# Specialist Mathematics 2019 v1.2

IA2 sample marking scheme

April 2023

## Examination (15%)

This sample has been compiled by the QCAA to model one possible approach to allocating marks in an examination. It matches the examination mark allocations as specified in the syllabus (~ 60% simple familiar, ~ 20% complex familiar and ~ 20% complex unfamiliar) and ensures that all assessment objectives are assessed.

### Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.

# Instrument-specific marking guide (ISMG)

## Criterion: Foundational knowledge and problem-solving

### Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.

| The student work has the following characteristics: | Cut-off | Marks |
|---|---|---|
| • consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions, and prove propositions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations. | > 93% | 15 |
| | > 87% | 14 |
| • correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions, and prove propositions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations. | > 80% | 13 |
| | > 73% | 12 |
| • thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions, and prove propositions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations. | > 67% | 11 |
| | > 60% | 10 |
| • selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations. | > 53% | 9 |
| | > 47% | 8 |

| The student work has the following characteristics: | Cut-off | Marks |
|---|---|---|
| • some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques. | > 40% | 7 |
| | > 33% | 6 |
| • infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques. | > 27% | 5 |
| | > 20% | 4 |
| • isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques. | > 13% | 3 |
| | > 7% | 2 |
| • isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions. | > 0% | 1 |
| • does not satisfy any of the descriptors above. | | 0 |

# Task

See the sample assessment instrument for IA2: Examination (15%) available on the QCAA Portal.

# Sample marking scheme

| Criterion | Marks allocated | Results |
|---|---|---|
| **Foundational knowledge and problem-solving**<br>Assessment objectives 1, 2, 3, 4, 5 and 6 | 15 | — |
| **Total** | **15** | — |

The annotations are written descriptions of the expected response for each question and are related to the assessment objectives.

Note: ✓ = $\frac{1}{2}$ mark

# Marking scheme

# Simple familiar

## Question 1 (5 marks) SF

**1a.**

comprehend:

- the connection between parametric and Cartesian form ✓
- the requirement to eliminate the parameter ✓

select and use:

- the relevant Pythagorean identity ✓
- a procedure to produce a Cartesian equation ✓
- a suitable procedure to the required result ✓✓

a. $\boldsymbol{r}(t) = 3\sin t\,\hat{\boldsymbol{\imath}} + 4\cos t\,\hat{\boldsymbol{\jmath}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$

$x = 3\sin t$ …(1)

$y = 4\cos t$ …(2) ✓

From (1) $\Rightarrow$ $\sin t = \frac{x}{3}$

From (2) $\Rightarrow$ $\cos t = \frac{y}{4}$ ✓

Substituting these results into the identity

$\sin^2 t + \cos^2 t = 1$ ✓

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

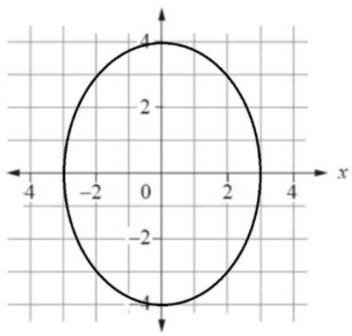
$\frac{x^2}{9} + \frac{y^2}{16} = 1$ ✓

$144\left(\frac{x^2}{9} + \frac{y^2}{16}\right) = 144$

$16x^2 + 9y^2 = 144$ (as required) ✓✓

b. Using the result from above $\frac{x^2}{9} + \frac{y^2}{16} = 1$

The graph of the path of the particle can be modelled by an ellipse ✓✓ with $x$-intercepts at $\pm 3$ ✓ and $y$-intercepts at $\pm 4$. ✓




**1b.**

recall and use:

- the definition of general form of an ellipse ✓✓
- the procedure to find intercepts of the ellipse ✓✓

## Question 2 (11 marks) SF

a. $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$ ✓

$= \begin{pmatrix} 3 \\ 4 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix}$ ✓

$= \begin{pmatrix} 5 \\ 7 \\ -6 \end{pmatrix}$ ✓

**2a.**

recall and use:

