Specialist Mathematics 2019 v1.2

Unit 1 Topic 1 high-level annotated sample response

May 2021

Problem-solving and modelling task

This sample has been compiled by the QCAA to assist and support teachers to match evidence in student responses to the characteristics described in the assessment objectives.

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 1 Topic 1
2. comprehend mathematical concepts and techniques drawn from Unit 1 Topic 1
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 1 Topic 1.

Task

Context
In 1870, Mark Twain wrote a humorous story called ‘Science vs. Luck’ about a gambler accused of illegal involvement in a ‘game of chance’. The gambler later escaped punishment by ‘proving’ that his gambling game was in fact a ‘game of science’ based on his understanding of mathematics rather than just luck. These days, gambling is often portrayed as a source of entertainment and an opportunity to gain quick financial rewards. However, while the mathematical probabilities and associated rewards of the ‘winning outcomes’ vary for each game of chance, the reality is that gambling odds always favour the long-term profitability of the gambling venue or ‘house’. Many long-term players have difficulty limiting the amount of money and time they spend gambling. This can harm the individual, their family and friends, and the wider community. In Australia, the long-term expected return to the casino (the ‘house edge’) varies for each gambling activity as regulated by law. Electronic gaming machines (‘pokies’) are required to meet the regulation of a house edge of between 10% and 15%.[1] [1]**Source:** Tasmanian Government, Department of Health and Human Services, *Know Your Odds*, ‘The house edge’, http://knowyourodds.net.au/house-edge.
Task
Your task is to design your own unique ‘game of science’, clearly outlining background information and rules. Your game design must be interesting enough to motivate people to play it. Decide on at least three winning outcomes in your game from which a player can gain appropriate financial rewards commensurate with their relative probability of occurrence. Across these winning outcomes, you must: • demonstrate an understanding of permutations and/or combinations calculations of the theoretical probabilities • show evidence that you have applied the addition principle and multiplication principle. Your game design must meet the required regulation of a house edge, as described in the context above, using the rule: $\text{House edge} = \frac{\text{expected return to the casino}}{\text{amount spent by gamblers}} \times 100\%$ Simulate relevant observations of your game using technology. Compare the experimental results from your simulation against the theoretical probabilities associated with the winning outcomes of both the player and the house, and against the theoretical house edge.

Sample response

Criterion	Marks allocated	Result
Formulate Assessment objectives 1, 2, 5		
Solve Assessment objectives 1, 6		
Evaluate and verify Assessment objectives 4, 5		
Communicate Assessment objective 3		
Total		

Annotations	Response
Communicate coherent and concise organisation of the response introduction clearly describes what the task is about and concisely outlines the intent of the writer	# 1 Introduction ### 1.1 Purpose I would like to introduce an exciting, new game to your casino called 'Big Numbers — Big Money'. It promises to deliver a fresh and motivating gaming experience to the player while providing a 'long-term' profit to your establishment. Included in this report are the details of the game, a full mathematical analysis of the probabilities of five winning outcomes for the player and the house based on theoretical calculations and experimental data as well as a statistical analysis confirming a house edge of between 10% and 15%. ### 1.2 Planning behind the game design My plan was to design a unique game that was based on simple rules for players to understand, inexpensive for a casino to establish and sufficiently motivating to promote a popular following. 'Big Numbers — Big Money' requires a screen that will generate a randomly ordered set of three digits from 0–9, with no duplicates permitted.
Formulate accurate translation of aspects of the problem by identifying mathematical concepts and techniques identification of the application of permutations, multiplication and addition principles	### 1.3 Planning behind the mathematics of the game design My game was designed with five 'player-winning' outcomes, allowing sufficient 'house-winning' outcomes to produce an acceptable long-term profit to your casino. In deciding the initial monetary return to the player for each of the five winning outcomes, whilst the amounts were randomly assigned, I ensured that they were commensurate with their relative probability of occurrence (based on a $1 bet). By assuming randomness and the non-duplication of digits in the 3-digit display of my game design, I could implement the use of permutations and the multiplication principle in determining the number of different ways each of the winning outcomes occurred. Furthermore, by ensuring that at least one of the winning outcomes was described using an 'or' scenario, the application of the addition principle could be demonstrated. These results were used to calculate the corresponding theoretical probabilities for each of the winning outcomes for both the player and the house. Utilising these probabilities, in conjunction with my chosen odds, the theoretical house edge percentage was determined using the rule $House\ edge = \frac{expected\ return\ to\ the\ casino}{amount\ spent\ by\ gamblers} \times 100\%$
Formulate accurate translation of aspects of the problem by identifying mathematical concepts and techniques	I then considered whether a house edge of between 10% and 15% was met and modified the monetary return to the player until the required range was attained. To verify these theoretical results, I designed a spreadsheet to simulate a large number of observations of the game. This required the application of relevant spreadsheet functions including random number generation, conditional logic formulas and lookup tables. The experimental probabilities and house edge corresponding to each of my theoretical calculations were then determined. Subsequently, I calculated the percentage error between the theoretical and experimental results using a tolerance of less than 5% as a measure of the reasonableness of my results[1].

