

# Specialist Mathematics 2019 v1.2

## Unit 2 sample assessment instrument

October 2018

### Examination

This sample has been compiled by the QCAA to assist and support teachers in planning and developing assessment instruments for individual school settings.

Schools develop internal assessments for each senior subject, based on the learning described in Units 1 and 2 of the subject syllabus. The examination must ensure that all assessment objectives are assessed.

### Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 2 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 2 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 2 topics.

<b>Subject</b>	Specialist Mathematics
<b>Technique</b>	Examination — short response
<b>Unit</b>	2: Complex numbers, trigonometry, functions and matrices
<b>Topic</b>	1: Complex numbers 1 2: Trigonometry and functions 3: Matrices

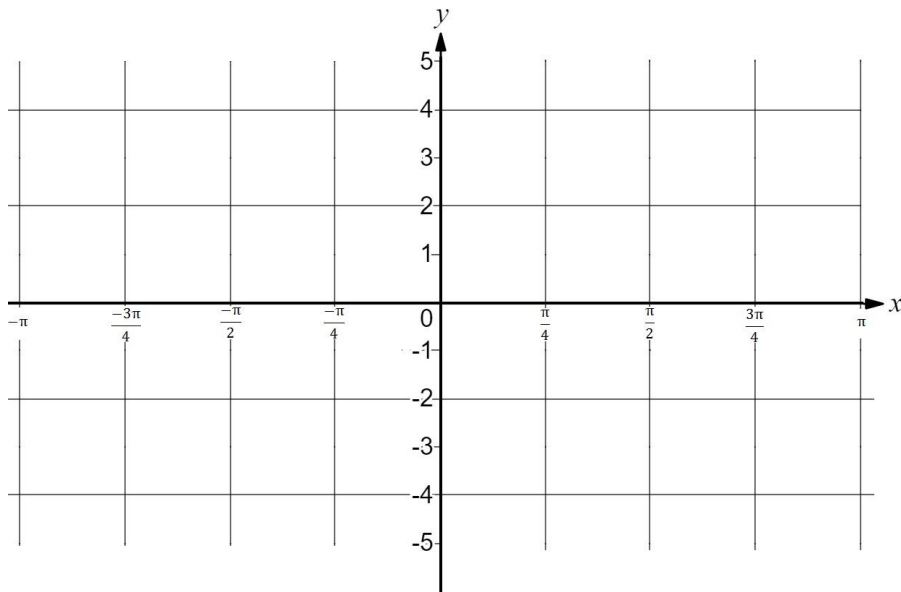
<b>Conditions</b>			
<b>Response type</b>	Short response		
<b>Time</b>	Paper 1: 60 minutes Paper 2: 60 minutes	<b>Perusal</b>	Paper 1: 2.5 minutes Paper 2: 2.5 minutes
<b>Other</b>	<ul style="list-style-type: none"> <li>• QCAA formula sheet must be provided</li> <li>• Approved non-CAS graphics calculator</li> </ul>		
<b>Instructions</b>			
<ul style="list-style-type: none"> <li>• Show all working in the spaces provided.</li> <li>• Write responses using black or blue pen.</li> <li>• Use of a non-CAS graphics calculator is permitted in Paper 2 (technology-active) only.</li> </ul>			
<b>Feedback</b>			

**Question 1 (1 mark)**

Simplify  $3i^2 + 2$ .

**Question 2 (6 marks)**

a. Sketch and label  $y = \cos(2x)$  over  $-\pi \leq x \leq \pi$  on the Cartesian plane below.