- the rule to determine the displacement vector ✓
- the procedure to result ✓✓

**2b.**

select and use:

- a rule to find the vector equation of a line ✓
- a suitable procedure to result ✓✓

**2c.**

comprehend the link between the cross product and the normal to the plane ✓

select and use:

- a rule to find the vector equation of a plane ✓✓✓

recall and use:

- the scalar product
- a suitable procedure to result ✓✓

**2d.**

recall and use:

- a procedure to produce an augmented matrix ✓✓
- the Gaussian elimination method ✓✓✓✓
- a suitable procedure to result ✓✓

b. Vector equation of line *AB* is

$$\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$$

$$= \boldsymbol{a} + k\overrightarrow{AB} \ ✓$$

$$= \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} ✓ + \ k\begin{pmatrix} 5 \\ 7 \\ -6 \end{pmatrix} \ ✓$$

(where *k* is a scalar)

c. $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 42 \\ -12 \\ 21 \end{pmatrix}$ represents $\boldsymbol{n}$, (normal to $P_1$) ✓

containing points $P(x, y, z)$

A vector in the plane is $\overrightarrow{AP} = \boldsymbol{p} - \boldsymbol{a}$ ✓

The equation of $P_1$ is

$$\overrightarrow{AP}.\ \boldsymbol{n} = 0 \quad ✓$$

$$\begin{pmatrix} x+2 \\ y+3 \\ z-3 \end{pmatrix} . \begin{pmatrix} 42 \\ -12 \\ 21 \end{pmatrix} = 0 \ ✓$$

$$42(x+2) - 12(y+3) + 21(z-3) = 0 \ ✓$$

$$42x + 84 - 12y - 36 + 21z - 63 = 0$$

$$42x - 12y + 21z = 15$$

$$14x - 4y + 7z = 5 \quad ✓$$

d. Expressing the 3 planes as an augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & -5 \\ -1 & 1 & 1 & 7 \\ 14 & -4 & 7 & 5 \end{array}\right) \quad ✓✓$$

$$= \left(\begin{array}{ccc|c} 1 & -2 & 1 & -5 \\ 0 & -1 & 2 & 2 \\ 0 & 10 & 21 & 103 \end{array}\right) \quad \begin{array}{c} R_1: R_1 \\ R_2: R_2 + R_1 \\ R_3: R_3 + 14R_2 \end{array}$$

$$= \left(\begin{array}{ccc|c} 1 & -2 & 1 & -5 \\ 0 & -1 & 2 & 2 \\ 0 & 0 & 41 & 123 \end{array}\right) \quad \begin{array}{c} R_1: R_1 \\ R_2: R_2 \\ R_3: R_3 + 10R_2 \end{array}$$

$$= \left(\begin{array}{ccc|c} 1 & -2 & 1 & -5 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right) \quad \begin{array}{c} R_1: R_1 \\ R_2: -R_2 \\ R_3: R_3 \div 41 \end{array}$$

$$= \left(\begin{array}{ccc|c} 1 & -2 & 0 & -8 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right) \quad \begin{array}{c} R_1: R_1 - R_3 \\ R_2: R_2 + 2R_3 \\ R_3: R_3 \end{array}$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right) \quad \begin{array}{c} R_1: R_1 + 2R_2 \\ R_2: R_2 \\ R_3: R_3 \end{array} \quad ✓✓✓✓$$

The intersection point is (0, 4, 3). ✓✓

e. Substituting this result into $x - 2y + z = -5$

LHS = 0 – 8 + 3

= –5

= RHS ✓✓

**2e.**

evaluate the reasonableness of the solution ✓✓

## Question 3 (4 marks) SF

a. $\omega$ = 3 rev per min

$= \frac{3 \times 2\pi}{60}$ rad s$^{-1}$ ✓✓

$= \frac{\pi}{10}$ ✓ rad s$^{-1}$

b. $v = \omega r$ ✓

$= \frac{\pi}{10} \times 15$ ✓

$= \frac{3\pi}{2}$ ✓ m s$^{-1}$ ✓✓

**3a.**

recall and use the required conversion of units ✓✓ expressing in simplified form, ✓

