Queensland Curriculum and Assessment Authority

Specialist Mathematics 2019 v1.2

IA3: Sample assessment instrument

Examination (15%)

This sample has been compiled by the QCAA to assist and support teachers in planning and developing assessment instruments for individual school settings.

Student name

Student number

Teacher

Exam date

Marking summary

Criterion	Marks allocated	Provisional marks
Foundational knowledge and problem solving	15	
Overall	**15**	

Conditions

Technique	Examination
Unit	Unit 4: Further calculus and statistical inference
Topic/s	Topic 1: Integration and applications of integration Topic 2: Rates of change and differential equations Topic 3: Statistical inference
Time	2 hours + 5 minutes perusal
Seen / unseen	Unseen questions
Other	Only the QCAA formula sheet must be provided. Notes are not permitted. Use of technology is required; schools must specify the technology used.

Instructions

- Show all working in the spaces provided.
- Write responses using black or blue pen.
- Unless otherwise instructed, all numerical answers should be given to 2 decimal places.
- This assessment instrument is comprised of two papers — Paper 1 is technology free and Paper 2 is technology-active.
- Use of a non-CAS graphics calculator is required for Paper 2 (technology-active) only, unless an analytic procedure is required.

Paper 1: Technology free

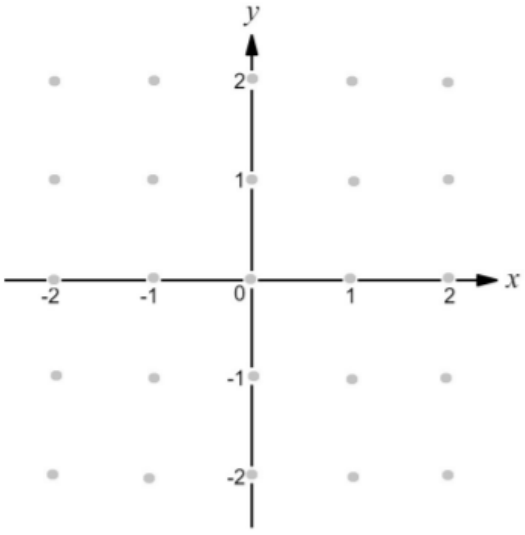
Total marks: 35

Question 1 (3 marks)

Use an appropriate substitution to determine an antiderivative of the function $f(x) = 2xe^{x^2+3}$.

Question 2 (4 marks)

Using the grid below:



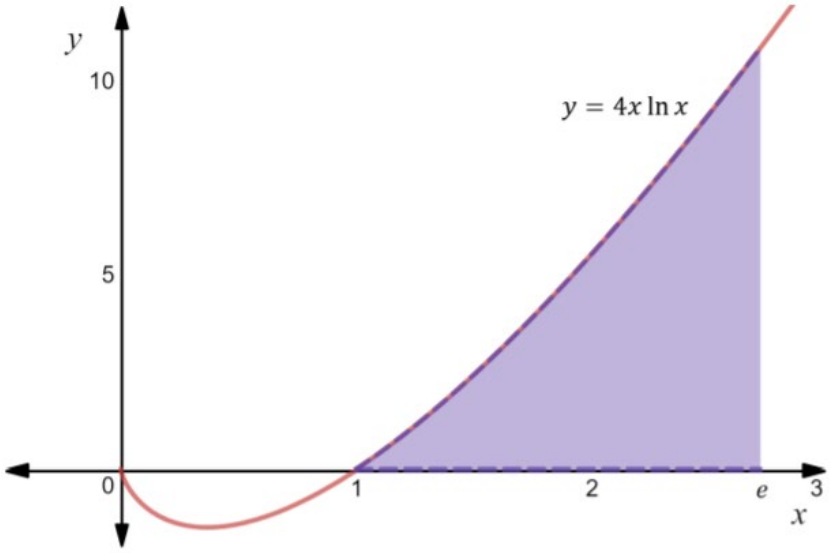
a. sketch a slope field for the differential equation $\frac{dy}{dx} = 1 - y$ using $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$.

b. use these results to sketch an appropriate solution curve for $\frac{dy}{dx} = 1 - y$ given that $y = -2$ when $x = -1$.

Question 3 (3, 2 marks)

a. Use an appropriate integration technique to show that $\int 4x\ln(x)dx = 2x^2\ln(x) - x^2 + c$.

b. Show that the shaded area in the sketch below is equal to $e^2 + 1$ square units.



