post-cycle report on the summative assessment program is not simply being produced as a matter of record. It is intended that it will play an active role in future assessment cycles by providing observations and findings in a way that is meaningful and helpful to support the teaching and learning process, provide future students with guidance to support their preparations for summative assessment, and promote transparency and accountability in the broader education community. Reflection and research are necessary for the new system to achieve stability and to continue to evolve. The annual subject report is a key medium for making it accessible to schools and others.

Background

Purpose

The annual subject report is an analysis of the previous year's full summative assessment cycle. This includes endorsement of summative internal assessment instruments, confirmation of internal assessment marks and external assessment.

The report provides an overview of the key outcomes of one full teaching, learning and assessment cycle for each subject, including:

- information about the application of the syllabus objectives through the design and marking of internal and external assessments
- information about the patterns of student achievement in each subject for the assessment cycle.

It also provides advice to schools to promote continuous improvement, including:

- identification of effective practices in the design and marking of valid, accessible and reliable assessments
- identification of areas for improvement and recommendations to enhance the design and marking of valid, accessible and reliable assessment instruments
- provision of tangible examples of best practice where relevant, possible and appropriate.

Audience and use

This report should be read by school leaders, subject leaders and teachers to inform teaching and learning and assessment preparation. The report is to be used by schools and teachers to assist in assessment design practice, in making assessment decisions and in preparing students for external assessment.

The report is publicly available to promote transparency and accountability. Students, parents, community members and other education stakeholders can learn about the assessment practices and outcomes for General subjects (including alternative sequences and Senior External Examination subjects, where relevant) and General (Extension) subjects.

Report preparation

The report includes analyses of data and other information from the processes of endorsement, confirmation and external assessment, and advice from the chief confirmer, chief endorser and chief marker, developed in consultation with and support from QCAA subject matter experts.

Subject data summary

Subject enrolments

Number of schools offering the subject: 325.

Completion of units	Unit 1	Unit 2	Units 3 and 4*
Number of students completed	3228	3255	3252

*Units 3 and 4 figure includes students who were not rated.

Units 1 and 2 results

Number of students	Satisfactory	Unsatisfactory	Not rated
Unit 1	3134	90	4
Unit 2	3070	181	4

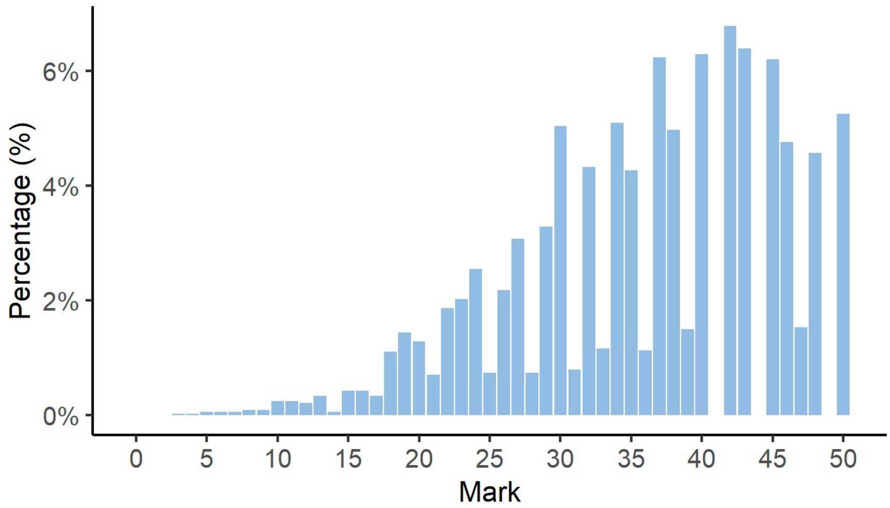
Units 3 and 4 internal assessment results

2020 COVID-19 adjustments

To support Queensland schools, teachers and students to manage learning and assessment during the evolving COVID-19 pandemic in 2020, the QCAA Board approved the removal of one internal assessment for students completing Units 3 and 4 in General and Applied subjects.

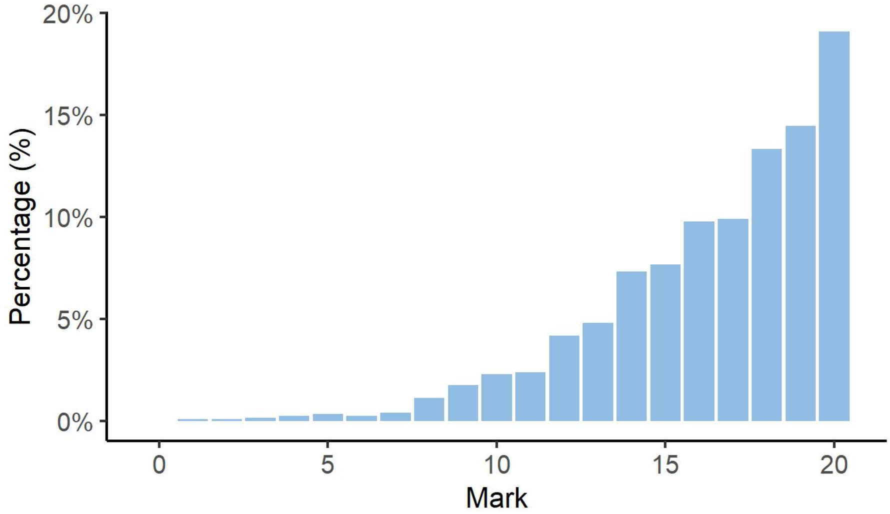
In General subjects, students completed two internal assessments and an external assessment. Schools made decisions based on QCAA advice and their school context. Therefore, across the state some instruments were completed by most schools, some completed by fewer schools and others completed by few or no schools. In the case of the latter, the data and information for these instruments has not been included.