**3b.**

recall and use the rule for finding the speed of the circular motion ✓✓

communicate using:

- relevant units ✓✓
- exact values in simplified form ✓

## Question 4 (7 marks) SF

a. Using GDC:

$$z_1 = 2\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right) = 2\operatorname{cis}\left(\frac{\pi}{3}\right) \quad ✓✓✓✓$$

b.
$$\frac{z_1}{z_2} = \frac{2\operatorname{cis}\left(\frac{\pi}{3}\right)}{4\operatorname{cis}\left(\frac{\pi}{4}\right)}$$

$$= \frac{2}{4}\operatorname{cis}\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \quad ✓✓$$

$$= \frac{1}{2}\operatorname{cis}\left(\frac{\pi}{12}\right) \quad ✓✓$$

c.
$$w = \left(\frac{z_1}{z_2}\right)^6$$

$$= \left(\frac{1}{2}\operatorname{cis}\left(\frac{\pi}{12}\right)\right)^6$$

$$= \left(\frac{1}{2}\right)^6 \operatorname{cis}\left(6 \times \frac{\pi}{12}\right) \quad ✓✓$$

$$= \left(\frac{1}{2}\right)^6 \operatorname{cis}\left(\frac{\pi}{2}\right) \quad ✓$$

$$= \frac{1}{64}\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$$

$$= \frac{1}{64}i$$

Since $Re(w) = 0$ ✓✓ is

$\therefore w$ a pure imaginary number. ✓

**4a.**

recall and use a suitable procedure to:

- find the modulus ✓
- find the argument ✓

communicate the result using:

- suitable polar form ✓
- exact values in simplified form ✓

**4b.**

recall and use a suitable complex number division procedure to:

- divide the moduli ✓
- subtract the arguments ✓

communicate the result using exact values in simplified form ✓✓

**4c.**

select and use:

- De Moivre's theorem ✓✓
- a suitable procedure to determine $w$ ✓

communicate that:

- $Re(w)$ is zero ✓✓
- $w$ is purely imaginary ✓

**5.**

identify:

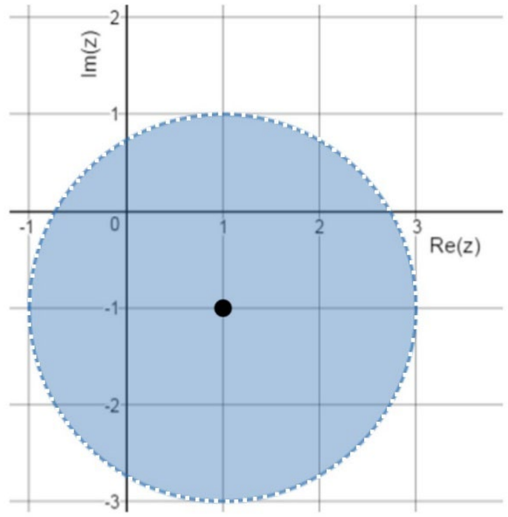
- a circular shape ✓✓

- that the centre of the circle is $(1, -1)$ ✓✓
- that the radius of the circle is 2 ✓

recognise subset of the complex plane is the shaded region inside the circle ✓✓ exclusive of its circumference ✓

## Question 5 (4 marks) SF

✓✓✓✓✓✓✓✓




**6a.**

comprehend the concept of the root of an equation by showing the substitution step ✓

use technology (or otherwise) to show that $P(-2)$ equals zero ✓

## Question 6 (5 marks) SF

a. $P(-2)$

$= (-2)^3 + (2-6i)(-2)^2 - (9+12i)(-2) - 18$ ✓

$= 0$ ✓

b.

$$\begin{array}{r}
z^2 \quad - 6iz \quad - 9 \checkmark \\
\checkmark \quad z+2\,\overline{\big)\,z^3 + (2-6i)z^2 + (-9-12i)z - 18} \\
\underline{z^3 + \quad 2z^2} \quad \checkmark \\
-6iz^2 + (-9-12i)z - 18 \quad \checkmark \\
\underline{-6iz^2 + \quad -12i\,z} \quad \checkmark \\
-9z - 18 \quad \checkmark \\
\underline{-9z - 18} \quad \checkmark \\
0
\end{array}$$