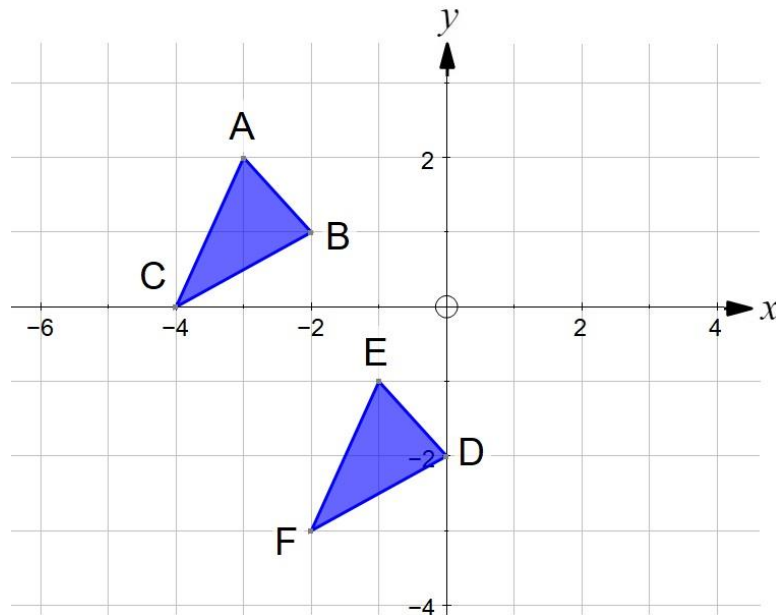
- b. Consider  $y = \sec(2x)$ . Briefly discuss how you could sketch this function based on its relationship with  $y = \cos(2x)$ .
- c. Sketch and label  $y = \sec(2x)$  over  $-\pi \leq x \leq \pi$  on the same Cartesian plane above. Ensure all key features of the graph of this function are evident.

Question 3 (2 marks)

Given  $z = -\sqrt{5} + 7i$ , determine the exact value of the modulus of  $z$ .

Question 4 (6 marks)

Triangle DEF is the result of a translation of triangle ABC as shown on the Cartesian plane below.



a. State the translation vector.

b. Determine the vertices of triangle LMN, which represent the image of triangle DEF under the linear transformation  $\mathbf{T} = \begin{bmatrix} -2 & -1 \\ -3 & 1 \end{bmatrix}$ .

c. Determine how many times larger the area of triangle LMN is than the area of triangle ABC.

**Question 5 (3 marks)**

Prove  $\cos(3\theta) \cos(\theta) = \cos^2(\theta) - \sin^2(2\theta)$ .

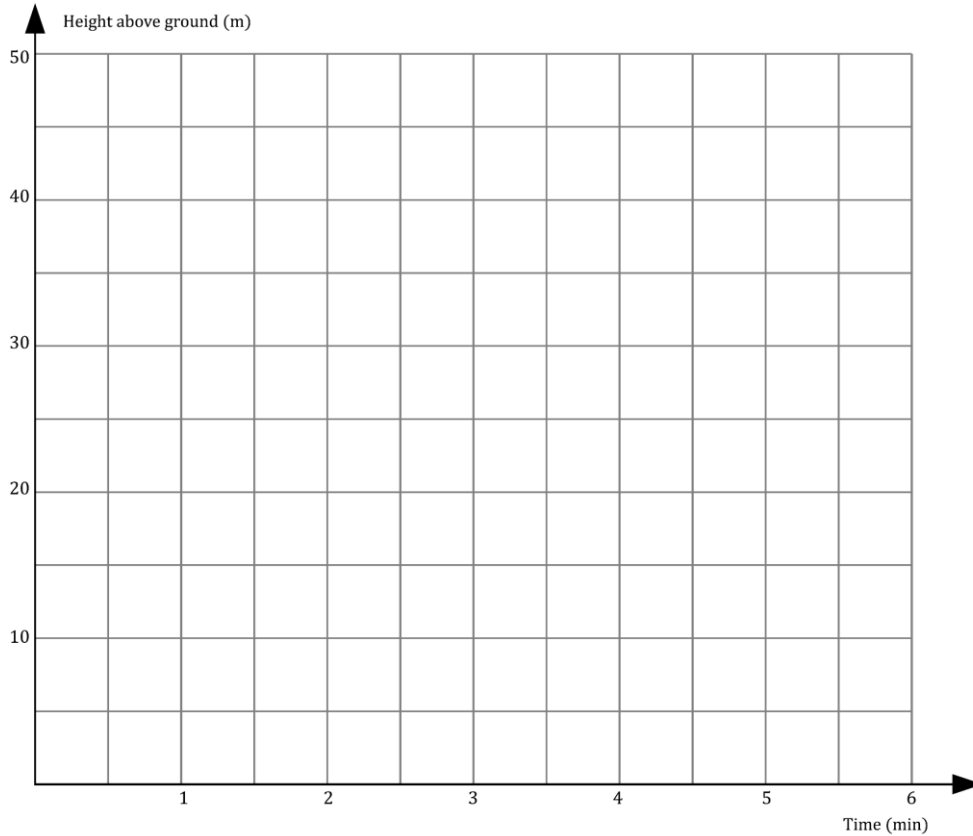
Question 6 (4 marks)

