

Specialist Mathematics 2019 v1.2

IA1 mid-level annotated sample response

September 2018

Problem-solving and modelling task (20%)

This sample has been compiled by the QCAA to assist and support teachers to match evidence in student responses to the characteristics described in the instrument-specific marking guide (ISMG).

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topic 2
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topic 2
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topic 2.

Instrument-specific marking guide (ISMG)

Criterion: Formulate

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topic 2
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topic 2
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
<ul style="list-style-type: none"> • documentation of appropriate assumptions • accurate documentation of relevant observations • accurate translation of all aspects of the problem by identifying mathematical concepts and techniques. 	3–4
<ul style="list-style-type: none"> • statement of some assumptions • statement of some observations • translation of simple aspects of the problem by identifying mathematical concepts and techniques. 	1–2
<ul style="list-style-type: none"> • does not satisfy any of the descriptors above. 	0

Criterion: Solve

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topic 2
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topic 2

The student work has the following characteristics:	Marks
<ul style="list-style-type: none"> • accurate use of complex procedures to reach a valid solution • discerning application of mathematical concepts and techniques relevant to the task • accurate and appropriate use of technology. 	6–7
<ul style="list-style-type: none"> • use of complex procedures to reach a reasonable solution • application of mathematical concepts and techniques relevant to the task • use of technology. 	4–5
<ul style="list-style-type: none"> • use of simple procedures to make some progress towards a solution • simplistic application of mathematical concepts and techniques relevant to the task • superficial use of technology. 	2–3
<ul style="list-style-type: none"> • inappropriate use of technology or procedures. 	1
<ul style="list-style-type: none"> • does not satisfy any of the descriptors above. 	0

Criterion: Evaluate and verify

Assessment objectives

4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
<ul style="list-style-type: none">• evaluation of the reasonableness of solutions by considering the results, assumptions and observations• documentation of relevant strengths and limitations of the solution and/or model• justification of decisions made using mathematical reasoning.	4–5
<ul style="list-style-type: none">• statements about the reasonableness of solutions by considering the context of the task• statements about relevant strengths and limitations of the solution and/or model• statements about decisions made relevant to the context of the task.	2–3
<ul style="list-style-type: none">• <u>statement about a decision and/or the reasonableness of a solution.</u>	1
<ul style="list-style-type: none">• does not satisfy any of the descriptors above.	0

Criterion: Communicate

Assessment objective

3. communicate using mathematical, statistical and everyday language and conventions

The student work has the following characteristics:	Marks
<ul style="list-style-type: none">• correct use of appropriate technical vocabulary, procedural vocabulary, and conventions to develop the response• coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet.	3–4
<ul style="list-style-type: none">• <u>use of some appropriate language and conventions to develop the response</u>• <u>adequate organisation of the response.</u>	1–2
<ul style="list-style-type: none">• does not satisfy any of the descriptors above.	0

Task

Context

Each week in the media, sporting commentators give their ‘expert tips’ on the likely winners of upcoming games, but how accurate are these predictions? According to Daniel Colasimone, reporter and producer for *ABC Grandstand*, ‘The world of sport never fails to surprise us, which is why trying to make predictions about it is a fool’s game’.

Colasimone, D 2015, ‘Unreliable 2016 sporting predictions: Tim Cahill, cricketing Mitchells, Nat Fyfe and Sharni Layton star’, *ABC News*, www.abc.net.au/news/2015-12-31/2016-sporting-predictions/7060172.

Task

You will be given a link to a website that contains data about every round of a completed sports competition. Use an appropriate sample of the data to develop a model that will enable you to ‘predict’ the winning teams in the subsequent three rounds of the competition.

This task poses the challenge — can a mathematics student predict a set of sporting results more accurately than the so-called ‘experts’?

Sample response

Criterion	Allocated marks	Marks awarded
Formulate Assessment objectives 1, 2, 5	4	1
Solve Assessment objectives 1, 6	7	3
Evaluate and verify Assessment objectives 4, 5	5	1
Communicate Assessment objective 3	4	2
Total	20	7

The annotations show the match to the instrument-specific marking guide (ISMG) performance-level descriptors.

Communicate [1–2]

adequate organisation of the response

Basic introduction evident

Formulate [1–2]

translation of simple aspects of the problem

Student recognises the suitability of Dominance matrices in ranking procedures.

Formulate [1–2]

statement of some observations

Results are not provided.

There is no documentation or statement of assumptions used.

Formulate [1–2]

translation of simple aspects of the problem

Student set up a suitable Dominance matrix.

Introduction

The purpose of this report is to successfully predict the winning teams in a historical sporting competition.