Total results for internal assessment

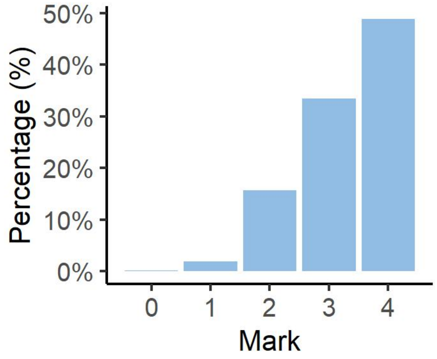


IA1 results

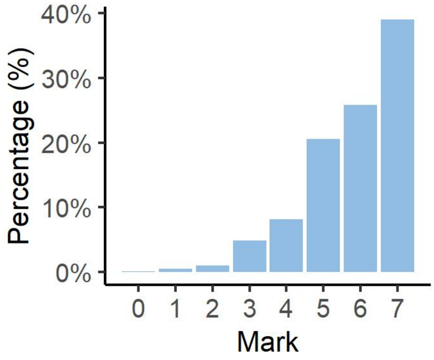
IA1 total



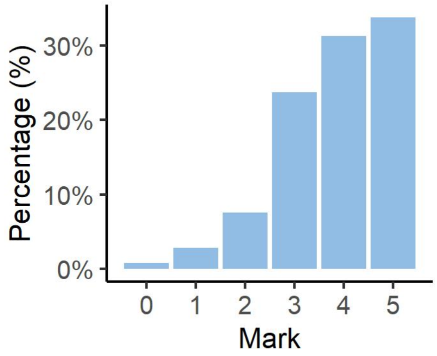
IA1 Criterion 1



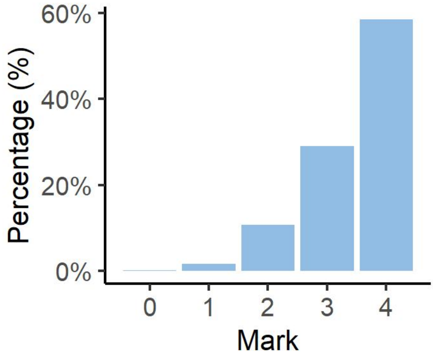
IA1 Criterion 2



IA1 Criterion 3

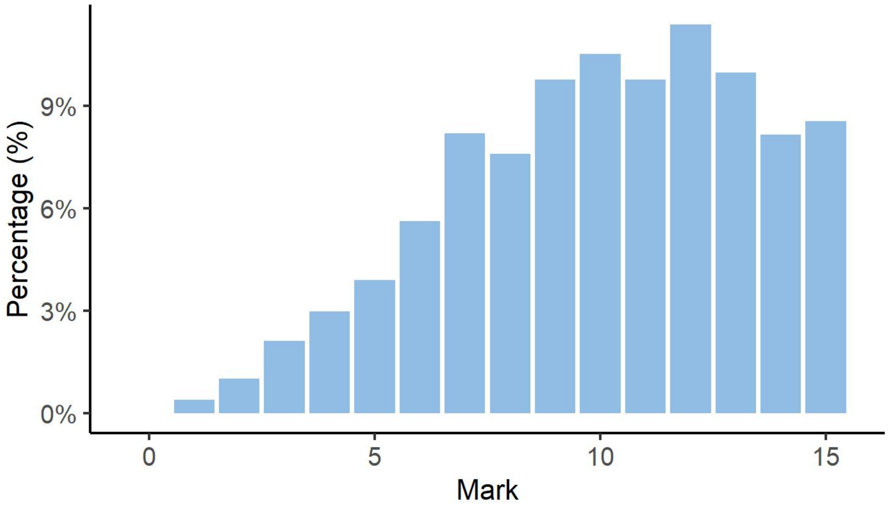


IA1 Criterion 4

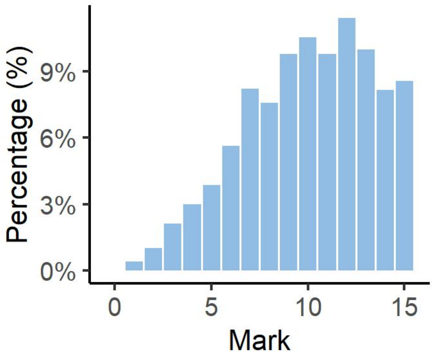


IA2 results

IA2 total

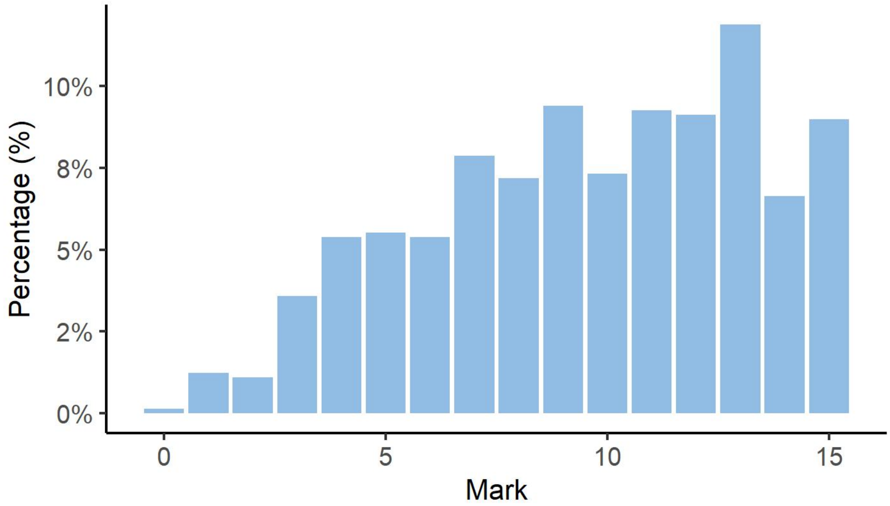


IA2 Criterion 1

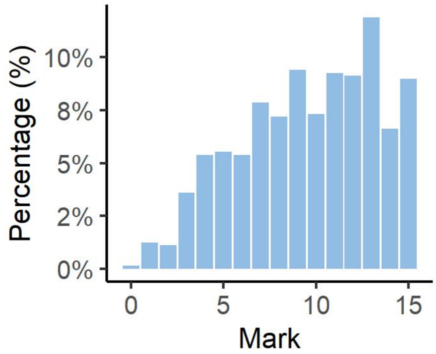


IA3 results

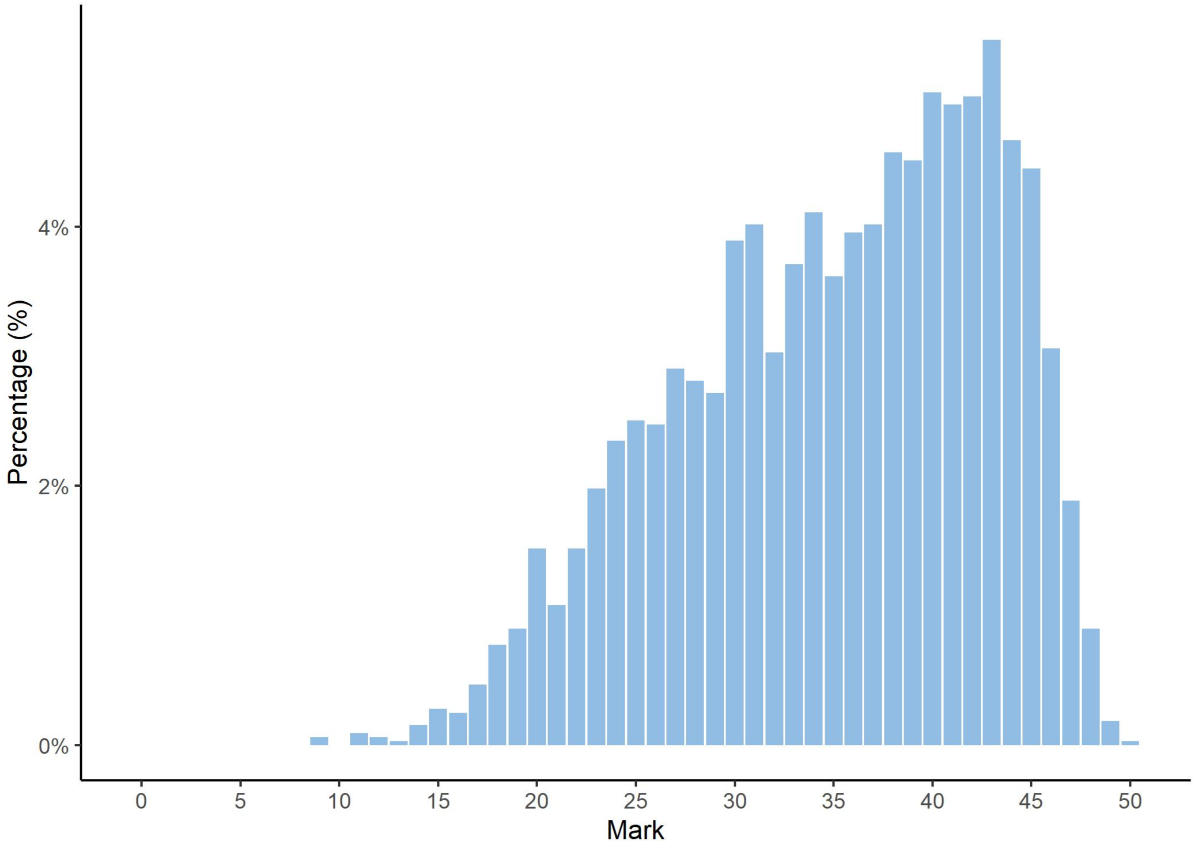
IA3 total



IA3 Criterion 1



External assessment results



Final standards allocation

The number of students awarded each standard across the state are as follows.

Standard	A	B	C	D	E
Number of students	1067	1173	802	190	1

Grade boundaries

The grade boundaries are determined using a process to compare results on a numeric scale to the reporting standards.

Standard	A	B	C	D	E
Marks achieved	100–81	80–64	63–45	44–21	20–0

Internal assessment

The following information and advice pertain to the assessment design and assessment decisions for each IA in Units 3 and 4. These instruments have undergone quality assurance processes informed by the attributes of quality assessment (validity, accessibility and reliability).

Endorsement

Endorsement is the quality assurance process based on the attributes of validity and accessibility. These attributes are categorised further as priorities for assessment and each priority can be further broken down into assessment practices. Data presented in the assessment design sections identifies the reasons why IA instruments were not endorsed at Application 1, by the priority for assessments. An IA may have been identified more than once for a priority for assessment, e.g. it may have demonstrated a misalignment to both subject matter and to the assessment objective. Refer to the quality assurance tools for detailed information about the assessment practices for each assessment instrument.

Total number of items endorsed in Application 1

Number of items submitted each event	IA1	IA2	IA3
Total number of instruments	331	331	331
Percentage endorsed in Application 1	23	13	21

Confirmation

Confirmation is the quality assurance process based on the attribute of reliability. Teachers make judgments about the evidence in students' responses using the instrument-specific marking guide (ISMG) to indicate the alignment of students' work with performance-level descriptors and determine a mark for each criterion. These are provisional criterion marks. The QCAA makes the final decision about student results through the confirmation processes. Data presented in the assessment decisions section identifies the level of agreement between provisional and final results.

Number of samples reviewed at initial, supplementary and extraordinary review

IA	Number of schools	Number of samples requested	Supplementary samples requested	Extraordinary review	School review	Percentage agreement with provisional
1	325	1460	79	9	13	99.09
2	238	1125	0	0	0	99.76
3	88	343	0	0	0	99.86

Internal assessment 1 (IA1)

Problem-solving and modelling task (20%)

The problem-solving and modelling task must use subject matter from Unit 3 Topic 2: Vectors and matrices and/or Topic 3: Complex numbers 2 (AS unit 3 Topic 1: Matrices and applications of matrices and/or Topic 2: Complex numbers 1). A problem-solving and modelling task is an assessment instrument developed in response to a mathematical investigative scenario or context. It requires students to respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams. Students must provide a response to a specific task or issue that is set in a context that highlights a real-life application of mathematics. The task requires students to use relevant stimulus material involving the selected subject matter and must have sufficient scope to allow students to address all the stages of the problem-solving and modelling approach. Technology must be used.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment — validity practices

Validity priority	Number of times priority was identified in decisions*
Alignment	151
Authentication	56
Authenticity	65
Item construction	19
Scope and scale	83

*Total number of submissions: 331. Each priority might contain up to four assessment practices.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that featured:

- relevant and useful stimulus material
- clear instructions to students about the requirements of the task, including identification of the topics being assessed
- opportunity for students to develop a unique response, e.g. providing an open-ended task such that students chose how to use the data and what concepts and techniques were relevant to develop the model and solve the problem
- realistic contexts that were accessible to students, e.g. predicting future population growth.

Practices to strengthen

It is recommended that assessment instruments:

- focus on subject matter from Unit 3 Topic 2: Vectors and matrices and/or Topic 3: Complex numbers 2 (AS unit 3 Topic 1: Matrices and applications of matrices and/or Topic 2: Complex numbers 1) and not subject matter outside the scope of the syllabus
- avoid scaffolding or task instructions that indicate to students how to solve the problem as this interferes with students' ability to demonstrate their knowledge and understanding of the relevant criteria and to provide a unique, authentic response
- focus on interpretation, analysis and evaluation of ideas and information rather than having research (locate, gather, record and analyse information to develop understanding) as a focus
- consider contexts for developing models beyond Dominance matrices, Leslie matrices and cryptography (using matrices)
- are sufficiently different to textbook practice assessments and QCAA sample assessment instruments to ensure responses are not rehearsed and that work submitted is the student's own
- include checkpoints that reflect the school's assessment policy and clearly indicate when and how teachers provide feedback on one draft.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment — accessibility practices

Accessibility priority	Number of times priority was identified in decisions*
Transparency	18
Language	48
Layout	10
Bias avoidance	4

*Total number of submissions: 331. Each priority might contain up to four assessment practices.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that featured:

- a scenario or context that was directly related to the task and accessible to students
- a specific task or issue that
  - was written in a straightforward manner and explicit about the nature of the problem
  - used appropriate language, diagrams and images
- text that was free from punctuation, grammatical, spelling and typographical errors.