[1] https://socratic.org/questions/what-percent-error-is-too-high

Formulate

statement of some assumptions

no evidence of documentation of assumptions

1.4 Assumptions

- The computer for the casino game and the spreadsheet program used to simulate the game generate random numbers with uniform distribution.
- Filtering out solutions from a list that allows repetition yields the same probabilities as generating digits without repetition.
- 72 000 simulations are sufficient for the experimental probabilities to approximate the theoretical probabilities.
- The spreadsheet is a suitable analogue for the game.

Communicate

correct use of appropriate technical vocabulary, procedural vocabulary, and conventions

coherent and concise organisation of the response

clear connection between discussion and table

diagram and table concisely summarises the discussion and the specifications of the winning outcomes

1.5 Specifications of the game of 'Big Numbers — Big Money'

The game requires a screen designed to display three digits each ranging from 0 to 9 with no digit appearing more than once. In the exemplar below, the 3-digit display forms the number 295. Each digit is selected independently of the other digits, with each digit appearing a maximum of once in the 3-digit display and randomly generated with equal chance of occurring.

First digit	Second digit	Third digit
2	9	5

Rules of 'Big Numbers — Big Money'

- The ***house wins*** if the 3-digit display forms a number in the range of less than 600 (as shown in the shaded row in Table 1).
- The ***player wins*** if the 3-digit display forms a number which is 600 or above. Table 1 indicates the subsequent breakdown of this range of numbers into the five winning outcomes for my game design. As the name of the game suggests, the bigger the number, the bigger the financial gain will be to the player.

Table 1

Winning outcomes P = player, H = house	3-digit range
1 (P)	960–999
2 (P)	900–959
3 (P)	820–899
4 (P)	720–819
5 (P)	600–719
1 (H)	000–599

2 Results and statistical analysis

2.1 Theoretical calculations of winning outcomes

Total number of 3-digit numbers where duplication is allowed
$= 10 \times 10 \times 10 = 1000$

Formulate

accurate translation of aspects of the problem

stated variables

Let X represent the number formed by the 3-digit display.

There are 10 digits possible for the first column, 9 digits for the second column, and 8 for the third column (due to the restriction that no digit is duplicated).

Total number of different values of X = $^{10}P_3$ = 720

Communicate

correct use of appropriate technical vocabulary, procedural vocabulary, and conventions

Winning outcomes for the house

Table 2 shows the total number of different values of X which produce a winning outcome for the house (the 3-digit displays that form a number less than 600).