If  $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is non-singular, prove  $|\mathbf{A}| = \frac{1}{|\mathbf{A}^{-1}|}$ .

**Question 7 (9 marks)**

A Ferris wheel 40 metres in diameter rotates once every 4 minutes. The centre axle of the Ferris wheel is 25 metres from the ground. Assume that a passenger is seated in a carriage located on the circumference of the wheel.

- a. Sketch the height of the passenger above the ground over the first 6 minutes of motion, assuming the passenger is initially positioned at the bottom of the wheel, using the Cartesian plane below.



- b. From your sketch, determine the amplitude, angular frequency and vertical shift.

c. Generate a suitable mathematical model to determine the exact height of the passenger after 40 seconds of motion.

d. Evaluate the reasonableness of your results.



**Question 8 (7 marks)**

Corresponding operations can sometimes be performed using properties from two different areas of mathematics that produce equivalent results.

- a. By considering the geometric effect of the product of complex numbers, determine a single transformation matrix  $\mathbf{T}$  that corresponds to multiplying any complex number by  $w$  where  $w = 3\text{cis}(0.5)$ .

- b. Consider an example of the product of  $w$  with another complex number  $z = 2\text{cis}(0.3)$ . It can easily be shown that  $wz = 6\text{cis}(0.8)$  using complex multiplication.  
Evaluate the reasonableness of the transformation matrix  $\mathbf{T}$  by calculating a result that corresponds to the product  $wz$ .

**Question 9 (4 marks)**

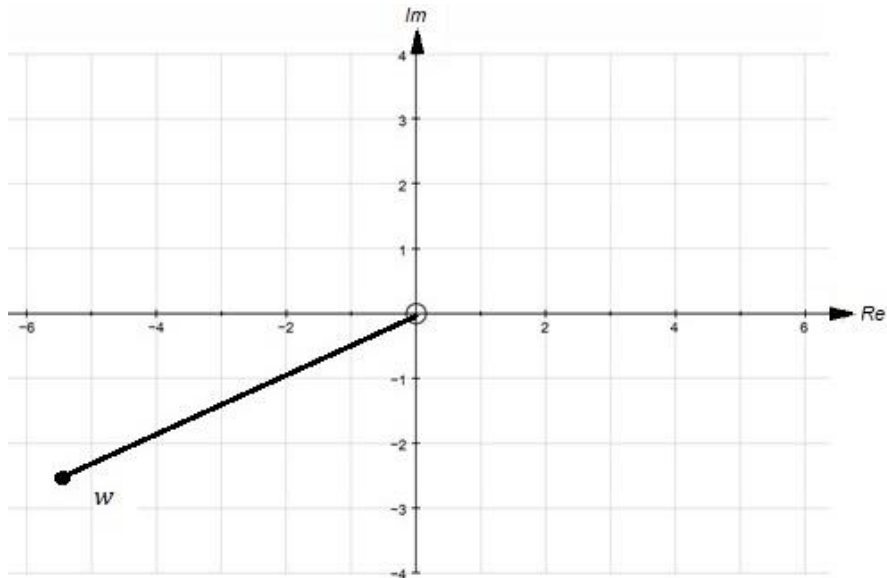
Consider the complex number  $w = 1 - 2i$ .

a. Determine the argument of  $w$  to the nearest degree.

b. Determine  $\text{Im}(w^6)$ .