Practices to strengthen

It is recommended that assessment instruments:

- only include information relevant to the problem-solving and modelling task to ensure students focus on what is required and are not distracted by extraneous material
- are viewed using the Print Preview button prior to submission, to ensure that the layout is clear and not distracting (e.g. avoiding misaligned text), and that items such as tables appear in their entirety on the page.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional
1	Formulate	99.21	0.69	0.09
2	Solve	99.18	0.57	0.25
3	Evaluate and verify	99.03	0.69	0.28
4	Communicate	99.21	0.28	0.50

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- making appropriate judgments about the documentation of assumptions and observations within the Formulate criterion. High-level responses provided statements of appropriate assumptions and relevant observations, and clearly supported these statements with evidence, such as decisive information and written references
- judgment of ‘accurate and appropriate use of technology’ vs ‘use of technology’ was clearly differentiated within the Solve criterion
- making judgments within the upper performance level of the Communication criterion, student responses adhered to appropriate mathematical, statistical and everyday language conventions and were structured, coherent and organised such that they were able to be read and interpreted independently of the instrument task sheet
- student responses were annotated to show the evidence schools used to make decisions
- either students met syllabus conditions for response length (up to 10 pages, maximum of 2000 words) when developing ideas and synthesising information (e.g. by placing repeated calculations and/or spreadsheet data into an appendix) or, where responses exceeded syllabus conditions, teachers consistently adhered to the school’s assessment policy by clearly indicating the evidence that was used to determine the result.

Samples of effective practices

The following excerpts are from responses that illustrate the characteristics for the criterion at the performance level indicated. The samples may provide evidence for more than one criterion. The characteristics highlighted may not be the only time the characteristics have occurred throughout the response.

Formulate (3–4 marks) **Documentation of appropriate assumptions** This response demonstrates evidence supporting assumptions that are appropriate, based on the logic of a proposed solution and/or model.	Assumptions 1. It is assumed an adequate amount of sample data is taken to produce accurate results. Therefore, Sample data will be taken from all 7 competition rounds to accurately deduce whether there is a home ground advantage. In an agility dominated and fast pace sport like netball (Australian Sports Camps 2020), analysing a substantial amount of data is vital in ensuring enough information is gathered. 2. It is assumed all venues have an adequate number of individuals specifically supporting the home team. A home ground advantage is dependent on the fact that there will be supporters viewing the game (Runciman 2008). 3. It is assumed the same team members play every game. Each team is assumed to maintain consistent results due to having the same members playing on court. This creates reason for future success. 4. Relating to point 4, no considerations about a drop of skill level from a highly ranked away team will be analysed. It will be assumed that the drop in skill level will be due to the home ground advantage. Reference List Australian Sports Camps 2020, *Advanced Netball Drills Easily Practiced at Home,* [Online], Available: https://australiansportscamps.com.au/blog/advanced-netball-drills-young-netballers/ [Date Accesses: March 1st, 2020] Runciman. D 2008, *Home sweet home?* [Online], Available: https://www.theguardian.com/sport/2008/feb/03/features.sportmonthly16 [Date Accesses: March 5th, 2020]

Solve (6–7 marks)
Accurate and appropriate use of technology

This response demonstrates the appropriate use of technology (graphics calculator and spreadsheet software).

Table 3: Potoroo Population Over Time 2

After *'a'* Years	Total Population	% Change from Previous Year
Initial Year	40	-
1	48	+20%
2	54	+12.5%
3	74	+37.03%
4	92	+24.32%
5	118	+28.26%
6	148	+25.42%
7	190	+28.37%
8	238	+25.26%
9	302	+26.89%
10	382	+26.49%
15	1232	+26.49%
20	3946	+26.15%
25	12640	+26.19%
30	40458	+26.2%

**The percentage change has been worked out from the year previous in terms of overall time (eg. from after 14 years to after 15 years) not from the year previous to it in the table*

Figure 3: Potoroo Population Graph 2

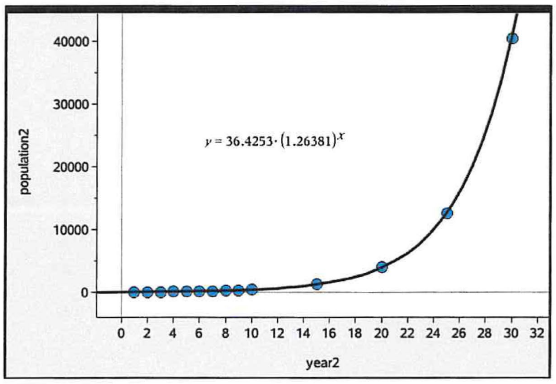


Figure 4: Statistical Calculations 2

Title	Exponen...
RegEqn	a*b^x
a	36.4253
b	1.26381
r²	0.999857
r	0.999929
Resid	{1.96514...
ResidTra...	{0.04180...

Communicate (3–4 marks)
Coherent and concise organisation of the response

This response demonstrates evidence of a suitable introduction and conclusion, which can be read independently of the task sheet.

Introduction

The purpose of this investigation is to use the understanding of matrices and their real-life applications in order to solve a problem relating to the population of a species over time. The particular species to be analysed in this investigation is the Potoroo. Initial data inclusive of age categories, number of potoroos alive in those categories, death rate and mean number of off-spring, are given:

Conclusion

It can therefore be concluded that through the use of wildlife sanctuaries, the survival rate of 0- to 1-year old potoroos will be raised, allowing the population to grow significantly over time. An appropriate range of years has been given in which these potoroos can begin being distributed to other areas of Australia where this process can be replicated and allow the population to become much more substantial within 30 years. This solution offers a much more favourable outcome than seen in the original condition, in which the potoroos in that habitat went extinct within 6 years.

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG in this IA, it is recommended that:

- within the Formulate criterion, distinguish between the terms ‘documentation’ and ‘statement’, and ‘assumption’ and ‘observation’
  - Provisional judgments at the higher performance level were not supported where responses only provided statements and lacked the supporting evidence to be classified under ‘documentation of appropriate assumptions’ or ‘accurate documentation of relevant observations’. Additionally, where samples grouped assumptions and observations together, there needed to be alignment to the definitions of ‘assumption’ and ‘observation’.
  - Responses that unambiguously demonstrated ‘documentation of appropriate assumptions’, included assumptions related to the model and evidence to support the assumptions, e.g. explanation of the likely effect of important assumptions and how this is considered in the model/solution, or the impact of not making the assumption.
  - Responses that unambiguously demonstrated ‘accurate documentation of relevant observations’ provided evidence to support observations (information/data) used in a student’s model/solution, e.g. explaining how the observations were collected, the source of the observations, what made the observations valid and reliable, or identifying a specific feature of an observation that made it relevant to the model/solution.
- within the Solve criterion, judgments reflect the terms ‘discerning’, ‘use of complex procedures’ and ‘valid and reasonable solution’
  - Responses that unambiguously demonstrated ‘discerning application of mathematical concepts and techniques relevant to the task’ made thoughtful and astute choices as to which concepts and techniques to apply in order to enhance the solution. There was clear evidence in the student work of a discriminating selection of the mathematical methods used and that each technique demonstrated was used to add value to the solution.
  - Responses that demonstrated ‘accurate use of complex procedures to reach a valid solution’ attained a valid solution using methods made up of multiple elements and/or interconnected parts. In general, using a trial-and-error approach to develop a function against certain restraints did not demonstrate use of complex procedures.
  - Teachers annotate the ISMG to show the typical characteristics evident in student work and not necessarily an entire descriptor, e.g. a response may ‘reach a reasonable solution’ from the mid-level descriptor and show ‘use of simple procedures’ from the lower descriptor.
- within the Evaluate and verify criterion, judgments align to the performance-level descriptors and their associated characteristics
  - evaluation of the reasonableness of solutions by considering the results, assumptions and observations
    - To demonstrate these characteristics, students needed to show that they had considered the results, assumptions and observations to appraise and justify their solutions. Any assumptions and observations introduced throughout the report could be used while evaluating the reasonableness of solutions.
    - The format of the evaluation varied depending on the situation, but could include the use of technology to verify solutions or the use of both mathematical and everyday language to justify solutions.
  - documentation of relevant strengths and limitations of the solution and/or model

    - A strength is an aspect or feature of the solution and/or model that makes it useful. A limitation is an aspect or feature of the solution and/or model that limits its usefulness; a weakness. Limitations are often a direct consequence of assumptions (making less restrictive assumptions gives opportunities for refinement).
    - For ‘documentation’ to be demonstrated there needed to be evidence in the student work of why elements of their solution and/or model were strengths or limitations.
    - Responses that unambiguously demonstrated these characteristics considered whether the solution and/or model fulfilled its intended purpose, how it compared to the real world, its generalisability and overall utility, and/or any caveats or conditions where the solution and/or model was no longer useful.

Additional advice

It is recommended that:

- schools develop their own solutions for problem-solving and modelling tasks prior to endorsement. This does not necessarily mean writing a full report, but at a minimum considering what an expected response would demonstrate for all criteria. The advantages of this include
  - ensuring that the task allows students to address all criteria at all performance levels within syllabus conditions
  - promoting consistency in the marking of student work within the school and a suitable method of quality assuring judgments
- teachers use and annotate the correct QCAA-formatted ISMG downloaded from the QCAA Portal by clearly highlighting or underlining relevant characteristics and indicating the subsequent mark allocation.