Table 2

Range of X	Restrictions by column	Number of different winning values of X
000 - 599	First digit: any of the 6 digits from 0-5 Second digit: any of the remaining 9 digits Third digit: any of the remaining 8 digits	$^6P_1 \times {}^9P_2$ = 432

Communicate

correct use of appropriate technical vocabulary, procedural vocabulary, and conventions

coherent and concise organisation of the response

clear connection between discussion and labelled tables

table concisely summarises the calculations and results

Solve

application of mathematical concepts and techniques relevant to the task

applies subject matter drawn from Unit 1 Topic 1: Combinatorics

Winning outcomes for the player

Table 3 shows the total number of different values of X that correspond to the five winning outcomes for the player.

Table 3

Range of X	Restrictions by column	Number of different winning values of X
960–999	First digit is 9 Second digit: any of the 3 digits from 6–8 Third digit: any of the remaining 8 digits	$^1P_1 \times {}^3P_1 \times {}^8P_1$ = 24
900–959	First digit is 9 Second digit: any of the 6 digits from 0–5 Third digit: any of the remaining 8 digits	$^1P_1 \times {}^6P_1 \times {}^8P_1$ = 48
820–899	First digit is 8 Second digit: any of the remaining 7 digits from 2–9 excluding 8 Third digit: any of the remaining 8 digits	$^1P_1 \times {}^7P_1 \times {}^8P_1$ = 56
720–819	First digit is 7 Second digit: any of the remaining 7 digits from 2–9 excluding 7 Third digit: any of the remaining 8 digits **OR** First digit is 8 Second digit: any of the 2 digits from 0–1 Third digit: any of the remaining 8 digits	$^1P_1 \times {}^7P_1 \times {}^8P_1$ + $^1P_1 \times {}^2P_1 \times {}^8P_1$ = 56 + 16 = 72
600–719	First digit is 6 Second digit: any of the 9 digits from 0–9 excluding 6 Third digit: any of the remaining 8 digits **OR** First digit is 7 Second digit: any of the 2 digits from 0–1 Third digit: any of the remaining 8 digits	$^1P_1 \times {}^9P_2$ + $^1P_1 \times {}^2P_1 \times {}^8P_1$ = 72 + 16 = 88

Evaluate and verify evaluation of the reasonableness of solutions by considering the results	**Evaluating the reasonableness** of these results: Total outcomes = Total number of different winning values of X for the house + Total number of different winning values of X for the player = 432 + (24 + 48 + 56 + 72 + 88) = 720 The calculations above were reasonable as the total number of outcomes corresponded to the total number of different values of X given on page 3.
Communicate correct use of appropriate technical vocabulary, procedural vocabulary, and conventions	**2.2 Theoretical calculations of the house edge** My game design produces a house edge of 11.7%. Table 4 summarises the monetary return to the house, based on a $1 bet for each of the winning outcomes introduced in Table 1. For house edge calculations, it was necessary to assign a positive value to represent a return to the house and a negative value to represent a return to the player. The returns chosen for each outcome were finally decided upon after consideration of house edge calculations from several preliminary trials.

Table 4

Winning outcomes P = player, H = house	Range of X	Return to house (based on $1 bet)
1 (P)	960–999	–$4
2 (P)	900–959	–$2
3 (P)	820–899	–$1.50
4 (P)	720–819	–$1
5 (P)	600–719	$0
1 (H)	000–599	$1

For example, for a $1 bet, the three-digit display of 982 would return $4 (plus the original $1 bet) to the player, while a display of 547 would result in the player losing their $1 bet to the house. A display of 607 would just return the $1 bet to the player.

Formulate accurate translation of aspects of the problem stated variables identified mathematical concepts and techniques	The mathematics underpinning the house edge calculations is shown below. Let the number of values in the range of X = n Let the return for each winning outcome = R Let the probability of the occurrence of a 3-digit number in the range of X = $p = \frac{n}{720}$ Expected return for each winning outcome = $p \times R$
Solve discerning application of mathematical concepts and techniques relevant to the task shows good judgment regarding house edge calculations	**Modifying the house edge formula** Using the given definition of the house edge: $$\text{House edge} = \frac{\text{expected return to the casino}}{\text{amount spent by gamblers}} \times 100\%$$ For a $1 bet, this definition simplifies to: $$\text{House edge} = \frac{\Sigma\, pR}{1} \times 100\% = \Sigma\, pR \times 100\%$$ (where $\Sigma\, np$ represents the total expected return for the house)

