Specialist Mathematics 2019 v1.2

Unit 1 sample assessment instrument

October 2018

Examination

This sample has been compiled by the QCAA to assist and support teachers in planning and developing assessment instruments for individual school settings.

Schools develop internal assessments for each senior subject, based on the learning described in Units 1 and 2 of the subject syllabus. The examination must ensure that all assessment objectives are assessed.

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 1 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 1 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 1 topics.

181245

Subject	Specialist Mathematics		
Technique	Examination		
Unit	1: Combinatorics, vectors and proof		
Topic	1: Combinatorics 2: Vectors in a plane 3: Introduction to proof		
Conditions			
Response type	Short response		
Time	120 minutes	**Perusal**	5 minutes
Other	• QCAA formula sheet • Notes are not permitted • Non-CAS graphics calculator		
Instructions			
• Show all working in the space provided. • Write responses using a black or blue pen. • Use of appropriate technology is permitted unless an analytical procedure is specified. • Unless otherwise instructed, give answers to **two decimal places.**			
Feedback			

Part A
Simple familiar — total marks: 39

Question 1 (2 marks)

If 24 horses run in the Melbourne Cup race, in how many different ways can the first three places be filled?

Question 2 (1½, 4½ marks)

A student committee consisting of five members is to be chosen from a group of 15 students.

a. Determine the number of different committees that could be organised if there are no restrictions on membership.

b. Three of the 15 students are school captains. Determine the probability that a committee selected at random includes at least two of the school captains.

Question 3 (3 marks)

At the front of a building there are 10 garage doors in a single row. Of the 10 doors, two are to be painted red, three painted green, one painted blue and the remaining four painted orange.

Determine the number of different arrangements of door colours if the two red doors must be next to each other.

Question 4 (3 marks)

Consider the two coordinates $A(-3, -1)$ and $B(-7, 2)$.

Determine the displacement vector $\overrightarrow{AB}$, giving your answer in polar form.

Question 5 (3½, 2½ marks)

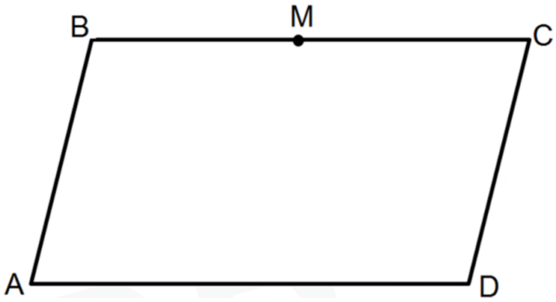
Consider the two vectors $\boldsymbol{a} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$.

a. Determine the smallest angle between these two vectors. Round your answer to the nearest degree.

b. Determine the projection of $\boldsymbol{a}$ on $\boldsymbol{b}$. Give your answer in exact form with a rational denominator.

Question 6 (2 marks)

ABCD is a parallelogram as shown in the diagram below. M is the midpoint of BC.



Given $\overrightarrow{AB} = \boldsymbol{a}$ and $\overrightarrow{AD} = \boldsymbol{b}$, write an expression for $\overrightarrow{DM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

Question 7 (1½, 3½ marks)

Consider the set of odd numbers {1, 3, 5, 7, … }.

a. Show that the following statement is false by providing a counter-example:

'The square of an odd number is always an even number.'

b. Prove that the square of any odd number always produces another odd number.

Question 8 (2 marks)

Consider the following statement:

'If two vectors are perpendicular, then their scalar product is zero.'

a. State the contraposition of this statement.

b. State whether the contraposition statement is true or false. Briefly justify your decision.