Internal assessment 2 (IA2)

Examination — short response (15%)

The examination assesses the application of a range of cognitions to a number of items using a representative sample of subject matter from all Unit 3 (AS unit 3) topics. Where relevant, the focus of this assessment should be on subject matter not assessed in the problem-solving and modelling task. Subject matter from Units 1 and 2 (AS units 1 and 2) is considered assumed knowledge. It is also assumed that work covered in Mathematical Methods will be known before it is required in Specialist Mathematics. Student responses must be completed individually, under supervised conditions, and in a set timeframe (120 minutes plus 5 minutes perusal). The percentage allocation of marks must match the degree of difficulty specifications (~20% Complex unfamiliar, ~20% Complex familiar, ~60% Simple familiar).

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment — validity practices

Validity priority	Number of times priority was identified in decisions*
Alignment	239
Authentication	0
Authenticity	70
Item construction	12
Scope and scale	94

*Total number of submissions: 331. Each priority might contain up to four assessment practices.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that featured:

- questions that assessed a selection of subject matter that accurately reflected the intended learning of all topics in Unit 3. For example, including a matrix application question is sufficient representation of the sub-topic of Applications of matrices. A representative sample does not require a question from every sub-topic with the following exceptions for the Specialist Mathematics General syllabus
  - Topic 2: Subject matter from the sub-topic Vector calculus must be assessed.
  - Topic 3: Subject matter from at least one of the sub-topics of Roots of complex numbers or Factorisation of polynomials must be assessed. Furthermore, if the sub-topic of Factorisation of polynomials is assessed, the subject matter must go beyond that of the Mathematical Methods course by using complex numbers in the solution.
- questions that explicitly provided opportunities to address all assessment objectives
- stimulus, where used, that was relevant to the question and necessary to solve the problem

- a balance of items requiring both technology-free and technology-active responses, which could be identified by specifying the questions (or sections of the paper) requiring a technology-free or technology-active approach
- an appropriate number of questions that matched the degree of difficulty specifications in the syllabus and allowed students to respond within the time conditions
- a correct marking scheme that indicated clearly how marks would be allocated as this assists schools to check the scope and scale of the assessment and promotes consistency in the awarding of marks
- QCAA approval for any amendment to an endorsed instrument to uphold the integrity of school-based assessment practices.

Practices to strengthen

It is recommended that assessment instruments:

- require students to demonstrate knowledge and understanding of Unit 3 subject matter and do not solely assess subject matter from Units 1 and 2, e.g. a question that only involves basic matrix operations (AS units 1 and 2, e.g. a question that only involves Simpson's rule)
- include questions that align with the Unit 3 description, e.g. extend students' knowledge of calculus from Mathematical Methods to explore vector equations and vector calculus in two- and three-dimensional space (AS unit 3 description, e.g. extend students' knowledge of quadratic equations from Mathematical Methods to explore complex conjugate solutions of real quadratic equations)
- provide complex unfamiliar opportunities such that
  - relationships and interactions have a number of elements, e.g. an algebraic component included in a technology-active question (if required)
  - all the information to solve the problem is not immediately identifiable, by avoiding scaffolding, e.g. not providing a series of parts that step through a problem, cues that indicate the procedure to use, or diagrams or graphs in technology-active questions that simplify the nature of the problem
- provide opportunities for students to respond to assessment objective 4: 'evaluate the reasonableness of solutions'
- provide opportunities for students to respond to assessment objective 5: 'justify procedures and decisions by explaining mathematical reasoning'
- are sufficiently different to QCAA sample questions, textbook questions and practice assessments to ensure responses are authentic and not rehearsed.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment — accessibility practices

Accessibility priority	Number of times priority was identified in decisions*
Transparency	51
Language	77
Layout	27
Bias avoidance	10

*Total number of submissions: 331. Each priority might contain up to four assessment practices.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that featured:

- simple familiar questions where what was being asked was clearly identifiable
- the language of the assessment objectives, e.g. ‘evaluate the reasonableness of ...’ rather than ‘discuss limitations of …’
- correct language conventions, and were free of punctuation, grammatical, spelling and typographical errors
- correct mathematical notation, e.g. the vector notation of $\hat{\imath}$ and not $\underset{\sim}{i}$ or i
- limited use of bold and italics
- adequate response space for each question
- clear, relevant images where appropriate.

Practices to strengthen

It is recommended that assessment instruments:

- are designed such that solutions can be obtained under the conditions of the examination, e.g. a complete solution can be obtained under technology-free conditions
- are reviewed using the Print Preview button, prior to submission, to ensure that the layout is clear and not distracting (e.g. avoiding misaligned text), and that items such as tables appear in their entirety on the page.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and final results

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional
1	Foundational knowledge and problem-solving	99.76	0	0.24

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- there was clear alignment between a school's submitted marking scheme and the awarded marks, which was most effective where schools provided marking schemes that detailed where marks were awarded
- annotations were used by teachers within the response to indicate where marks were awarded
- schools recorded on the ISMG the total possible and awarded marks for the examination and the calculated percentage
- the 'greater than x%' cut-offs were correctly applied to the percentage calculations to determine accurate provisional marks, for example
  - results are not rounded to the nearest percentage before applying the ISMG
  - a student who receives > 80% is allocated 13/15, whereas a student who receives 80% (exactly) is allocated 12/15
- only the percentage cut-offs in the ISMG were used to award the mark out of 15, not the performance-level descriptors. The performance-level descriptors are not used to directly make a judgment or allocate a mark. As the percentage allocation of marks in endorsed examinations must match the degree of difficulty specifications (~20% Complex unfamiliar, ~20% Complex familiar, ~60% Simple familiar), a student who is awarded 14 marks, for example, meets the performance-level descriptor for that performance level.

Samples of effective practices

The following example illustrates appropriate application of the percentage cut-off ISMG to determine the correct mark allocation.

Foundational knowledge and problem-solving

This example shows an annotated ISMG that clearly indicates the response was awarded 60 marks out of a possible 75 marks. This equates to 80% (exactly) and the response is therefore allocated 12 marks.

Instrument-specific marking guide (IA2): Examination (15%)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics

$\frac{60}{75} = 80\%$

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and	> 67%	11

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG in this IA, it is recommended that:

- schools update marking schemes for an endorsed instrument to correctly allocate marks to student responses when it is necessary to correct errors in questions or sample responses, change mark allocations and/or accept alternative solutions. The communication of these changes in a timely manner to the QCAA, either through the amendment process or at the time of confirmation submission, is required to assist confirmers to make appropriate decisions during the confirmation process
- the correct marking scheme is used and submitted for a comparable assessment
- schools use internal practices to quality assure judgments and check mark totals, percentage calculations and correct application of the percentage cut-off ISMG.

Internal assessment 3 (IA3)

Examination — short response (15%)

The examination assesses the application of a range of cognitions to a number of items using a representative sample of subject matter from all Unit 4 (AS unit 4) topics. Subject matter from Units 1, 2 and 3 (AS units 1, 2 and 3) is considered assumed knowledge. It is also assumed that work covered in Mathematical Methods will be known before it is required in Specialist Mathematics. Student responses must be completed individually, under supervised conditions, and in a set timeframe (120 minutes plus 5 minutes perusal). The percentage allocation of marks must match the degree of difficulty specifications (~20% Complex unfamiliar, ~20% Complex familiar, ~60% Simple familiar).

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment — validity practices

Validity priority	Number of times priority was identified in decisions*
Alignment	216
Authentication	0
Authenticity	3
Item construction	24
Scope and scale	83

*Total number of submissions: 331. Each priority might contain up to four assessment practices.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that featured:

- questions that assessed a selection of subject matter that accurately reflects the intended learning of all topics in Unit 4 (AS unit 4).
- questions that explicitly provided opportunities to address all assessment objectives
- realistic contexts where appropriate, which generally occurred in technology-active items
- stimulus, where used, that was relevant to the question and necessary to solve the problem
- a balance of items requiring both technology-free and technology-active responses, which could be identified by specifying the questions (or sections of the paper) requiring a technology-free or technology-active approach
- an appropriate number of questions that matched the degree of difficulty specifications in the syllabus and allowed students to respond within the time conditions
- a correct marking scheme that indicated how marks would be allocated and provided solutions that used methods from the syllabus

- QCAA approval for any amendment to an endorsed instrument to uphold the integrity of school-based assessment practices.

Practices to strengthen

It is recommended that assessment instruments:

- are explicitly aligned to the subject matter from Unit 4 (AS unit 4) and do not solely reference subject matter from Units 1, 2 and 3 (AS units 1, 2 and 3) or Mathematical Methods, e.g. calculating the area under a polynomial (AS example, determining linear factors of real quadratic polynomials).
- representatively sample subject matter across all three topics in Unit 4 (AS unit 4), including Topic 3 Statistical inference. In particular, for AS unit 4,
  - Topic 2: Subject matter from at least one of the sub-topics Roots of complex numbers or Factorisation of polynomials must be assessed. Furthermore, if the Factorisation of polynomials sub-topic is assessed, the subject matter must go beyond that of Mathematical Methods by using complex numbers in the solution.
- assess subject matter within the scope and scale of the syllabus
- include questions (and corresponding marking scheme) that align with the unit descriptions of Unit 4 (AS unit 4), e.g. students use calculus techniques in contexts such as kinematics rather than using physics formulas
- provide complex unfamiliar opportunities such that
  - relationships and interactions have a number of elements, e.g. an algebraic component included in a technology-active question, if required
  - all the information to solve the problem is not immediately identifiable, by avoiding scaffolding, e.g. not providing a series of parts that step through a problem, cues that indicate the procedure to use, or diagrams or graphs in technology-active questions that simplify the nature of the problem
- provide opportunities for students to respond to assessment objective 4: ‘evaluate the reasonableness of solutions’
- provide opportunities for students to respond to assessment objective 5: ‘justify procedures and decisions by explaining mathematical reasoning’
- are sufficiently different to QCAA sample questions, textbook questions and practice assessments to ensure responses are authentic and not rehearsed.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment — accessibility practices

Accessibility priority	Number of times priority was identified in decisions*
Transparency	43
Language	87
Layout	27
Bias avoidance	11

*Total number of submissions: 331. Each priority might contain up to four assessment practices.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that featured:

- simple familiar questions where what was being asked was clearly identifiable
- the language of the assessment objectives, e.g. 'evaluate the reasonableness of ...' rather than 'explain the associated effect of this assumption …'
- questions that did not reference specialised language or non-accessible contexts
- correct language conventions, and were free of punctuation, grammatical, spelling and typographical errors
- correct mathematical notation and symbols, e.g. $\frac{d^2y}{dx^2}$ instead of d2y/dx2
- limited use of bold and italics
- adequate response space for each question
- clear, relevant images where appropriate.