**Question 10 (4 marks)**

The complex number  $w$  is shown on the Argand plane below.



a. Sketch the conjugate of  $w$  on the diagram above.

b. Based on an estimation that  $|w| = 6.0$  and  $\arg(w) = -147.5^\circ$ , express  $w$  in Cartesian form rounded to two decimal places. Briefly discuss the reasonableness of this estimation.

**Question 11 (3 marks)**

Given that  $\operatorname{cosec}(\alpha) = 1.25$ , use a suitable trigonometric identity to determine the value/s of  $\cot(\alpha)$  for  $\frac{\pi}{2} \leq \alpha \leq \pi$ .

**Question 12 (3 marks)**

Consider the matrices  $\mathbf{A} = \begin{bmatrix} -2 & -3 \\ 3 & 2 \end{bmatrix}$  and  $\mathbf{B} = \begin{bmatrix} 2 & 5 \\ -4 & 0 \end{bmatrix}$ .

a. State the matrix element equal to  $b_{2,1}$ .

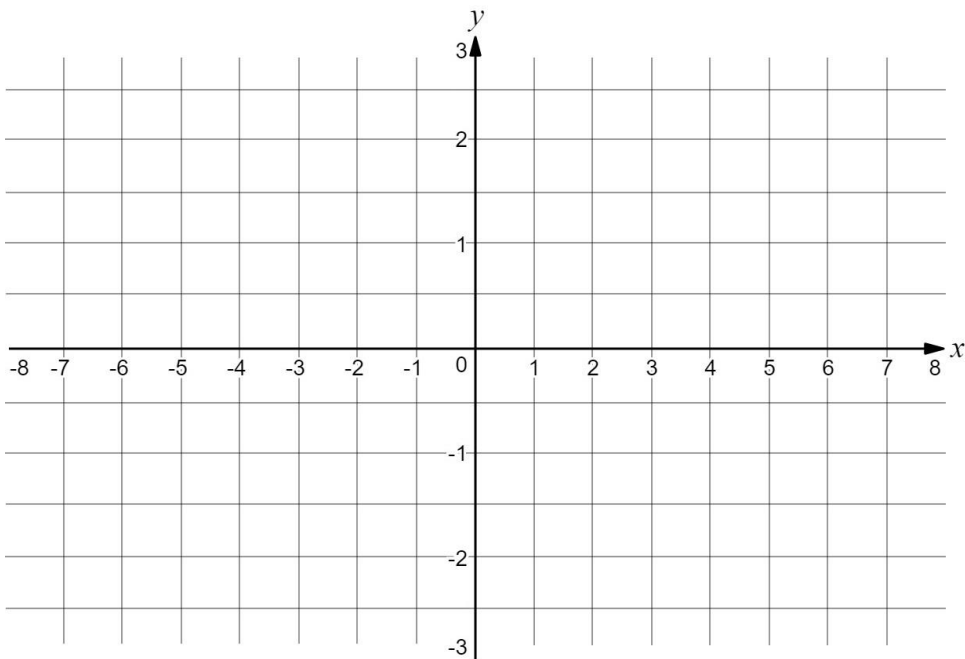
b. Determine  $2\mathbf{A} - \mathbf{B}^2$ .