Question 9 (4 marks)

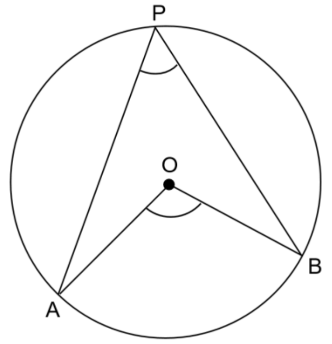
The initial part of a proof by contradiction to show that $\sqrt{2}$ is an irrational number is given below.

$$\textit{Assume that } \sqrt{2} \textit{ is a rational number.}$$
$$\textit{Then } \sqrt{2} \textit{ can be written as a ratio of integers such that } \sqrt{2} = \frac{a}{b}, a, b \in Z, b \neq 0$$
$$\textit{where } a \textit{ and } b \textit{ only have } 1 \textit{ as a common factor}$$
$$\therefore \quad 2 = \frac{a^2}{b^2}$$
$$\Rightarrow a^2 = 2b^2$$
$$\Rightarrow a^2 \textit{ must be even}$$
$$\Rightarrow a \textit{ must be even}$$
$$\Rightarrow a = 2k, \ k \in Z$$
$$\dots$$

Complete the remainder of this proof by contradiction.

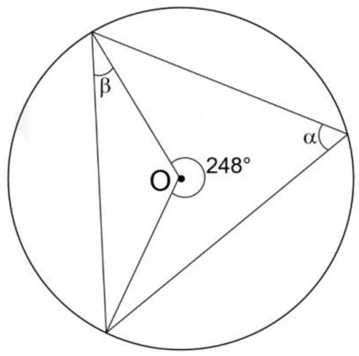
Question 10 (3, 3 marks)

a. Prove that the angle at the centre subtended by an arc of a circle is twice the angle at the circumference subtended by the same arc, as described in the diagram below.



b. Determine the value of angles α and β in the diagram below.

Assume O is the centre of the circle.
Include suitable angle codes to support your mathematical calculations.



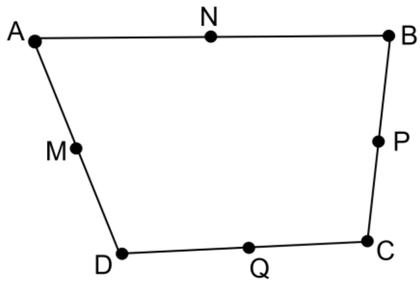
Not to scale

Part B
Complex familiar — total marks: 13

Question 11 (6 marks)

Consider the quadrilateral ABCD below.

Points M, N, P and Q are midpoints of the respective sides of the quadrilateral.



Use vector methods to prove that points M, N, P and Q join to form a parallelogram.

Question 12 (7 marks)

A yacht captain sets a sailing course at 8.00 knots through the water at a true bearing of 125.0°. However, as a result of a strong current, the yacht actually travels at 8.04 knots at a true bearing of 146.6°.

The yacht captain estimates that the direction of the current is running towards the south-west.