I chose to use Dominance matrices for the basis of my ranking model as the mathematics behind dominance theory considers this issue. My data came from the Women's AFL Premiership of 2017. I organised the results of all the games from the first 6 rounds into the table below. I used the results from these first 6 round games to predict the results of the four round 7 matches.

Results

I used these results to set up the table below, allocating 2 points for a win, 1 point for a draw and 0 points for a loss.

	Adelaide (A)	Bulldogs (B)	Carlton (C)	Demons (D)	Fremantle (F)	Giants (G)	Lions (L)	Magpies (M)
Adelaide (A)		2	2	0	2	2	0	
Bulldogs (B)	0		0	0	2		0	2
Carlton (C)	0	2		0	0	2		2
Demons (D)	2	2	2			0	0	2
Fremantle (F)	0	0	2			1	0	0
Giants (G)	0		0	2	1		0	0
Lions (L)	2	2		2	2	2		2
Magpies (M)		0	0	0	2	2	0	

The following dominance matrix, X was formed from this table.

$X =$

0	2	2	0	2	2	0	0
0	0	0	0	2	0	0	2
0	2	0	0	0	2	0	2
2	2	2	0	0	0	0	2
0	0	2	0	0	1	0	0
0	0	0	2	1	0	0	0
2	2	0	2	2	2	0	2
0	0	0	0	2	2	0	0

Solve [4–5]

application of mathematical concepts and techniques relevant to the task

Student uses first-order Dominance matrix calculations.

Calculations

Using dominance calculations

$$\begin{matrix}
 & \begin{matrix} 0 & 2 & 2 & 0 & 2 & 2 & 0 & 0 \end{matrix} \\
 & \begin{matrix} 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 \end{matrix} \\
 & \begin{matrix} 0 & 2 & 0 & 0 & 0 & 2 & 0 & 2 \end{matrix} \\
 \mathbf{X1} = & \begin{matrix} 2 & 2 & 2 & 0 & 0 & 0 & 0 & 2 \end{matrix} \\
 & \begin{matrix} 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \end{matrix} \\
 & \begin{matrix} 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \end{matrix} \\
 & \begin{matrix} 2 & 2 & 0 & 2 & 2 & 2 & 0 & 2 \end{matrix} \\
 & \begin{matrix} 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 \end{matrix}
 \end{matrix}
 \quad = \quad
 \begin{matrix}
 \begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{matrix} & \begin{matrix} 8 \\ 4 \\ 6 \\ 8 \\ 3 \\ 3 \\ 12 \\ 4 \end{matrix} & \begin{matrix} A \\ B \\ C \\ D \\ F \\ G \\ L \\ M \end{matrix}
 \end{matrix}$$

Communicate [1–2]

use of some appropriate language and conventions to develop the response

Solve [2–3]

superficial use of technology

There is no evidence of matrix calculations using technology.

The matrix 1 is a column matrix with all values equal to 1.

As I needed to perform matrix multiplications, I used a spreadsheet.

	A	B	C	D	E	F	G	H	I	J	K
12		0	2	2	0	2	2	0	0		1
13		0	0	0	0	2	0	0	2		1
14		0	2	0	0	0	2	0	2		1
15	$\mathbf{X} =$	2	2	2	0	0	0	0	2	$\mathbf{1} =$	1
16		0	0	2	0	0	1	0	0		1
17		0	0	0	2	1	0	0	0		1
18		2	2	0	2	2	2	0	2		1
19		0	0	0	0	2	2	0	0		1

Evaluate and verify [1]

statement about a decision

No evidence of decisions made relevant to the context of the task.

I now used more dominance calculations.

$$\begin{matrix}
 & \begin{matrix} 0 & 2 & 2 & 0 & 2 & 2 & 0 & 0 \end{matrix} \\
 & \begin{matrix} 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 \end{matrix} \\
 & \begin{matrix} 0 & 2 & 0 & 0 & 0 & 2 & 0 & 2 \end{matrix} \\
 \mathbf{X+0.1X^2} = & \begin{matrix} 2 & 2 & 2 & 0 & 0 & 0 & 0 & 2 \end{matrix} \\
 & \begin{matrix} 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \end{matrix} \\
 & \begin{matrix} 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \end{matrix} \\
 & \begin{matrix} 2 & 2 & 0 & 2 & 2 & 2 & 0 & 2 \end{matrix} \\
 & \begin{matrix} 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 \end{matrix}
 \end{matrix}
 \quad +0.1 \quad
 \begin{matrix}
 \begin{matrix} 0 & 4 & 4 & 4 & 6 & 6 & 0 & 8 \end{matrix} \\
 \begin{matrix} 0 & 0 & 4 & 0 & 4 & 6 & 0 & 0 \end{matrix} \\
 \begin{matrix} 0 & 0 & 0 & 4 & 10 & 4 & 0 & 4 \end{matrix} \\
 \begin{matrix} 0 & 8 & 4 & 0 & 12 & 12 & 0 & 8 \end{matrix} \\
 \begin{matrix} 0 & 4 & 0 & 2 & 1 & 4 & 0 & 4 \end{matrix} \\
 \begin{matrix} 4 & 4 & 6 & 0 & 0 & 1 & 0 & 4 \end{matrix} \\
 \begin{matrix} 4 & 8 & 12 & 4 & 14 & 10 & 0 & 8 \end{matrix} \\
 \begin{matrix} 0 & 0 & 4 & 4 & 2 & 2 & 0 & 0 \end{matrix}
 \end{matrix}$$