Solve

accurate use of complex procedures to reach a valid solution

the solution involves a combination of parts that are interconnected

Evaluate and verify

evaluation of the reasonableness of solutions by considering the results

documentation of relevant strengths of the solution

justification of decisions made using mathematical reasoning

Table 5 shows the calculations of the probabilities and expected returns associated with the various game outcomes.

Table 5

Range of X	*n*	*p*	*R*	Expected house return (in $'s) = *pR*
960–999	24	$\frac{24}{720}=\frac{1}{30}$	–$4	$-4\times\frac{1}{30}=\frac{-2}{15}$
900–959	48	$\frac{48}{720}=\frac{1}{15}$	–$2	$-2\times\frac{1}{15}=\frac{-2}{15}$
820–899	56	$\frac{56}{720}=\frac{7}{90}$	–$1.50	$-1.5\times\frac{7}{90}=\frac{-7}{60}$
720–819	72	$\frac{72}{720}=\frac{1}{10}$	–$1	$-1\times\frac{11}{90}=\frac{-1}{10}$
600–719	88	$\frac{88}{720}=\frac{11}{90}$	$0	$0\times\frac{11}{90}=0$
000–599	432	$\frac{432}{720}=\frac{3}{5}$	$1	$1\times\frac{3}{5}=\frac{3}{5}$

Calculating the house edge:

$\text{House edge} = \Sigma\, pR\ \times\ 100\%$

$$= \left(\frac{-2}{15}+\frac{-2}{15}+\frac{-7}{60}+\frac{-1}{10}+0+\frac{3}{5}\right)\times 100\% \approx 11.7\%$$

Evaluating the reasonableness of the results:

Evaluating the reasonableness of these results with regards to the associated probabilities:

$$\Sigma\, p = \frac{1}{30}+\frac{1}{15}+\frac{7}{90}+\frac{1}{10}+\frac{11}{90}+\frac{3}{5}=1$$

This showed that the results were reasonable as the sum of the probabilities was 1.

The overall result of a house edge of 11.7% represented a reasonable solution as the regulation of a result between 10% and 15% was met.

Preliminary trials

To ensure that my game complied with the required regulation of the house edge between 10% and 15%, my modelling based on the player returns was refined many times. I have included two such trials which demonstrate the path to my valid solution.

Trial 1

Table 6 is a variation on the expected return to the player used in Table 4.

Table 6

Winning outcomes P = player, H = house	Range of X	Return to house (based on $1 bet)
1 (P)	960–999	–$5
2 (P)	900–959	–$4
3 (P)	820–899	–$3
4 (P)	720–819	–$2
5 (P)	600–719	–$1
1 (H)	000–599	$1

$\text{House edge} =\ \Sigma\, pR\ \times\ 100\%$

$$= \left(-5\times\frac{1}{30}+-4\times\frac{1}{15}+-3\times\frac{7}{90}+-2\times\frac{1}{10}+-1\times\frac{11}{90}+1\times\frac{3}{5}\right)\times 100\%$$

$$\approx -38.9\%$$

Evaluate and verify documentation of relevant limitations of the solution justification of decisions made using mathematical reasoning	As this house edge was a negative value, the player return used did not produce a suitable solution as it meant that the casino would be losing money in the long-term.

Trial 2

To change this house edge into a positive value between 10% and 15%, the expected return to the player was reduced as shown in Table 7.