Question 4 (5 marks)

Determine the volume generated by rotating the curve $y = \cos(x)$ between $x = 0$ and $x = \frac{\pi}{4}$ about the x-axis. Leave your answer in exact form.

Question 5 (4 marks)

A spherical balloon is inflated such that the radius increases at a constant rate of 3 centimetres per minute.

Determine the rate at which the volume in increasing when the radius of the balloon is 5 centimetres. Leave your answer in exact form.

Question 6 (5 marks)

The velocity of a particle as a function of its position is given by $v(x) = 9 + x^2 (\text{ms}^{-1})$.

If the particle is initially at $x = -3$ metres, show that $\frac{\pi}{12}$ seconds later the particle is at the origin.

Question 7 (4, 5 marks)

A rock is dropped from a 20-metre high platform. Simultaneously, a particle is launched upwards from ground level, directly below the rock, at 12 ms^{-1}. Assume g = 10 ms^{-2}.

a. An observer estimates that the particle collided with the rock at approximately 9 metres above the ground. Evaluate the reasonableness of this estimation.

b. Determine the exact speed at which the particle would have to be launched so that the collision with the rock occurred 10 metres above the ground.

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Paper 2: Technology-active

Total marks: 30

Question 8 (4 marks)

The time t (in minutes) to perform haircuts at a local hairdresser is a random variable with probability density function $f(t) = 0.04e^{-0.04t}$, $t \geq 0$.

Determine the probability that a haircut takes less than 10 minutes, expressing your answer as a percentage to the nearest whole number.

Question 9 (1.5, 4.5 marks)

House prices in a large regional Queensland city are distributed such that the mean price is $280 000 with a standard deviation of $80 000.

Fifty samples of house prices are randomly collected from the city. Each sample contains prices for 200 houses.

a. Determine the mean and standard deviation of the sampling distribution of the house prices

b. Determine in how many samples the mean price would be expected to be above $290 000.

Question 10 (4 marks)

A union claims that the mean of its members' weekly salaries has fallen to $1200. A statistician subsequently conducts a random sample of 100 members' weekly salaries and calculates the mean of the sample data to be $1280 with a standard deviation of $125.

Use a 99% confidence interval to assess the validity of the union's claim, based on the statistician's data.

Question 11 (4 marks)

The cross-sectional area of a road cutting is closely modelled by the area in m^2 between the x-axis and the function $f(x) = \frac{\ln(x+0.1)}{2.1} - x + 7.1$ for $0 \leq x \leq 8$ (m).

Use Simpson's rule with four strips to determine the approximate area of the cross-section.

Question 12 (4 marks)

Airline data shows that on Brisbane to Los Angeles flights, luggage mass averages 17.3 kilograms per passenger with a standard deviation of 3.1 kilograms.

Given the probability of the average luggage mass being less than 16 kilograms for a tour group of size n is 0.0179, determine the value of n. Assume that luggage mass is normally distributed.

Question 13 (8 marks)

A certain function $y = f(x)$ has a gradient of $\frac{y^2-11y+30}{\cos^2(x)}$ (for $\cos^2(x) \neq 0$) and a y-intercept of 7.

Determine the smallest positive value of x that satisfies $f(x) = 10$.

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Examination marks summary

Paper 1 (technology free)	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
1	3		
2	4		
3	5		
4	5		
5	4		
6		5	
7		4	5
Totals	**21**	**9**	**5**

Paper 2 (technology-active)	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
8	4		
9	6		
10	4		
11	4		
12		4	
13			8
Totals	**18**	**4**	**8**

Combined papers	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)	Across all levels
Totals	**39**	**13**	**13**	**65**
Percentage	**60%**	**20%**	**20%**	**100%**

Instrument-specific marking guide
(IA3): Examination — short response (15%)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions, and prove propositions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	>93%	15
	>87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions, and prove propositions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	>80%	13
	>73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions, and prove propositions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	>67%	11
	>60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	>53%	9
	>47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	>40%	7
	>33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques;	>27%	5

The student work has the following characteristics:	Cut-off	Marks
some description of the reasonableness of results; and infrequent application of mathematical concepts and techniques.	>20%	4
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of results; and disjointed application of mathematical concepts and techniques.	>13%	3
	>7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.	>0%	1
• does not satisfy any of the descriptors above.		0

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