So $Q(z) = z^2 - 6iz - 9$ ✓

**6b.**

translate information into a suitable mathematical format ✓

select and use a suitable long division polynomial procedure (or similar) ✓✓✓✓✓✓

communicate the polynomial *Q(z)* clearly ✓

**7.**

communicate using a clear diagram ✓

translate information into a mathematical model for acceleration ✓

solve by:

- determining a general solution for $\boldsymbol{v}$ ✓
- using the given conditions to find $\boldsymbol{v}$ ✓

solve by:

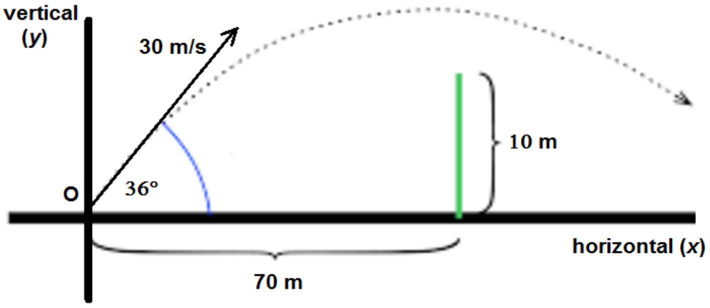
- determining a general solution for $\boldsymbol{r}$ ✓
- using the given conditions to find $\boldsymbol{r}$ ✓

justify procedure to:

- represent the vector position of the wall
- find the time of flight ✓
- find the vertical displacement at the determined time ✓

# Complex familiar

## Question 7 (6 marks) CF




✓

Assume that the origin is at the initial projection point:

Acceleration $\boldsymbol{a} = -g\,\hat{\boldsymbol{j}}$ ✓

Finding $\boldsymbol{v}$:

Therefore, $\boldsymbol{v} = \int -g\,\hat{\boldsymbol{j}}\,dt$

$= -g\,t\hat{\boldsymbol{j}} + c$ ✓

Given the initial velocity $\boldsymbol{v}(0) = 30\cos 36°\,\hat{\boldsymbol{i}} + 30\sin 36°\,\hat{\boldsymbol{j}}$

$\therefore\ c = 30\cos 36°\,\hat{\boldsymbol{i}} + 30\sin 36°\,\hat{\boldsymbol{j}}$

$\therefore\ \boldsymbol{v} = 30\cos 36°\,\hat{\boldsymbol{i}} + (30\sin 36° - gt)\,\hat{\boldsymbol{j}}$ ✓

Finding position vector $\boldsymbol{r}$:

$\boldsymbol{r} = \int \boldsymbol{v}\ dt$

$= \int 30\cos 36°\,\hat{\boldsymbol{i}} + (30\sin 36° - gt)\,\hat{\boldsymbol{j}}\,dt$

$= 30\cos 36°t\,\hat{\boldsymbol{i}} + \left(30\sin 36°t - \frac{1}{2}gt^2\right)\hat{\boldsymbol{j}} + k$ ✓

Assuming $t = 0$, $\boldsymbol{r} = 0\,\hat{\boldsymbol{i}} + 0\,\hat{\boldsymbol{j}}$
$\therefore\ k = 0\,\hat{\boldsymbol{i}} + 0\,\hat{\boldsymbol{j}}$

$\therefore\ \boldsymbol{r} = 30\cos 36°t\,\hat{\boldsymbol{i}} + \left(30\sin 36°t - \frac{1}{2}gt^2\right)\hat{\boldsymbol{j}}$ ✓

At the wall, the horizontal displacement is $\boldsymbol{r}_x = 70\,\hat{\boldsymbol{i}}$ ✓

Equating $\boldsymbol{r}_x$ components:

$70 = 30\cos 36°t$

$t = \dfrac{70}{30\cos 36°}$

evaluate the reasonableness of solution ✓

communicate:

- the outcome that the ball will clear the wall ✓
- using suitable linking statements throughout the solution ✓

8.