Table 7

Winning outcomes P = player, H = house	Range of X	Return to house (based on $1 bet)
1 (P)	960–999	–$4
2 (P)	900–959	–$3
3 (P)	820–899	–$1.50
4 (P)	720–819	–$1
5 (P)	600–719	$0
1 (H)	000–599	$1

Evaluate and verify

justification of decisions made using mathematical reasoning

evaluation of the reasonableness of solutions by considering the results

$$\text{House edge} = \Sigma\, pR \times 100\%$$
$$= \left(-4 \times \frac{1}{30} + -3 \times \frac{1}{15} + -1.5 \times \frac{7}{90} + -1 \times \frac{1}{10} + 0 \times \frac{11}{90} + 1 \times \frac{3}{5}\right) \times 100\%$$
$$= 5\%$$

In this trial, while a positive house edge was achieved, it still did not represent a reasonable solution as the 10% to 15% regulation was not met. I then continued with my trials until attaining the required range.

Formulate

accurate documentation of relevant observations

accurate translation of aspects of the problem by identifying mathematical concepts and techniques

Communicate

correct use of appropriate technical vocabulary, procedural vocabulary, and conventions to develop the response

clear connection between Table 4, Table 8 and required house edge

Solve

accurate and appropriate use of technology

calculation of the allowable 3-digit number

calculation of the player return

2.3 Experimental probability and house edge calculations using technology

I designed a spreadsheet to simulate 100 000 trials of 'Big Numbers — Big Money' to calculate the relevant experimental probabilities. Simulations resulting in duplicate digits were not included in the calculations. I noticed that this reduced the overall sample size, on average, to around 72 000 usable data which I still considered a sufficient sample size to produce reliable comparisons.

This number of usable data seemed reasonable as it reflected the earlier results (see Section 2.1) that a suitable 3-digit number should occur 720 times out of 1000 on average.

Table 8 shows an excerpt of five trials of my game and the resulting house or player return e.g. in Trial 1, the randomly generated three digits formed the number 835 which would return $1.50 to the player (see Table 4). Note that cells E4 and E5 are blank due to digit duplication which discounted these trials as usable observations. A full version of this spreadsheet is given in Appendix 1(a).

Table 8

	A	B	C	D	E	F
1	Game Trial	Hundreds	Tens	Units	Allowable 3 digit number (X)	Player return
2	1	8	3	5	835	-1.5
3	2	3	7	2	372	1
4	3	6	2	2		
5	4	2	7	7		
6	5	9	7	0	970	-4

The corresponding spreadsheet formulas are shown in Table 9.

Solve

accurate and appropriate use of technology

use of conditional formulas

use of a vlookup table to allow simple modification to the player and house return values

Table 9

	A	B	C	D	E	F
1	Game Trial	Hundreds	Tens	Units	Allowable 3 digit number (X)	Player return
2	1	=RANDBETWEEN(0,9)	=RAND	=RAND	=IF(OR(B2=C2,B2=D2,C2=D2),"",100*B2+10*C2+D2)	=IF(E2="", "",VLOOKUP(E2,$H$2:$I$7,2,TRUE))
3	2	=RANDBETWEEN(0,9)	=RAND	=RAND	=IF(OR(B3=C3,B3=D3,C3=D3),"",100*B3+10*C3+D3)	=IF(E3="", "",VLOOKUP(E3,$H$2:$I$7,2,TRUE))
4	3	=RANDBETWEEN(0,9)	=RAND	=RAND	=IF(OR(B4=C4,B4=D4,C4=D4),"",100*B4+10*C4+D4)	=IF(E4="", "",VLOOKUP(E4,$H$2:$I$7,2,TRUE))
5	4	=RANDBETWEEN(0,9)	=RAND	=RAND	=IF(OR(B5=C5,B5=D5,C5=D5),"",100*B5+10*C5+D5)	=IF(E5="", "",VLOOKUP(E5,$H$2:$I$7,2,TRUE))
6	5	=RANDBETWEEN(0,9)	=RAND	=RAND	=IF(OR(B6=C6,B6=D6,C6=D6),"",100*B6+10*C6+D6)	=IF(E6="", "",VLOOKUP(E6,$H$2:$I$7,2,TRUE))