Solve [4–5]

application of mathematical concepts and techniques relevant to the task

Student shows use of second-order Dominance matrix calculations and evidence of parameter weighting considerations.

Communicate [1–2]

use of some appropriate language and conventions to develop the response

Student organised the predictions in a table.

$$X + 0.1X^2 = \begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 2.4 & 2.4 & 0.4 & 2.6 & 2.6 & 0 & 0.8 \\ \hline 0 & 0 & 0.4 & 0 & 2.4 & 0.6 & 0 & 2 \\ \hline 0 & 2 & 0 & 0.4 & 1 & 2.4 & 0 & 2.4 \\ \hline 2 & 2.8 & 2.4 & 0 & 1.2 & 1.2 & 0 & 2.8 \\ \hline 0 & 0.4 & 2 & 0.2 & 0.1 & 1.4 & 0 & 0.4 \\ \hline 0.4 & 0.4 & 0.6 & 2 & 1 & 0.1 & 0 & 0.4 \\ \hline 2.4 & 2.8 & 1.2 & 2.4 & 3.4 & 3 & 0 & 2.8 \\ \hline 0 & 0 & 0.4 & 0.4 & 2.2 & 2.2 & 0 & 0 \\ \hline \end{array}$$

Multiplying $X + 0.1X^2$ by matrix 1 created the following rankings.

$$(X+0.1X^2)1 = \begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 2.4 & 2.4 & 0.4 & 2.6 & 2.6 & 0 & 0.8 \\ \hline 0 & 0 & 0.4 & 0 & 2.4 & 0.6 & 0 & 2 \\ \hline 0 & 2 & 0 & 0.4 & 1 & 2.4 & 0 & 2.4 \\ \hline 2 & 2.8 & 2.4 & 0 & 1.2 & 1.2 & 0 & 2.8 \\ \hline 0 & 0.4 & 2 & 0.2 & 0.1 & 1.4 & 0 & 0.4 \\ \hline 0.4 & 0.4 & 0.6 & 2 & 1 & 0.1 & 0 & 0.4 \\ \hline 2.4 & 2.8 & 1.2 & 2.4 & 3.4 & 3 & 0 & 2.8 \\ \hline 0 & 0 & 0.4 & 0.4 & 2.2 & 2.2 & 0 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 1 & 11.2 & A \\ \hline 1 & 5.4 & B \\ \hline 1 & 8.2 & C \\ \hline 1 & 12.4 & D \\ \hline 1 & 4.5 & F \\ \hline 1 & 4.9 & G \\ \hline 1 & 18 & L \\ \hline 1 & 5.2 & M \\ \hline \end{array}$$

The rank order of teams was:

Rank	Team
1	L
2	D
3	A
4	C
5	B
6	M
7	G
8	F

Evaluate and verify [1]

statement about a decision

Formulate [1–2]

statement of some observations

There is no documentation of the relevant observations.

Evaluate and verify [1]

statement about the reasonableness of a solution

There is no evidence of the limitations of the solution and/or model.

Solve [2–3]

use of simple procedures to make some progress towards a solution

A reasonable solution has not been reached, as the 'predicted' results for the subsequent three rounds have not been addressed, as required in the task.

Communicate [1–2]

Little evidence of appropriate technical vocabulary and procedural vocabulary to develop the response. No evidence of a conclusion.

Predictions

My predictions for the round 7 matches were:

Match	Predicted winner
Demons (D) vs Fremantle (F)	D
Giants (G) vs Bulldogs (B)	B
Magpies (M) vs Adelaide (A)	A
Carlton (C) vs Lions (L)	L

The winners of the actual games were D, B, A and there was a tie between C and L. This is a very accurate result.

My model was almost perfect in predicting the winning teams of the next round.

Note: The student did not include any appendixes with this response.