QCAA

Queensland Curriculum and Assessment Authority

Specialist Mathematics 2019 v1.2

IA2: Sample assessment instrument

Examination (15%)

This sample has been compiled by the QCAA to assist and support teachers in planning and developing assessment instruments for individual school settings.

Student name

Student number

Teacher

Exam date

Marking summary

Criterion	Marks allocated	Provisional marks
Foundational knowledge and problem solving	15	
Overall	**15**	

Conditions

Technique	Examination
Unit	Unit 3: Mathematical induction, and further vectors, matrices and complex numbers
Topic/s	1: Proof by mathematical induction 2: Vectors and matrices 3: Complex numbers 2
Time	2 hours + 5 minutes perusal
Other	Only the QCAA formula sheet must be provided. Notes are not permitted. Use of technology is required; schools must specify the technology used.

Instructions

- Show all working in the spaces provided.
- Write responses using black or blue pen.
- Use of a non-CAS graphics calculator is permitted unless an analytical procedure is required.

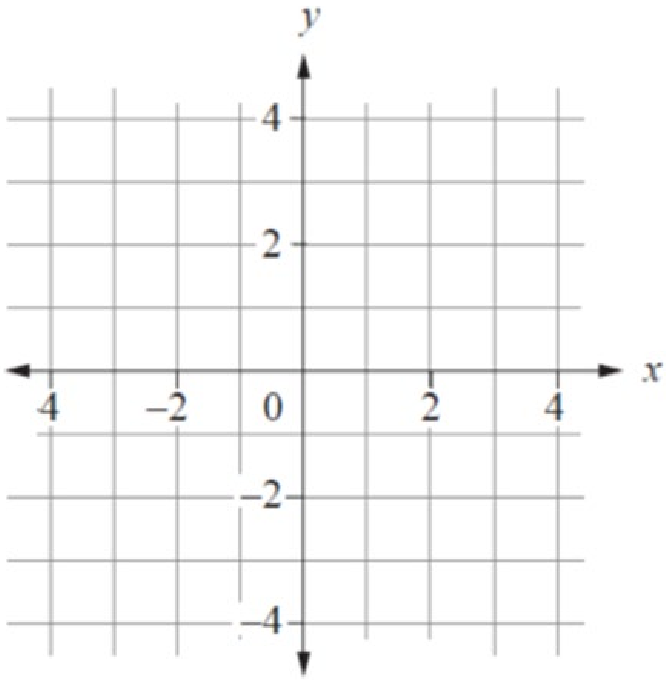
Simple familiar — total marks: 36

Question 1 (5 marks)

The position vector function for a particle is given by $\boldsymbol{r} = 3\sin t\ \hat{\boldsymbol{\imath}} + 4\cos t\ \boldsymbol{j}$.

a. Use an algebraic method to show that the corresponding Cartesian equation of the path of the particle is given by $16x^2 + 9y^2 = 144$.

b. Use the Cartesian plane below to sketch a graph of the path of the particle.



Question 2 (11 marks)

A(−2, −3, 3), *B*(3, 4, −3) and *C*(0, 4, 3) are three points on plane P_1.

a. Determine the displacement vector $\overrightarrow{AB}$.

b. Determine the vector equation of the line that passes through *A* and *B*.

c. Given $\overrightarrow{AB}$ x $\overrightarrow{AC}$ = $\begin{pmatrix} 42 \\ -12 \\ 21 \end{pmatrix}$, show that the equation of the plane P_1 is $14x - 4y + 7z = 5$.

d. Two other planes, P_2 and P_3, are given by $x - 2y + z = -5$ and $-x + y = z = 7$. Use Gaussian elimination calculations to show that planes P_1, P_2 and P_3 intersect at one point.

e. Evaluate the reasonableness of your result for part (d).

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Question 3 (4 marks)

A Ferris wheel with a radius of 15 metres completes three revolutions in one minute

a. Convert the angular velocity of the wheel into rad s^{-1}.

b. Determine the speed of a carriage on the Ferris wheel, stating your answer in exact form, including relevant units.

Question 4 (7 marks)

Two complex numbers are $z_1 = 1 + \sqrt{3i}$ and $z_2 = 4(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$.