	H	I
1	Lower limit of X	Player Return
2	0	$1.00
3	600	$0.00
4	720	-$1.00
5	820	-$1.50
6	900	-$2.00
7	960	-$4.00

Solve

discerning application of mathematical concepts and techniques relevant to the task

design of game simulation demonstrates intellectual perception

This simulated data was used to calculate the experimental probabilities for each winning outcome (for both player and house) and the house edge. The results were then compared against the original theoretical probabilities and house edge calculations using percentage error as the statistical measure

$$\text{Percentage error} = \left|\frac{T - E}{T}\right| \times 100\%$$

T and *E* represent the theoretical and experimental values respectively[2]. The results of one simulation involving 71 857 games are shown in Table 10.

Table 10

K	L	M	N	O	P
	House Edge comparisons - Experimental vs Theoretical				
	Long-term house profit	Allowable games	Experimental house edge (%)	Theoretical house edge (%)	% error in house edge
	$8,365.50	71857	11.642	11.667	0.2%
	Winning Outcomes probability comparisons - Experimental vs Theoretical				
Winning Outcome	Number of Winning Outcomes	Number of allowable games	Experimental Probability	Theoretical Probability	Difference in Probability
0 ≤ X < 600	43179	71857	0.601	0.600	0.2%
600 ≤ X < 720	8692	71857	0.121	0.122	-1.0%
720 ≤ X < 820	7198	71857	0.100	0.100	0.2%
820 ≤ X < 900	5609	71857	0.078	0.078	0.4%
900 ≤ X < 960	4757	71857	0.066	0.067	-0.7%
960 ≤ X < 999	2422	71857	0.034	0.033	1.1%

Solve

accurate and appropriate use of technology

calculation of the number of winning outcomes

calculation of the long-term profit

use of conditional formulas

The corresponding spreadsheet formulas are shown in Table 11.

Table 11

	K	L	M	N	O	P
1		House Edge comparisons - Experimental vs Theoretical				
2		Long-term	Allowable	Experimental	Theoretical	% error in
3		house profit	games	house edge (%)	house edge (%)	house edge
4		=SUM(F:F)	=COUNT(F:F)	=L4/M4*100	11.667	=ABS(O4-N4)/O4
5						
6		Winning Outcomes probability comparisons - Experimental vs Theoretical				
7	Winning	Number of	Number of	Experimental	Theoretical	% Difference
8	Outcome	Winning	allowable	Probability	Probability	in Probability
9	0 ≤ X < 600	=COUNTIF(F:F,I2)	=COUNT(F:F)	=L9/M9	=3/5	=ABS(O9-N9)/O9
10	600 ≤ X < 720	=COUNTIF(F:F,I3)	=COUNT(F:F)	=L10/M10	=11/90	=ABS(O10-N10)/O10
11	720 ≤ X < 820	=COUNTIF(F:F,I4)	=COUNT(F:F)	=L11/M11	=1/10	=ABS(O11-N11)/O11
12	820 ≤ X < 900	=COUNTIF(F:F,I5)	=COUNT(F:F)	=L12/M12	=7/90	=ABS(O12-N12)/O12
13	900 ≤ X < 960	=COUNTIF(F:F,I6)	=COUNT(F:F)	=L13/M13	=1/15	=ABS(O13-N13)/O13
14	960 ≤ X < 999	=COUNTIF(F:F,I7)	=COUNT(F:F)	=L14/M14	=1/30	=ABS(O14-N14)/O14