Practices to strengthen

It is recommended that assessment instruments:

- are designed such that solutions can be obtained under the conditions of the examination, e.g. a technology-free response can be obtained under technology-free conditions
- are reviewed using the Print Preview button, prior to submission, to ensure that the layout is clear and not distracting (e.g. avoiding misaligned text), and that items such as tables appear in their entirety on the page.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and final results

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional
1	Foundational knowledge and problem-solving	99.86	0.14	0

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- there was clear alignment between a school's submitted marking scheme and the awarded marks, which was most effective where schools provided marking schemes that detailed where marks were awarded
- annotations were used by teachers within the response to indicate where marks were awarded
- schools recorded on the ISMG the total possible and awarded marks for the examination and the calculated percentage

- the ‘greater than x%’ cut-offs were correctly applied to the percentage calculations to determine accurate provisional marks, for example:
  - results are not to be rounded to the nearest percentage before applying the ISMG
  - a student who receives > 80% is allocated 13/15, whereas a student who receives 80% (exactly) is allocated 12/15
- only the percentage cut-offs in the ISMG were used to award the mark out of 15, not the performance-level descriptors. The performance-level descriptors are not used to directly make a judgment or allocate a mark. As the percentage allocation of marks in endorsed examinations must match the degree of difficulty specifications (~20% Complex unfamiliar, ~20% Complex familiar, ~60% Simple familiar), a student who is awarded 14 marks, for example, meets the performance-level descriptor for that performance level.

Samples of effective practices

The following example illustrates appropriate application of the percentage cut-off ISMG to determine the correct mark allocation.

Foundational knowledge and problem-solving

This example shows an annotated ISMG that clearly indicates the response was awarded 74.5 marks out of a possible 80 marks. This equates to 93.1% and the response is therefore allocated 15 marks. The calculated percentage is not rounded to the nearest whole number when using the ISMG.

Instrument-specific marking guide (IA2): Examination (15%)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics

$\frac{74\frac{1}{2}}{80} = 93.1\%$

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG in this IA, it is recommended that:

- schools update marking schemes for an endorsed instrument to correctly allocate marks to student responses when it is necessary to correct errors in questions or sample responses, change mark allocations and/or accept alternative solutions. The communication of these changes in a timely manner to the QCAA, either through the amendment process or at the time of confirmation submission, is required to assist confirmers to make appropriate decisions during the confirmation process
- the correct marking scheme is used and submitted for a comparable assessment
- schools use internal practices to quality assure judgments and check mark totals, percentage calculations and correct application of the percentage cut-off ISMG.

External assessment

Summative external assessment (EA): Examination (50%)

Assessment design

Assessment specifications and conditions

Summative external assessment is developed and marked by the QCAA. In Specialist Mathematics, it contributes 50% to a student’s overall subject result. Summative external assessment assesses learning from Units 3 and 4 (AS units 3 and 4). Subject matter from Units 1 and 2 (AS units 1 and 2) is assumed knowledge and may be drawn on, as applicable, in the development of the examination. The external assessment in Specialist Mathematics is common to all schools and administered under the same conditions, at the same time, on the same day.

Conditions

- Time
  - Paper 1 (technology-free, 25%); 90 minutes plus 5 minutes perusal
  - Paper 2 (technology-active, 25%); 90 minutes plus 5 minutes perusal.
- Length: the number of short-response items should allow students to complete the response in the set time.
- Short-response format, consisting of a number of items that ask students to respond to the following activities
  - calculating using algorithms
  - drawing, labelling or interpreting graphs, tables or diagrams
  - short items requiring multiple-choice, single-word, term, sentence or short-paragraph responses
  - justifying solutions using appropriate mathematical language where applicable
  - responding to seen or unseen stimulus materials
  - interpreting ideas and information.
- Other
  - the QCAA formula sheet will be provided for both papers
  - no calculator or technology of any type is permitted in Paper 1 (technology-free); access to a QCAA-approved handheld graphics calculator (no CAS functionality) is a requirement for Paper 2 (technology-active) of the external assessment, and scientific calculators may also be used.

The assessment instrument consisted of two papers: technology-free (Paper 1) and technology-active (Paper 2). The examination assesses the application of a range of cognitions to a number of items drawn from Units 3 and 4 (AS units 3 and 4). Student responses must be completed individually, under supervised conditions, and in a set timeframe. This assessment was used to determine student achievement in the following assessment objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4 (AS units 3 and 4)
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4 (AS units 3 and 4)
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4 (AS units 3 and 4)

Paper 1 Section 1 was 10 multiple-choice questions.

Paper 1 Section 2 was 9 short-response questions.

Paper 2 Section 1 was 10 multiple-choice questions.

Paper 2 Section 2 was 9 short-response questions.

Assessment decisions

Overall, students responded well to the following assessment aspects:

- assessment objectives 1, 2 and 3
- demonstrating knowledge and understanding of the subject matter and application of skills in simple familiar problems
- calculating using algorithms in simple familiar situations.

Effective practices

The following samples were selected to illustrate highly effective student responses in some of the assessment objectives of the syllabus.

Multiple-choice item response

Item: Question 9 — Paper 1 (Technology free)

Assessment objectives: 1 and 2 — Simple familiar

The following question highlights the validity arguments of the distractors when designing multiple-choice items for possible student responses in a technology-free examination.

This question has been included to illustrate the following subject matter:

- use simulation related to variations in confidence intervals between samples.

Foundational knowledge and problem solving	**QUESTION 9** The scores on a test are assumed to be normally distributed. Researchers use the results from a random sample of scores to calculate a confidence interval for the population mean. However, a shorter confidence interval width is required so the researchers decide to use a second sample for their calculations. Assuming that the standard deviations for both samples are the same, the researchers can ensure that a shorter confidence interval width is produced by (A) decreasing the sample size and decreasing the confidence level. (B) decreasing the sample size and increasing the confidence level. (C) increasing the sample size and decreasing the confidence level. (D) increasing the sample size and increasing the confidence level.

Validity arguments for the options:

(A) Decreasing the value of z shortens the interval; however, decreasing the value of n lengthens the interval. So, these combined changes *may not necessarily* shorten the confidence interval for the mean.

(B) Decreasing the value of n and increasing the value of z both lengthen the interval. So, these combined changes would *not* shorten the confidence interval for the mean.

(C) Decreasing the value of z and increasing the value of n both shorten the interval length. So, these combined changes would ensure that a shorter confidence interval for the mean is produced. This is the correct response.

(D) Increasing the value of n shortens the interval; however, decreasing the value of n lengthens the interval. So, these combined changes *may not necessarily* shorten the confidence interval for the mean.

Item: Question 6 — Paper 2 (Technology active)

Assessment objectives: 1 and 2 — Simple familiar

The following question highlights the validity arguments of the distractors when designing multiple-choice items for possible student responses in a technology-active examination.

This question has been included to illustrate the following subject matter:

- solve systems of linear equations using matrix algebra.

The question also illustrates an opportunity for students to make decisions regarding the efficient use of technology over an analytic procedure.

Foundational knowledge and problem solving	**QUESTION 6** Solve the matrix equation for **X**. $$\begin{bmatrix}0 & 1\\2 & 3\end{bmatrix}\mathbf{X}\begin{bmatrix}4 & 5\\6 & 7\end{bmatrix}=\begin{bmatrix}8 & 9\\0 & 1\end{bmatrix}$$ (A) $\begin{bmatrix}-9 & -9\\4 & 4\end{bmatrix}$ (B) $\begin{bmatrix}3 & -4\\-1 & 2\end{bmatrix}$ (C) $\begin{bmatrix}13 & -14\\-11 & 12\end{bmatrix}$ (D) $\begin{bmatrix}54 & 56\\-28 & -29\end{bmatrix}$

The value of matrix **X** is calculated using matrix algebra involving the use of the multiplicative inverse of matrices A and B as well as recognising that matrix multiplication is not commutative, as shown in the solution below.

Let $\mathbf{A}=\begin{bmatrix}0 & 1\\2 & 3\end{bmatrix}$ $\mathbf{B}=\begin{bmatrix}4 & 5\\6 & 7\end{bmatrix}$ and $\mathbf{C}=\begin{bmatrix}8 & 9\\0 & 1\end{bmatrix}$

$$\mathbf{AXB}=\mathbf{C}$$

$$\mathbf{A}^{-1}\mathbf{AXBB}^{-1}=\mathbf{A}^{-1}\mathbf{CB}^{-1}$$

$$\mathbf{IXI}=\mathbf{A}^{-1}\mathbf{CB}^{-1}$$

$$\mathbf{X}=\mathbf{A}^{-1}\mathbf{C}\,\mathbf{B}^{-1}$$

$$=\begin{pmatrix}3 & -4\\-1 & 2\end{pmatrix}\quad \text{(using calculator)}$$

Validity arguments for the options:

(A) Calculated $\mathbf{X}=\mathbf{A}^{-1}(\mathbf{C}-\mathbf{B})$. Students incorrectly used the additive inverse for $\mathbf{B}$.