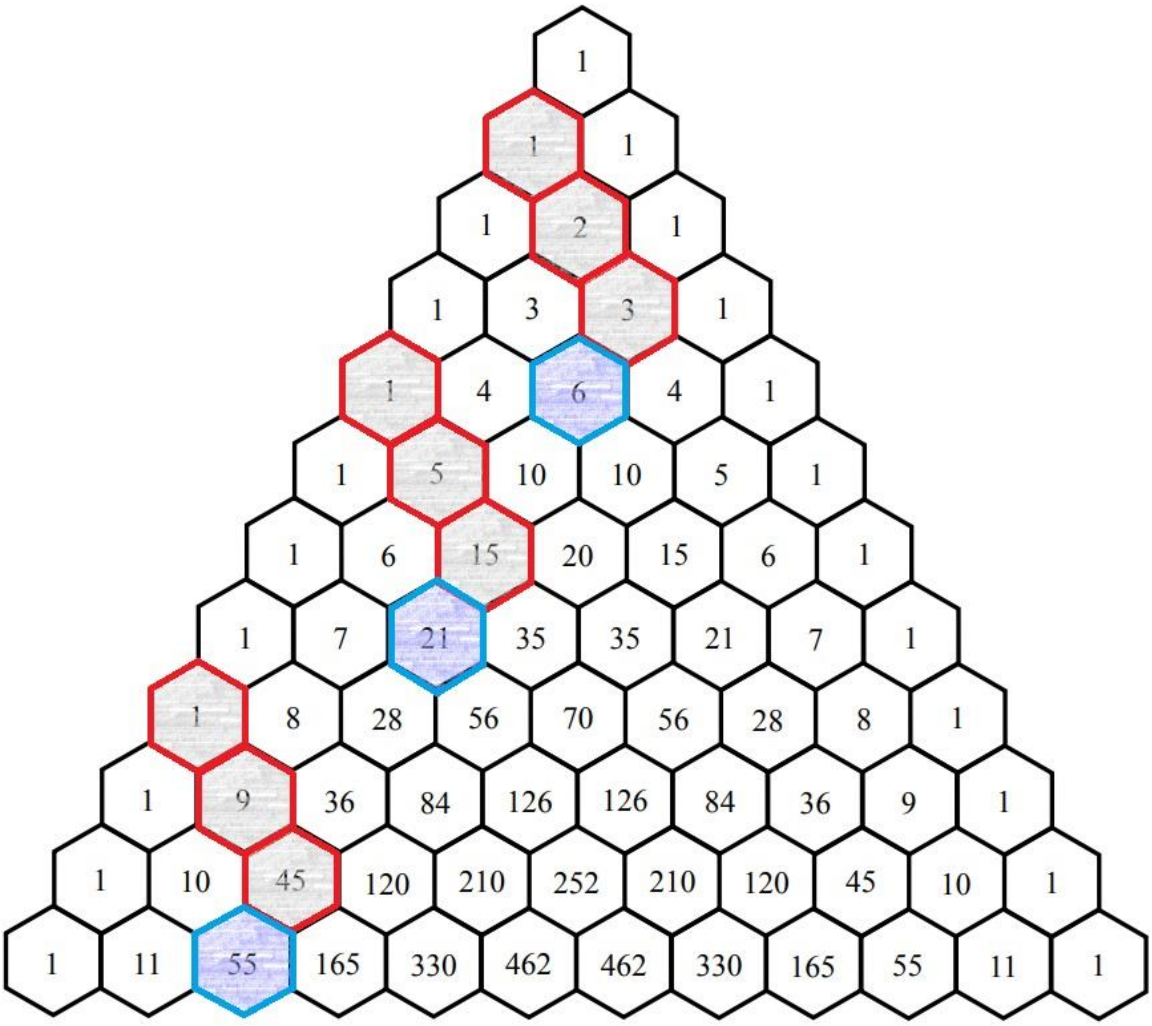
Evaluate the reasonableness of the estimation skills of the captain using a vector method.

Part C
Complex unfamiliar — total marks: 13

Question 13 (6 marks)

A pattern of numbers in Pascal's triangle is shown below. The pattern shows the sum of the numbers in the three red hexagons equals the number in the blue hexagon, e.g. 1 + 5 + 15 = 21.

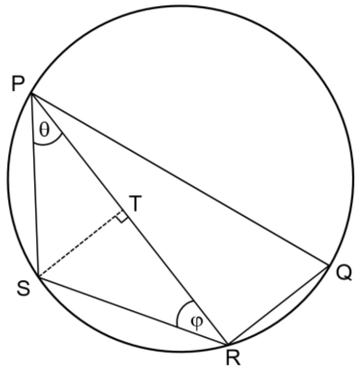
Use your knowledge of combinatorics to prove this pattern is true for any general case.



Question 14 (7 marks)

The points P, Q, R and S lie on a circle of radius r. PQ is a diameter of the circle. The point T is positioned on the diagonal PR so that ST is perpendicular to PR as shown in the diagram below.

Let $\angle$ RPS $= \theta$ and $\angle$ PRS $= \varphi$.



Show that PT $= 2r\cos\theta\sin\varphi$.

Examination marks summary

Question number	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
1	2		
2	6		
3	3		
4	3		
5	6		
6	2		
7	5		
8	2		
9	4		
10	6		
11		6	
12		7	
13			6
14			7

Combined papers	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)	Across all levels
Totals	39	13	13	65
Percentage	60%	20%	20%	100%