$= 2.88\ \text{s}$ ✓

comprehend by making connections with the requirement of the proof ✓

Determining the vertical displacement at $t = 2.88\ \text{s}$

$r_y = 30 \sin 36°(2.88) - \frac{1}{2}(9.8)(2.88)^2$

$= 10.1\ \text{m}$ ✓

prove the step 1 proposition using mathematical reasoning ✓✓

This is greater than the 10-metre height of the wall. ✓

So, the ball should just clear the wall. ✓ ✓

## Question 8 (6 marks) CF

Using mathematical induction to prove that, for $n \in Z^+$,

$$3^{2n+4} - 2^{2n} \text{ is divisible by 5.}$$

This statement can be rewritten as the proposition

$3^{2n+4} - 2^{2n} = 5A \text{ where } A \in Z^+$ ✓

Step 1:

RTP the proposition is true for $n = 1$.

comprehend the information by identifying the critical element of step 2 ✓✓

LHS $= 3^{2\times1+4} - 2^{2\times1}$

$= 3^6 - 2^2$

$= 725$

RHS $= 5 \times 145$

$= 725$

prove the step 3 proposition using mathematical reasoning ✓✓✓✓✓

= LHS

So, the proposition is proven true for $n = 1$. ✓✓

Step 2:

If the proposition is assumed to be true for $n = k$, then

$3^{2k+4} - 2^{2k} = 5A \text{ where } k, A \in Z^+$ ✓✓

Step 3:

Use this assumption to prove that the proposition is true

for $n = k + 1$,

$3^{2(k+1)+4} - 2^{2(k+1)} = 5A$ ✓

which simplifies to

communicate clearly:

$3^{2k+6} - 2^{2k+2} = 5A \text{ where } k, A \in Z^+$

| | | |
|---|---|---|
| • the three steps of the proof ✓<br>• the concluding statement ✓ | LHS $= 3^{2k+6} - 2^{2k+2}$<br>$= 3^2 \times 3^{2k+4} - 2^2 \times 2^{2k}$ ✓<br>$= 9 \times (5A + 2^{2k}) - 2^2 \times 2^{2k}$ ✓ (using assumption)<br>$= 45A + 9 \times 2^{2k} - 4 \times 2^{2k}$<br>$= 45A + 5 \times 2^{2k}$ ✓<br>$= 5(9A + 2^{2k})$ ✓ (where $9A + 2^{2k} \in \mathbb{Z}^+$)<br>= RHS<br>So the proposition is proven true for $n = k + 1$.<br>Hence, the original proposition is proven using the method of mathematical induction. ✓✓ | |
| **9.**<br>solve by finding a suitable expression for $\boldsymbol{v} \times \boldsymbol{B}$ ✓✓<br><br>solve by finding a suitable expression for $\boldsymbol{F}$ ✓✓<br><br>translate information into a mathematically workable format:<br>• simultaneous equations ✓<br>• matrix equation ✓ | | |

# Complex unfamiliar

## Question 9 (6 marks) CU

Finding the vector product,

$$\boldsymbol{v} \times \boldsymbol{B} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

$$= \begin{bmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ a & b & c \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \hat{\boldsymbol{\imath}} \begin{bmatrix} b & c \\ -2 & 1 \end{bmatrix} - \hat{\boldsymbol{\jmath}} \begin{bmatrix} a & c \\ 3 & 1 \end{bmatrix} + \hat{\boldsymbol{k}} \begin{bmatrix} a & b \\ 3 & -2 \end{bmatrix}$$

$$= (b + 2c)\,\hat{\boldsymbol{\imath}} + (-a + 3c)\,\hat{\boldsymbol{\jmath}} + (-2a - 3b)\,\hat{\boldsymbol{k}}$$

$$= \begin{pmatrix} b + 2c \\ -a + 3c \\ -2a - 3b \end{pmatrix} \quad ✓✓$$

Finding the force vector,

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

$$= 1\left(\begin{pmatrix} 2a \\ b \\ 3c \end{pmatrix} + \begin{pmatrix} b + 2c \\ -a + 3c \\ -2a - 3b \end{pmatrix}\right)$$

$$= \begin{pmatrix} 2a + b + 2c \\ -a + b + 3c \\ -2a - 3b + 3c \end{pmatrix} \quad ✓✓$$