(B) Calculated $\mathbf{X}=\mathbf{A}^{-1}\mathbf{C}\,\mathbf{B}^{-1}$. This is the correct response.

(C) Calculated $\mathbf{X}=\mathbf{B}^{-1}\mathbf{CA}^{-1}$. Students incorrectly assumed commutativity for matrices.

(D) Calculated $\mathbf{X}=\mathbf{A}^{-1}\mathbf{B}^{-1}\,\mathbf{C}$. Students incorrectly assumed commutativity for matrices.

Short response

Item: Question 11 — Paper 1 (Technology free)

Assessment objective: 1, 2, 3 and 4 — Simple familiar

This question has been included to illustrate the following subject matter:

- review Cartesian form
- examine the roots of unity and their position on the unit circle
- use De Moivre's theorem for integral powers.

The question also illustrates:

- that a question part worth more than 1 mark requires mathematical reasoning and/or working to be shown to support answers
- the need to read the question carefully to glean information, i.e. the vertices are positioned on a unit circle
- that the rule to convert $r\ \mathrm{cis}(\theta)$ into Cartesian form is given in the Specialist Mathematics formula book
- an example of the assessment objective 'Evaluate the reasonableness of solutions'.

Student sample of effective response

This excerpt has been included to illustrate the:

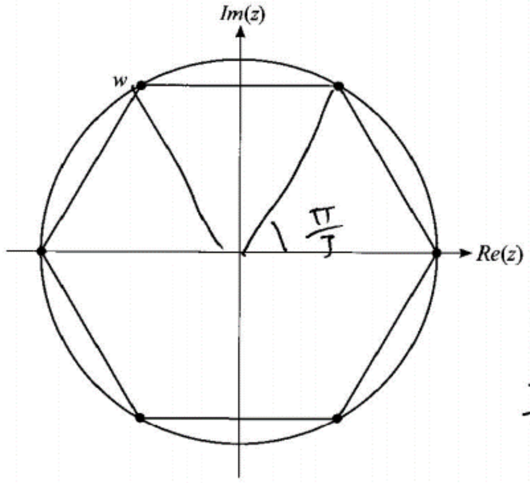
- recognition that six complex numbers equally positioned on a unit circle are $\frac{\pi}{3}$ $(60°)$ apart and of magnitude 1 unit from its centre
- correct expression of the given complex number w in $r\ \mathrm{cis}(\theta)$ form
- correct conversion of w into $r\cos(\theta) + r\sin(\theta)\, i$ form and use of exact values for $Re(w)$ and $Im(w)$
- recognition of the position of the roots of unity for $z^6 = 1$
- evaluation of the reasonableness of a solution using a substitution method (and De Moivre's theorem) rather than completing a full alternative method of solution of determining the general solution of the roots of unity for $z^6 = 1$.

Foundational knowledge and problem solving

QUESTION 11 (7 marks)

The vertices of a regular hexagon are positioned on the circumference of a unit circle as shown on the Argand plane.

6



$\frac{2\pi}{6}$

$\frac{\pi}{3}$

Consider the complex number *w*, as shown on the plane.

a) Determine *w*, expressing your answer in the form $r\,\text{cis}(\theta)$. *[1 mark]*

$w = 1\text{cis}\left(\frac{2\pi}{3}\right)$

b) Convert *w* into Cartesian form. *[2 marks]*

$w = \cos\left(\frac{2\pi}{3}\right) + \sin\left(\frac{2\pi}{3}\right)i$

$= -\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)i$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i.$

Each vertex of the hexagon is a solution of an equation of the form $z^n = a$ where $z \in C$.

c) State the value of n. *[1 mark]*

$n = 6$

d) State the value of a. *[1 mark]*

$a = 1$

e) Verify that w satisfies the equation $z^n = a$ using the results from 11c) and 11d). *[2 marks]*

RTP: $\left(1\text{cis}\left(\frac{2\pi}{3}\right)\right)^6 = 1$

$LHS = \left(1\text{cis}\left(\frac{2\pi}{3}\right)\right)^6$

$= 1^6 \text{cis}\left(\frac{2\times6\times\pi}{3}\right)$, by De Moivre's Theorem

$= 1\text{cis}(4\pi)$

$= 1\text{cis}(0)$

$= \cos(0) + \sin(0)i$

$= 1 + 0i$

$= 1 = RHS.$

$\therefore w$ satisfies the equation $z^6 = 1$

Item: Question 12 — Paper 1 (Technology free) (Specialist Mathematics EA only)

Assessment objectives: 1, 2, 3 and 4 — Simple familiar

This question has been included to illustrate the following subject matter:

- determine the Cartesian coordinates for a three-dimensional rectangular prism
- use the vector (cross) product to determine a vector normal to a given plane
- use unit vectors
- determine the Cartesian equation of a plane.

The question also illustrates:

- the need to read the given information carefully, i.e. the unit vector of the normal was required to be determined rather than a vector normal to the plane
- that the rule to determine the equation for the Cartesian equation of the plane is given in the Specialist Mathematics formula book
- an example of the assessment objective ‘Evaluate the reasonableness of solutions’.

Student sample of effective response

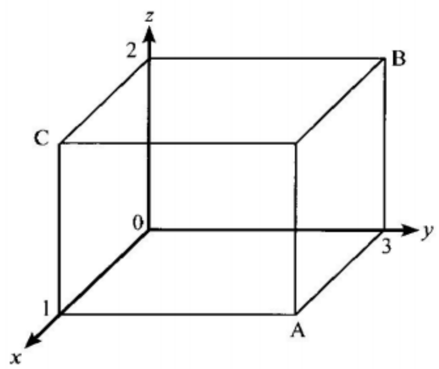
This excerpt has been included to illustrate the:

- understanding of the verb ‘state’ in the question
- correct calculation of a vector product representing a vector normal to the plane containing A, B and C and the subsequent calculation of a unit vector
- evaluation of the reasonableness of a solution using an appropriate verification procedure by showing that the unit vector normal to the plane is perpendicular to a given vector in the plane
- correct determining of the Cartesian equation of the plane containing A, B and C.

Foundational knowledge and problem solving

QUESTION 12 (8 marks)

Consider the vertices A, B and C of the rectangular prism as shown.



a) State the coordinates of A, B and C. *[1 mark]*

A(1, 3, 0) B(0, 3, 2) C(1, 0, 2)

b) Determine a unit vector, $\hat{n}$, that is normal to the plane containing A, B and C. *[3 marks]*

$\vec{AB} = (3j+2k) - (i+3j) = -i+2k$

$\vec{AC} = (i+2k) - (i+3j) = -3j+2k$

The normal vector to the plane is

$n = \vec{AB} \times \vec{AC} = (-i+2k)\times(-3j+2k) = 6i-(-2)j+3k = 6i+2j+3k$

$\hat{n} = \frac{n}{|n|} = \frac{6i+2j+3k}{\sqrt{6^2+2^2+3^2}} = \frac{1}{7}(6i+2j+3k)$

$\therefore$ the unit vector $\hat{n}$ is $\frac{1}{7}(6i+2j+3k)$.

c) Verify that $\hat{n}$ is perpendicular to $\overrightarrow{AB}$. *[2 marks]*

$\hat{n} \cdot \overrightarrow{AB} = \frac{1}{7}(6i+2j+3k) \cdot (-i+2k) = \frac{1}{7} \times (6 \times (-1) + 3 \times 2) = \frac{1}{7} \times (-6+6) = 0$

$\because \hat{n} \cdot \overrightarrow{AB} = 0$

$\therefore \hat{n}$ is perpendicular to $\overrightarrow{AB}$.

d) Determine the Cartesian equation of the plane that contains A, B and C. *[2 marks]*

$\because$ The normal vector to the plane containing A, B and C is $n = 6i+2j+3k$.

$r \cdot n = \ell$ (ℓ is a constant)

$r = xi + yj + zk$

$\therefore r \cdot n = (xi+yj+zk) \cdot (6i+2j+3k) = 6x+2y+3z = \ell$

$\because$ The plane passes through $A(1,3,0)$

$\therefore 6 \times 1 + 2 \times 3 + 3 \times 0 = \ell \quad \ell = 12$

$\therefore$ The Cartesian equation of the plane is $6x+2y+3z=12$

Item: Question 14 — Paper 1 (Technology free) (Specialist Mathematics AS EA only)

Assessment objectives: 1, 2 and 3 — Simple familiar

This question has been included to illustrate the following subject matter:

- use a basic linear transformation and the representation of this transformation by a 2×2 matrix
- apply a transformation to a point in the plane and geometric objects
- examine the relationship between the determinant and the effect of the linear transformation on area.

The question also illustrates:

- the use of prior learning related to the area of a parallelogram
- that the rule to determine the area of a parallelogram is given in the Specialist Mathematics formula book.

Student sample of effective response

This excerpt has been included to illustrate:

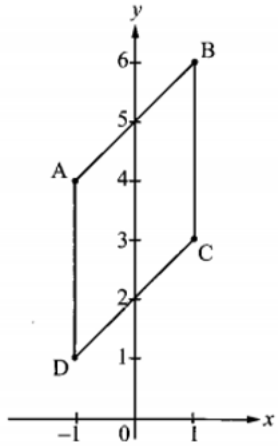
- the given linear transformation of the form $\mathbf{T}: (x, y) \rightarrow (ax + by, cx + dy)$ correctly expressed as a 2×2 matrix of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- the correct application of the transformation to a point on the plane
- the suitable use of coordinate notation (rather than vector notation)

- that the effect of a linear transformation on the area of a geometric shape requires consideration of the absolute value of the determinant of the corresponding 2×2 matrix.