**Question 13 (4 marks)**

Solve the matrix equation  $\mathbf{XA} = \mathbf{B}$  for  $\mathbf{X}$ , given that  $\mathbf{A} = \begin{bmatrix} -1 & 2 & 4 \\ -2 & 0 & 3 \\ 1 & -4 & -3 \end{bmatrix}$  and  $\mathbf{B} = [-2.2 \quad 10.4 \quad 9.3]$ .

**Question 14 (3 marks)**

Sketch the graph of  $y = -\frac{1}{5} |2x + 1| + 2$  for  $-8 \leq x \leq 8$  on the Cartesian plane below, clearly indicating the position of the vertex and  $x$ -intercepts.



**Question 15 (6 marks)**

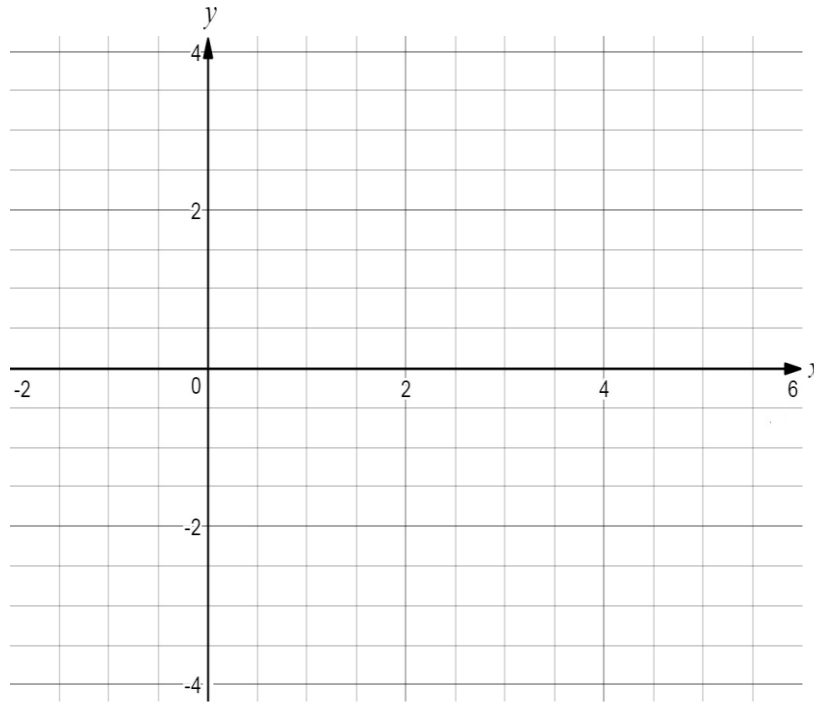
a. If one root of the real quadratic equation  $f(z) = 0$  is  $z = 2 - 3i$ , determine  $f(z)$ .  
Show your result in expanded form.

b. Evaluate the reasonableness of your result.

**Question 16 (10 marks)**

A particle is oscillating about a central point such that its displacement  $x$  (centimetres) at any time  $t$  (seconds) is modelled by  $x = \sin(t) + \cos(t) + \cos\left(t - \frac{\pi}{3}\right)$  for  $t \geq 0$ .

- a. Determine the initial time that the particle is positioned 0.5 centimetres from the central point using a graphical method of solution. Justify your solution using a relevant sketch on the Cartesian plane below.



- b. Use algebraic methods to verify the reasonableness of your solution.





Extra working space (if required)

# Examination marks summary

Paper 1 (technology-free)	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
1	1		
2	6		
3	2		
4	6		
5	3		
6		4	
7	4	5	
8			7
<b>Totals</b>	<b>22</b>	<b>9</b>	<b>7</b>

Paper 2 (technology-active)	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
9	4		
10	4		
11	3		
12	3		
13	4		
14	3		
15		6	
16	2		8
<b>Totals</b>	<b>23</b>	<b>6</b>	<b>8</b>

Combined papers	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)	Across all levels
<b>Totals</b>	45	15	15	75
<b>Percentage</b>	60%	20%	20%	100%