| | | |
|---|---|---|
| use technology to solve the matrix equation ✓✓<br><br>justify procedure to determine the speed of the particle ✓✓<br><br>evaluate the reasonableness of solution ✓<br><br>communicate the reasonableness of the statement ✓<br><br>**10.**<br>solve the problem to find the roots of unity ✓✓<br><br>justify decision for choosing the required root ✓<br><br>solve the problem to determine the three required roots of *P(z)* ✓✓<br><br>comprehend by expressing *P(z)* in factorised form ✓<br><br>solve the problem by expressing *P*(*z*) in expanded form ✓✓✓✓✓ | Equating to the given force, $\boldsymbol{F} = \begin{pmatrix} -2 \\ -6 \\ 1 \end{pmatrix}$ the following simultaneous equations are formed:<br>$2a + b + 2c = -2$<br>$-a + b + 3c = -6$<br>$-2a - 3b + 3c = 1$ ✓<br>Rewriting in matrix form<br>$\begin{pmatrix} 2 & 1 & 2 \\ -1 & 1 & 3 \\ -2 & -3 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \\ 1 \end{pmatrix}$ ✓<br>Solving the matrix equation (using GDC)<br>$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 & 1 & 2 \\ -1 & 1 & 3 \\ -2 & -3 & 3 \end{pmatrix}^{-1}\begin{pmatrix} -2 \\ -6 \\ 1 \end{pmatrix}$<br>$= \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ ✓✓<br>So, the velocity vector is $\boldsymbol{v} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$<br>Calculating the speed when $\boldsymbol{F} = \begin{pmatrix} -2 \\ -6 \\ 1 \end{pmatrix}$<br>$\lvert \boldsymbol{v} \rvert = \sqrt{1^2 + (-2)^2 + (-1)^2}$ ✓<br>$= \sqrt{6}$ ✓<br>The speed of the particle is $\sqrt{6}$ $(\approx 2.4)$ m s$^{-1}$ which is less than 3 m s$^{-1}$. ✓<br>So, the scientist's statement is not reasonable. ✓<br>**Question 10 (6 marks) CU**<br>The 3 roots of unity of $\omega^3 = 1$ are:<br>$\omega_1 = 1\,cis\,0^{\circ} \quad = 1 + 0\,i$<br>$\omega_2 = 1\,cis\,120^{\circ} = \cos 120^{\circ} + \sin 120^{\circ} i = \frac{-1}{2} + \frac{\sqrt{3}}{2} i$<br>$\omega_3 = 1\,cis\,240^{\circ} = \cos 240^{\circ} + \sin 240^{\circ} i = \frac{-1}{2} - \frac{\sqrt{3}}{2} i$ ✓✓<br>Given Im(ω) > 0, then the required root is $\omega_2$. ✓<br>Given $P(z) = z^3 + az^2 + bz + c$ has roots at 1, $-\omega_2$ and $-\overline{\omega_2}$, the three roots are: | |

$1, -\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)$ and $-\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)$

or 1, $\frac{1}{2}-\frac{\sqrt{3}}{2}i$ ✓ and $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ ✓

In factorised form:

$$P(z) = (z-1)\left(z-\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)\right)\left(z-\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\right) \quad ✓$$

In expanded form:

solve the problem by equating parts ✓

$$P(z) = (z-1)\left(z-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(z-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) \quad ✓$$

$$= (z-1)\left(\left(z-\frac{1}{2}\right)+\frac{\sqrt{3}}{2}i\right)\left(\left(z-\frac{1}{2}\right)-\frac{\sqrt{3}}{2}i\right)$$

$$= (z-1)\left(z-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2$$

$$= (z-1)\left(z^2-z+\frac{1}{4}+\frac{3}{4}\right)$$

$$= (z-1)((z^2-z+1) \quad ✓✓✓$$

$$= z^3-z^2+z-z^2+z-1$$

$$= z^3-2z^2+2z-1 \quad ✓$$

Equating with $P(z) = z^3+az^2+bz+c$,

$a = -2,\ b = 2,\ c = -1$ ✓