Foundational knowledge and problem solving

QUESTION 14 (9 marks)

The diagram shows a parallelogram, S, with vertices A(–1, 4), B(1, 6), C(1, 3) and D (–1, 1).



a) Determine the area of S. *[1 mark]*

$S_{area} = bh$

$= 2 \times 3$

$= 6$ units^2

The parallelogram S is transformed into the image S′ with vertices A′, B′, C′ and D′ under the linear transformation $\mathbf{T}_1 : (x, y) \to (-x + 2y,\ 4x - 3y)$.

b) State the transformation $\mathbf{T}_1$ in matrix form. *[1 mark]*

~~$T_1 = \begin{bmatrix} -1 & 4 \\ 2 & -3 \end{bmatrix}$~~ $\quad T_1 = \begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix}$

c) Use the results from 14a) and 14b) to determine the area of S′. *[2 marks]*

$S'_{area} = |\det|T|| \times S_{area}$ $\det|T| = ad - bc$

$= |-5| \times 6$ $= 3 - 8$

$= 30 \text{ units}^2$ $= -5$

d) Use the result from 14b) to determine the coordinates of A′. *[2 marks]*

~~$A' = \begin{bmatrix} -1 & 4 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x + 4y \\ 2x - 3y \end{bmatrix}$~~

$A(x, y) \rightarrow A'(x', y') = (-x + 2y, 4x - 3y)$

$A(-1, 4) \rightarrow A'(1 + 8, -4 - 12)$

$A(-1, 4) \rightarrow A'(9, -16)$

The area of parallelogram S remains unchanged under the different transformation $\mathbf{T}_2 = \begin{bmatrix} \frac{1}{3} & 1 \\ a & 6 \end{bmatrix}$

e) Determine the value/s of a. *[3 marks]*

$S'_{area} = |\det T_2| \, S_{area}$

$1 = |\det T_2|$

$1 = |ad - bc|$

$1 = |\frac{1}{3} \times 6 - 1 \times a|$

$1 = |2 - a|$

$a = 1, 3$

Item: Question 18 — Paper 1 (Technology free)

Assessment objective: 1, 2, 3, 5 and 6 — Complex unfamiliar

This question has been included to illustrate the following subject matter:

- apply the factor theorem for polynomials
- consider conjugate roots for polynomials with real coefficients
- review imaginary components of a complex number.

The question also illustrates the following assumed knowledge:

- use the general solution of real quadratic equations (Specialist Mathematics Unit 1)
- understand the role of the discriminant to determine the number of solutions to a quadratic equation (Mathematical Methods Unit 1).

Student sample of effective response

This excerpt has been included to illustrate:

- clear justification of procedures by explaining mathematical reasoning related to conjugate roots

- the correct application of the factor theorem using the given root of $z = -i$ to determine b where $b \in Z^+$
- the recognition that since the coefficients of the given polynomial were real, the conjugate root $z = i$ is another solution of $P(z) = 0$
- the well-constructed use of the two known roots to deduce that $(z + i)(z - i) = z^2 + 1$ is a factor of $P(z)$
- the efficient use of long division using the quadratic factor $z^2 + 1$ to determine the remaining quadratic factor of $z^2 + az + 4$
- the use of assumed knowledge to determine the condition of a by recognising that the determinant of the quadratic equation $z^2 + az + 4 = 0$ must be negative in order for the remaining roots of $P(z)$ to be imaginary
- the correct use of the given domain to determine all possible values of a where $a \in Z^+$.

Foundational knowledge and problem solving

QUESTION 18 (6 marks)

Consider the function $P(z) = 2z^4 + az^3 + 6z^2 + az + b$ where $a, b \in Z^+$

One of the roots of $P(z)$ is $z = -i$

Determine the possible value/s for a and b such that all remaining roots of $P(z)$ have an imaginary component.

If $z = -i$ is a root, then $z = i$ is also a root, by conjugate root theorem.

$\therefore (z-i)(z+i)$ are factors, hence (z^2+1) is a factor

$$\begin{array}{r} 2z^2 + az + 4 \\ z^2+1 \enclose{longdiv}{2z^4 + az^3 + 6z^2 + az + b} \\ -(2z^4 + 0z^3 + 2z^2) \\ az^3 + 4z^2 + az \\ -(az^3 + 0z^2 + az) \\ 4z^2 + 0z + b \\ -(4z^2 + 0z + 4) \\ b - 4 \end{array}$$

If z^2+1 is a factor, the remainder is 0

$\therefore b - 4 = 0$

$b = 4$.

Consider $Q(z) = 2z^2 + az + 4$.

For remaining 2 roots to be complex.

$\Delta < 0$, where $\Delta = b^2 - 4ac$, $a = 2, b = a, c = 4$

$b^2 - 4ac < 0$

$a^2 - 4 \times 2 \times 4 < 0$

$a^2 - 32 < 0$

$a^2 < 32$

$\therefore a < \pm\sqrt{32}$

$a < \pm 4\sqrt{2}$

But if $a \in Z^+$.

a must be 1, 2, 3, 4, 5

$\therefore a = 1$ or 2 or 3 or 4 or 5

$b = 4$ for the remaining roots of $P(z)$ to be imaginary.

Item: Question 16 — Paper 2 (Technology active)

Assessment objective: 1, 2, 3, 4 and 5 — Complex familiar

This question has been included to illustrate the following subject matter:

- use De Moivre's theorem for integral powers.

The question also illustrates the following assumed knowledge:

- use the general solution of real quadratic equations (Specialist Mathematics Unit 2)
- apply the Pythagorean identities (Specialist Mathematics Unit 2, AS unit 2)
- use the binomial theorem to prove and apply multi-angle trigonometric identities up to $\sin(4x)$ and $\cos(6x)$ (Specialist Mathematics Unit 2, AS unit 2).

Student sample of effective response

This excerpt has been included to illustrate the:

- appropriate use of the cue 'using De Moivre's theorem'
- recognition that $\cos(4\theta)$ required consideration of the real part of z^4
- correct use of brackets when expanding expressions, e.g. using $-6(1-\sin^2(\theta))$
- connection with the more familiar proof of this type through an additional application of a Pythagorean identity (by considering the right-hand side of the expression of the required proof).

Foundational knowledge and problem solving

QUESTION 16 (6 marks)

Consider the identity

$$\cos(4\theta) = A\cos^4(\theta) + B\sin^2(\theta) + C \text{ where } A, B \text{ and } C \in Z$$

a) Determine the values of A, B and C using De Moivre's theorem. *[5 marks]*

Let $z = \cos\theta + i\sin\theta$

$z^4 = (\cos\theta + i\sin\theta)^4 = \cos^4\theta + 4\times\cos^3\theta\times i\sin\theta + 6\cos^2\theta\cdot(i\sin\theta)^2 + 4\cos\theta\cdot(i\sin\theta)^3 + (i\sin\theta)^4$

$= \cos^4\theta + 4\cos^3\theta\sin\theta\, i - 6\cos^2\theta\sin^2\theta - 4\cos\theta\sin^3\theta\, i + \sin^4\theta$

$= \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta + (4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta)\, i$

$\because z = \cos\theta + i\sin\theta = cis(\theta)$

$\therefore z^4 = (cis(\theta))^4 = cis(4\theta) = \cos(4\theta) + i\sin(4\theta)$

Equating real parts,

$\cos(4\theta) = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

$= \cos^4(\theta) - 6(1-\sin^2\theta)\sin^2\theta + \sin^4\theta$

$= \cos^4(\theta) - 6\sin^2\theta + 6\sin^4\theta + \sin^4\theta$

$= \cos^4(\theta) - 6\sin^2(\theta) + 7\sin^4(\theta)$

$= \cos^4(\theta) - 6\sin^2(\theta) + 7(1-\cos^2\theta)^2$

$= \cos^4(\theta) - 6\sin^2(\theta) + 7(1 - 2\cos^2\theta + \cos^4\theta)$

$= \cos^4\theta - 6\sin^2\theta + 7 - 14\cos^2\theta + 7\cos^4\theta$

$= 8\cos^4\theta - 6\sin^2\theta + 7 - 14(1-\sin^2\theta)$

$= 8\cos^4\theta - 6\sin^2\theta + 7 - 14 + 14\sin^2\theta$

$= 8\cos^4\theta + 8\sin^2\theta - 7$

$= A\cos^4(\theta) + B\sin^2(\theta) + C$

$\therefore A = 8,\ B = 8,\ C = -7$

Item: Question 17 — Paper 2 (Technology active)

Assessment objective: 1, 2, 3, 4, 5 and 6 — Complex familiar

This question has been included to illustrate the following subject matter:

- use the formula $\int \frac{1}{x}\,dx = \ln|x| + c$, for $x \neq 0$
- solve simple first-order differential equations of the form $\frac{dy}{dx} = f(x)g(y)$ using separation of variables
- use differential equations

- consider and solve problems involving motion in a straight line with non-constant acceleration and the use of expressions $\frac{dv}{dt}$, $\frac{d^2x}{dt^2}$, $v\,\frac{dv}{dx}$ and $\frac{d(\frac{1}{2}v^2)}{dx}$ for acceleration.

Student sample of effective response

This excerpt has been included to illustrate the:

- correct use of algebraic skills in the development of the separation of variables stage within the differential equation
- recognition that $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$
- correct interpretation of the given assumptions in the solution, i.e. recognising that the downwards direction must be considered positive ($g = 9.8$) and that displacement is measured from ground level ($x = -100$ at $t = 0$)
- efficient use of technology to solve equations
- evaluation of the reasonableness of a solution by recognising that a quadratic equation produces two solutions. Subsequently, by correctly considering the given assumption (regarding the sign of the direction of motion) the negative solution can be rejected
- demonstration of logical setting out of the key steps.