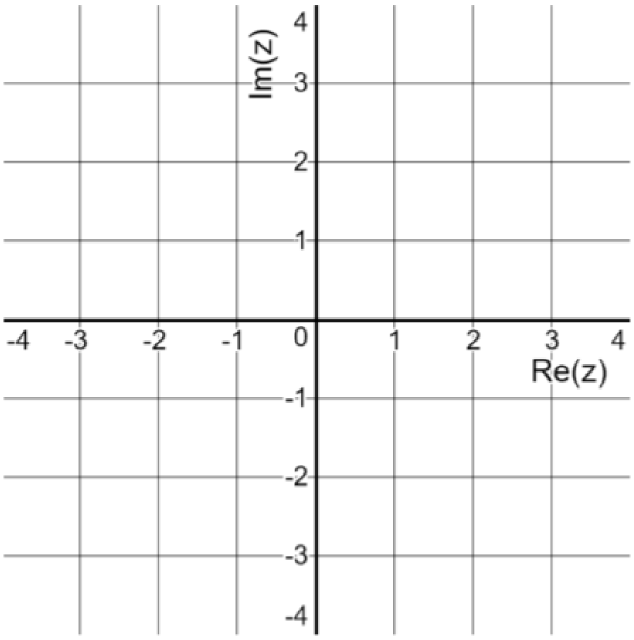
a. Express z_1 in polar form using exact values.

b. Use algebraic procedures to calculate $\frac{z_1}{z_2}$ leaving your result in polar form using exact values.

c. Given $w = \left(\frac{z_1}{z_2}\right)^6$, use De Moivre's theorem to show that w is a pure imaginary number.

Question 5 (4 marks)

On the diagram below, sketch the subset of the complex plane defined by $|z - 1 + i| < 2$.



Question 6 (5 marks)

Consider the polynomial $P(z) = z^3 + (2 - 6i)z^2 - (9 + 12i)z - 18$.

a. Show that $z = -2$ is a solution of $P(z) = 0$.

b. It follows that $P(z) = (z + 2)\,Q(z)$. Use long division or a similar procedure to determine the polynomial $Q(z)$.

Complex familiar — total marks: 12

Question 7 (6 marks)

A ball is projected from ground level at an initial speed of 30 metres per second at an angle of 36° above the horizontal. Use your knowledge of vector calculus to model the motion of the ball. Use this model to determine whether the ball will clear a 10-metre high vertical wall that is 70 metres horizontally away from the point of projection. Assume that the effect of air resistance is negligible and the acceleration due to gravity is $g = 9.8\ \text{m s}^{-2}$.

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Question 8 (6 marks)

Use proof by mathematical induction to prove that $3^{2n+4} - 2^{2n}$ is divisible by 5 for $n \in$ Z+.

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Complex unfamiliar — total marks: 12

Question 9 (6 marks)

The force exerted on a charged particle of magnitude q in an electromagnetic flow can be calculated using the Lorentz relation, which states $\boldsymbol{F} = q\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$, where $\boldsymbol{F}$ is the force, $\boldsymbol{E}$ is the electric field strength force, $\boldsymbol{v}$ is the velocity of the particle and $\boldsymbol{B}$ is the magnetic force.

The following quantities are constant: $q = 1$ and $\boldsymbol{B} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$.

The remaining vector quantities have a relationship based on parameters a, b and c, such that $\boldsymbol{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\boldsymbol{E} = \begin{pmatrix} 2a \\ b \\ 3c \end{pmatrix}$.

The electromagnetic flow becomes unstable for particle speeds above 3 ms^{-1}.

A scientist states that the electromagnetic flow will become unstable when $\boldsymbol{F} = \begin{pmatrix} -2 \\ -6 \\ 1 \end{pmatrix}$. Evaluate the reasonableness of the scientist's statement.

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Question 10 (6 marks)

Let ω be the complex number satisfying the following two conditions:

- $\omega^3 = 1$
- the imaginary component of $\omega > 0$.

The polynomial $P(z) = z^3 + az^2 + bz + c$ has roots at 1, $-\omega$ and $-\overline{\omega}$, given $a, b, c \in R$.

Determine the values of a, b and c.

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Examination marks summary

Question number	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
1	5		
2	11		
3	4		
4	7		
5	4		
6	5		
7		6	
8		6	
9			6
10			6
Totals	**36**	**12**	**12**
Percentage	**60%**	**20%**	**20%**

Instrument-specific marking guide (IA 2): Examination — short response (15%)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions, and prove propositions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	>93%	15
	>87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions, and prove propositions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	>80%	13
	>73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions, and prove propositions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	>67%	11
	>60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	>53%	9
	>47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	>40%	7
	>33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques;	>27%	5

The student work has the following characteristics:	Cut-off	Marks
some description of the reasonableness of results; and infrequent application of mathematical concepts and techniques.	>20%	4
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of results; and disjointed application of mathematical concepts and techniques.	>13%	3
	>7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.	>0%	1
• does not satisfy any of the descriptors above.		0