[2] http://www2.phy.ilstu.edu/~wenning/slh/Percent%20Difference%20Error.pdf

Annotations	Response
Evaluate and verify justification of decisions made using mathematical reasoning consideration of statistical measures of success in the evaluation process	**Evaluating the reasonableness of these results:** Repeated simulations showed that the theoretical probabilities and house edge calculations, for the revised game, outlined in Table 10, consistently matched their corresponding experimental results with a percentage error of less than 5%. These simulations also produced an experimental house edge consistently between the required range of between 10% to 15%. Based upon my tolerance margin, this confirmed that my theoretical calculations were reasonable. Similar results are evident in the two other trials of the simulation included in Appendix 1(b).
Communicate coherent and concise organisation of the response…including a suitable introduction, body and conclusion, which can be read independently of the task sheet conclusion provides an overview of the significance of the information presented **Evaluate and verify** evaluation of the reasonableness of solutions by considering the results, assumptions and observations documentation of relevant strengths and limitations of the solution	**3 Conclusion** Strengths of my solution The main strength in my solution is that the theoretical model compared well to the numerical model. An additional strength is that the use of a numerical model of the game was an efficient way to verify the theoretical predictions. A percentage error of consistently under 5%, considered an appropriate tolerance margin for the validity of this decision, was consistently met. Limitations of my solution A limitation in my game design is that it is designed using permutations which reduces the total number of possible 3-digit display outcomes from 1000 down to 720. Players may become concerned when they realise this feature of the game, for example, that there are only 24 winning numbers in the 960–999 range rather than the intuitive 40 numbers. A limitation of the model is that it uses a simulation of the game, rather than the game itself, to validate the combinatorial theory. So, if the assumption that the spreadsheet is a suitable analogue for the game is not appropriate, then the 'observations' used to judge the theoretical model are not valid. Another limitation is that the theoretical model relies on a large number of trials. So, if the game is unpopular, the theoretical house edge may not be obtained. In summary, my unique game, 'Big Numbers — Big Money', meets the requirement of a regulated house edge return of between 10% and 15%. Its simple rules and profitable payout would promote motivation for player participation. A recommendation of a further study could be to incorporate a 'giant jackpot payout' for the appearance of the biggest number possible (987). My spreadsheet could easily be refined to simulate any modification of the game, whilst ensuring that the house edge remained within the required range. **References** Percent difference – percent error http://www2.phy.ilstu.edu/~wenning/slh/Percent%20Difference%20Error.pdf What percent error is too high? https://socratic.org/questions/what-percent-error-is-too-high

4 Appendixes

Appendix 1(a)

Simulation Trial 1

Game Trial	Hundreds	Tens	Units	Allowable 3 digit	Player return
1	5	7	6	576	1
2	6	7	8	678	0
3	6	9	0	690	0
4	6	3	0	630	0
5	5	7	3	573	1
6	6	3	0	630	0
7	9	2	7	927	-2
8	0	4	8	48	1
9	5	7	5		
10	8	2	0	820	-1.5
11	8	1	5	815	-1
12	1	8	3	183	1
13	5	5	5		
14	8	6	4	864	-1.5
99991	7	6	5	765	-1
99992	3	2	2		
99993	4	6	7	467	1
99994	4	0	2	402	1
99995	3	7	9	379	1
99996	8	6	4	864	-1.5
99997	3	7	1	371	1
99998	5	6	7	567	1
99999	3	0	3		
100000	1	8	5	185	1

Lower limit of X	Player Return
0	$1.00
600	$0.00
720	-$1.00
820	-$1.50
900	-$2.00
960	-$4.00

Press F9 to simulate 100 000 games

House Edge comparisons - Experimental vs Theoretical				
Long-term house profit	Allowable games	Experimental house edge (%)	Theoretical house edge (%)	% error in house edge
$8,145.00	72090	11.298	11.667	3.2%

	Winning Outcomes probability comparisons - Experimental vs Theoretical				
Winning Outcome	Number of Winning	Number of allowable	Experimental Probability	Theoretical Probability	% Difference in Probability
$0 \leq X < 600$	43157	72090	0.599	0.600	0.2%
$600 \leq X < 720$	8848	72090	0.123	0.122	0.4%
$720 \leq X < 820$	7200	72090	0.100	0.100	0.1%
$820 \leq X < 900$	5608	72090	0.078	0.078	0.0%
$900 \leq X < 960$	4854	72090	0.067	0.067	1.0%
$960 \leq X < 999$	2423	72090	0.034	0.033	0.8%