Foundational knowledge and problem solving	**QUESTION 17 (7 marks)** An object is released from rest at a height of 100 m above the ground. The motion of the vertical descent of the object is modelled by $v\frac{dv}{dx} = 9.8 - 0.004v^2 \quad (v \geq 0)$ where v is the velocity (m s^{-1}) and x is the displacement from the ground (m). Determine the velocity of the object when it strikes the ground.

$v \, dv/dx = 9.8 - 0.004v^2$

$= 9.8 - \frac{v^2}{250}$

$v \, dv/dx = \frac{2450 - v^2}{250}$

$\int \frac{v}{2450 - v^2} \, dv = \int \frac{1}{250} \, dx$

$-\frac{1}{2} \ln|2450 - v^2| = \frac{1}{250}x + c_1$

when $x = -100$, $v = 0$ as ↓ is +ve

$-100 = -125 \ln|2450 - v^2| + c$

$= -125 \ln|2450| + c$

$c = -100 + 125 \ln|2450|$

$x = -125 \ln|2950 - v^2| - 100 + 125 \ln|2450|$

Find v when $x = 0$

Solve N $(0 = -125 \ln|2450 - v^2| - 100 + 125 \ln|2450|)$

$\therefore v = \pm 36.73$

Only accept +ve v

$\therefore$ velocity when object reaches ground is 36.73 m/s

Item: Question 18 — Paper 2 (Technology active)

Assessment objective: 1, 2, 3, 4, 5 and 6 — Complex unfamiliar

This question has been included to illustrate the following subject matter:

- simulate repeated random sampling from a variety of distributions and a range of sample sizes to illustrate the approximate standard normality of $\frac{\bar{X} - \mu}{(s/\sqrt{n})}$ where s is the sample standard deviation
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain μ
- use $\bar{x}$ and s to estimate μ and σ, to obtain approximate intervals covering desired proportions of values of a normal random variable, and compare with an approximate confidence interval for μ.

Student sample/s of effective responses

This excerpt has been included to illustrate the:

- efficient use of technology in determining the z scores from the probabilities associated with the first two random samples
- clear and concise generation of the two suitable simultaneous equations in terms of μ and σ
- efficient use of technology in determining the solutions of the simultaneous equations and the solution of the equation in terms of n
- correct consideration of the confidence interval length requirement of ‘not containing μ’
- evaluation of the reasonableness of solutions by recognising that the range of sample sizes must involve positive integer values
- demonstration of logical setting out of the key steps.

Foundational knowledge and problem solving

QUESTION 18 (6 marks)

The mass of a certain species of kangaroo is known to be normally distributed with a mean mass of μ kg and standard deviation of σ kg.

When one of the kangaroos is randomly selected, the probability that its mass is greater than 83.2 kg is 0.145.

When a sample of 12 kangaroos is randomly selected, the probability that the sample mean mass is less than 74.1 kg is 0.079.

A 90% approximate confidence interval for μ is calculated using a random sample of n of the kangaroos that has a sample mean mass of 79.1 kg and a sample standard deviation equal to σ.

Determine the possible range of values that n could have been, given that the confidence interval did not contain μ.

$P(X > 83.2) = 0.145 \rightarrow n = 1$

$P(X < 74.1) = 0.079 \rightarrow n = 12$

A 90% C.I is given by,

$$\left(79.1 \pm 1.645 \frac{\sigma}{\sqrt{n}}\right)$$

For $P(X > 83.2) = 0.145$,

$$1.058 = \frac{83.2 - \mu}{\frac{s}{\sqrt{1}}}$$

$$1.058 = \frac{83.2 - \mu}{\sigma} \quad (2)$$

$$\sigma = \frac{83.2 - \mu}{1.058}$$

For $P(X < 74.1) = 0.079$

$$-1.4118 = \frac{74.1 - \mu}{\left(\frac{s}{\sqrt{12}}\right)}$$

$$-1.4118 = \frac{\sqrt{12}\,(74.1 - \mu)}{s}$$

$$s = \frac{\sqrt{12}\,(74.1 - \mu)}{-1.4118} \quad ①$$

$$-1.4118 = \frac{74.1 - \mu}{\frac{\sigma}{\sqrt{12}}} \leftarrow \text{sub } \sigma = \frac{83.2 - \mu}{1.058}$$

Using SolveN, $\mu = 76.6305$ $\quad \sigma = 6.20935$

$79.1 - 76.63 = 2.47$

$\therefore E = 2.47$

$$\left(79.1 \pm 1.645\left(\frac{6.20935}{\sqrt{n}}\right)\right) =$$

$$2.47 = 1.645\left(\frac{6.20935}{\sqrt{n}}\right)$$

$$n = 17.10$$

Does not contain μ if $n > 17$

Practices to strengthen

It is recommended that when preparing students for external assessment, teachers consider:

- Assessment objective 1 — select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4 (AS units 3 and 4), in particular, the
  - appropriateness of rounding during multiple calculations
  - accurate use of numeracy and algebra skills, e.g. appropriate use of brackets, use of fractions
  - recall of assumed knowledge from prior learning (e.g. properties of similar triangles) and the Mathematical Methods course
  - recall of relevant assumed knowledge from Units 1 and 2 (AS units 1 and 2)
  - knowledge of exact values in trigonometry, $\text{e.g. } \sin(60°) = \frac{1}{2}$
  - selection and use of appropriate formulas from the examination formula book.
- Assessment objective 2 — comprehend concepts and techniques drawn from Units 3 and 4 (AS units 3 and 4), in particular, the
  - use of de Moivre's theorem and the binomial expansion to prove trigonometric propositions
  - appropriate manipulation of variable separable differential equations, e.g. motion of an object falling under gravity with air resistance
  - use of assumptions in the solution of problems involving motion
  - proof of geometrical results using circle properties identified in the syllabus (AS EA).
- Assessment objective 3 — communicate using mathematical, statistical and everyday language and conventions, in particular, the
  - correct use of mathematical notation, e.g. vector and calculus notation
  - identification of parameters and definition of variables in determining mathematical models
  - logical organisation of key steps for complex familiar and unfamiliar problems.
- Assessment objective 4 — evaluate the reasonableness of solutions, in particular, the
  - verification of a solution through substitution into an appropriate equation
  - use of mathematical reasoning to reject a possible solution, e.g. consideration of the given domain
  - supporting or refuting of statements/claims based on mathematical results or checking the reasonableness of calculations by considering the context of the situation.
- Assessment objective 5 — justify procedures and decisions by explaining mathematical reasoning, in particular,
  - explaining mathematical thinking/justification
  - the labelling of equations so that they can be easily referenced in later working, e.g. to communicate when equations are being equated
  - explicitly stating when substitution occurs and clearly linking to values or expressions that are being substituted
  - completing proofs of results for sums for any positive integer using mathematical induction by ensuring algebraic procedures involving one side of the proof are conducted independently of the other side

- constructing mathematical arguments and the provision of reasons for choices made and/or conclusions reached.

- Assessment objective 6 — solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4 (AS units 3 and 4), in particular, being able to
  - analyse, generalise and translate the given information into a mathematically workable format based on the context of the problem
  - identify when the use of technology obviates time consuming analytic procedures
  - use conceptual understanding of the subject matter of the Specialist Mathematics syllabus to make connections to new information.

Further recommendations for preparing students for external assessment include:

- supporting students in their familiarisation of words such as ‘show’, ‘determine’, ‘state’, ‘verify’ and ‘prove’. Glossary entries in the syllabus should also be brought to the attention of students
- ensuring students are aware that many simple familiar questions are scaffolded and subsequently recognise the significance of phrases such as ‘use the result from …’ or ‘show that the result is … ’ in providing guidance for their response to the following part/s of that question. Students should also be aware that questions worth more than 1 mark require evidence of suitable mathematical reasoning in order to gain full marks
- ensuring students are aware that if they are precluded from completing a verification of a result because of an error in their previous working, they can still be awarded a ‘follow through mark’ by including a mathematical statement indicating the intended verification requirement and/or a relevant comment. Furthermore, it also should be drawn to students’ attention that the intention of the cue to ‘state an appropriate method of verifying your result’ in a question is that only a brief explanation is required rather than a full verification procedure
- supporting students in responding to complex familiar and complex unfamiliar questions by presenting their working clearly using a logical sequence of key steps. Students should be encouraged to use explanatory and linking statements, use suitable mathematical notation and/or vocabulary of the syllabus, and arrange their responses in a top-to-bottom, left-to-right structure. Units should also be included in the final answer of questions, where appropriate
- enhancing students’ abilities in their use of a graphics calculator in the technology-active section of the examination. Such abilities include
  - graphing functions including parametric graphs
  - solving equations where one solution only is required, e.g. use of a numerical solve technique
  - solving equations where multiple solutions may be possible, e.g. use of a graphical technique or an advanced use of a numerical solve technique
  - solving simultaneous equations
  - solving equations involving calculus, e.g. solving for the unknown value of the upper limit of a definite integral
  - using statistical facilities, e.g. determining normal distribution probabilities, inverse normal calculations, confidence intervals calculations
  - matrix calculations, e.g. using matrix operations, finding determinants and/or inverse matrices
  - calculus calculations, e.g. finding the derivative at a known point on a given function, determining the value of a definite integral

  - the use of the appropriate mode setting (radian/degree) in order to respond to the requirements of a question

- providing students with opportunities to analyse the context of mathematical problems and make decisions about the concepts, techniques and technology that should be used in their solution. This is an important part of the problem-solving process and students should be supported in practising these skills independently
- providing students with opportunities to move beyond simply practising basic techniques to developing a deeper conceptual understanding of the subject matter, enhancing their ability to connect what they already know to new information.