Appendix 1(b)

Simulation Trial 2

Game Trial	Hundreds	Tens	Units	Allowable 3 digit	Player return
1	0	5	4	54	1
2	0	8	8		
3	8	7	7		
4	4	3	6	436	1
5	4	5	7	457	1
6	5	7	3	573	1
7	7	1	6	716	0
8	4	1	5	415	1
9	0	2	8	28	1
10	0	5	3	53	1
11	3	4	2	342	1
12	9	7	2	972	-4
13	0	7	6	76	1
99990	7	5	8	758	-1
99991	2	9	6	296	1
99992	9	7	7		
99993	5	6	8	568	1
99994	6	4	6		
99995	4	8	0	480	1
99996	0	3	4	34	1
99997	1	4	0	140	1
99998	4	3	1	431	1
99999	6	6	4		
100000	9	3	2	932	-2

Lower limit of X	Player Return
0	$1.00
600	$0.00
720	-$1.00
820	-$1.50
900	-$2.00
960	-$4.00

Press F9 to simulate 100 000 games

House Edge comparisons - Experimental vs Theoretical				
Long-term house profit	Allowable games	Experimental house edge (%)	Theoretical house edge (%)	% error in house edge
$8,363.50	71884	11.635	11.667	0.3%

	Winning Outcomes probability comparisons - Experimental vs Theoretical				
Winning Outcome	Number of Winning	Number of allowable	Experimental Probability	Theoretical Probability	% Difference in Probability
$0 \leq X < 600$	43131	71884	0.600	0.600	0.0%
$600 \leq X < 720$	8761	71884	0.122	0.122	0.3%
$720 \leq X < 820$	7206	71884	0.100	0.100	0.2%
$820 \leq X < 900$	5561	71884	0.077	0.078	0.5%
$900 \leq X < 960$	4840	71884	0.067	0.067	1.0%
$960 \leq X < 999$	2385	71884	0.033	0.033	0.5%

Simulation Trial 3

Game Trial	Hundreds	Tens	Units	Allowable 3 digit	Player return
1	0	2	0		
2	8	4	6	846	-1.5
3	1	9	2	192	1
4	5	5	2		
5	6	9	5	695	0
6	1	2	3	123	1
7	8	2	0	820	-1.5
8	3	2	4	324	1
9	9	7	2	972	-4
10	4	5	9	459	1
11	8	0	7	807	-1
12	8	3	6	836	-1.5
13	8	8	1		
14	1	3	7	137	1
99991	8	9	0	890	-1.5
99992	6	9	4	694	0
99993	5	4	1	541	1
99994	9	1	3	913	-2
99995	7	9	8	798	-1
99996	9	7	1	971	-4
99997	1	7	3	173	1
99998	0	5	6	56	1
99999	0	3	8	38	1
100000	5	8	6	586	1

Lower limit of X	Player Return
0	$1.00
600	$0.00
720	-$1.00
820	-$1.50
900	-$2.00
960	-$4.00

Press F9 to simulate 100 000 games

House Edge comparisons - Experimental vs Theoretical				
Long-term house profit	Allowable games	Experimental house edge (%)	Theoretical house edge (%)	% error in house edge
$8,271.50	72083	11.475	11.667	1.6%

	Winning Outcomes probability comparisons - Experimental vs Theoretical				
Winning Outcome	Number of Winning	Number of allowable	Experimental Probability	Theoretical Probability	% Difference in Probability
$0 \leq X < 600$	43231	72083	0.600	0.600	0.0%
$600 \leq X < 720$	8865	72083	0.123	0.122	0.6%
$720 \leq X < 820$	7192	72083	0.100	0.100	0.2%
$820 \leq X < 900$	5609	72083	0.078	0.078	0.0%
$900 \leq X < 960$	4695	72083	0.065	0.067	2.3%
$960 \leq X < 999$	2491	72083	0.035	0.033	3.7%