Specialist Mathematics 2019 v1.2

IA1: High-level annotated sample response 1

October 2021

Problem-solving and modelling task (20%) 1

This sample has been compiled by the QCAA to assist and support teachers to match evidence in student responses to the characteristics described in the instrument-specific marking guide (ISMG).

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules definitions and procedures drawn from Unit 3 Topics 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3

Queensland Government
Queensland Curriculum & Assessment Authority
For all Queensland schools
210950

Instrument-specific marking guide (ISMG)

Criterion: Formulate

Assessment objectives

1. select, recall and use facts, rules definitions and procedures drawn from Unit 3 Topics 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• documentation of appropriate assumptions • accurate documentation of relevant observations • accurate translation of all aspects of the problem by identifying mathematical concepts and techniques.	3–4
• statement of some assumptions • statement of some observations • translation of simple aspects of the problem by identifying mathematical concepts and techniques.	1–2
• does not satisfy any of the descriptors above.	0

Criterion: Solve

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3

The student work has the following characteristics:	Marks
• accurate use of complex procedures to reach a valid solution • discerning application of mathematical concepts and techniques relevant to the task • accurate and appropriate use of technology.	6–7
• use of complex procedures to reach a reasonable solution • application of mathematical concepts and techniques relevant to the task • use of technology.	4–5
• use of simple procedures to make some progress towards a solution • simplistic application of mathematical concepts and techniques relevant to the task • superficial use of technology.	2–3
• inappropriate use of technology or procedures.	1
• does not satisfy any of the descriptors above.	0

Criterion: Evaluate and verify

Assessment objectives

4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model • justification of decisions made using mathematical reasoning.	4–5
• statements about the reasonableness of solutions by considering the context of the task • statements of relevant strengths and limitations of the solution and/or model • statements about decisions made relevant to the context of the task.	2–3
• statement about a decision and/or the reasonableness of a solution.	1
• does not satisfy any of the descriptors above.	0

Criterion: Communicate

Assessment objective

3. communicate using mathematical, statistical and everyday language and conventions

The student work has the following characteristics:	Marks
• correct use of appropriate technical vocabulary, procedural vocabulary and conventions to develop the response • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet.	3–4
• use of some appropriate language and conventions to develop the response • adequate organisation of the response.	1–2
• does not satisfy any of the descriptors above.	0

Task

Each week in the media, sporting commentators give their 'expert tips' on the likely winners of upcoming games, but how accurate are these predictions? According to Daniel Colasimone, reporter and producer for *ABC Grandstand*, 'The world of sport never fails to surprise us, which is why trying to make predictions about it is a fool's game'.

Colasimone, D 2015, 'Unreliable 2016 sporting predictions: Tim Cahill, cricketing Mitchells, Nat Fyfe and Sharni Layton star', *ABC News*, www.abc.net.au/news/2015-12-31/2016-sporting-predictions/7060172. Used with permission.

You will be given a link to a website that contains data about every round of a completed sports competition. Use an appropriate sample of the data to develop a model that will enable you to 'predict' the winning teams in at least three subsequent rounds of the competition.

This task poses the challenge — can a mathematics student predict a set of sporting results more accurately than the so-called 'experts'?

See IA1: Examination — short response (25%) 1 (available on the QCAA Portal).

Sample response

Criterion	Marks allocated	Provisional marks
Formulate Assessment objectives 1, 2, 5	4	4
Solve Assessment objectives 1, 6	7	7
Evaluate and verify Assessment objectives 4, 5	5	5
Communicate Assessment objective 3	4	4
Total	**20**	**20**

The annotations show the match to the instrument-specific marking guide (ISMG) performance-level descriptors.

Communicate [3–4]

coherent and concise organisation of the response

The introduction clearly describes what the task is about and concisely outlines the intent of the writer, independent of the task sheet.

1 Introduction

The purpose of this task was to design a mathematical model for ranking teams using the data of the 2016 National Rugby League (NRL) competition. This report analysed sufficient results from a sample of the 40 competition matches to produce individual rankings for all 16 teams. These were subsequently used to predict results of the next 3 rounds.

A comparison test of the forecasts against those of a 'so-called expert' was developed which acted as an 'accuracy barometer'.

Formulate [3–4]

accurate translation of aspects of the problem by identifying mathematical concepts and techniques

These factors were considered:

- Not all teams had played each other the same number of times.
- Highly ranked teams may have played many lower ranked teams.

Dominance theory is used as the basis for the mathematical model as it considers the discrepancies related to these two factors. Based on the observations from the first five rounds of the 2016 NRL season (see match results in Appendix 1), the dominance ranking vectors were calculated using third-order dominance calculations with suitable weighting parameters to ensure individual ranking. Subsequently, winners from each match from rounds 6–8 were predicted by selecting the highest ranked team.

Formulate [3–4]

accurate documentation of relevant observations

The details of the competition and premiership ladder, from which the expert's opinion are derived, are documented.

The NRL competition consists of 16 teams that play 26 rounds in the regular season with each team playing 9 teams twice, 6 teams once and two byes. 'Expert' match tipping was done by picking the higher team on the points ladder after each round. The points system used by the ladder awards two points for a win or bye, one for a draw, and zero for a loss. Teams with the equal points are ranked based on a match points differential (match points scored less match points scored against) (Wikipedia, 2020). The expert's predictions and those generated by the dominance matrix were compared to the actual results in rounds 6–8.

Refinement of that model was considered necessary if either a high mean percentage of accurate forecasting was not achieved and/or the expert's predictions were better than those generated by the dominance model.

A spreadsheet was utilised for matrix representations, which simplified repeated calculations of large data and ease of transition to refined models.

Assumptions

Formulate [3–4]

documentation of appropriate assumptions

1. The first 5 rounds are adequate to demonstrate the form of each team allowing accurate predictions to be made. This is a reasonable assumption if the draw provides opportunity for each team to play against a range of opponents.
2. To rank teams who have not played each other, indirect win–loss results are a better ranking metric than the magnitude of head-to-head winning margins. This assumption promotes the use of dominance matrix techniques.
3. Home venue, game time and weather conditions do not affect the results. While these are variables commonly known to affect the outcome of sporting competitions, they are ignored here to simplify the model formulation.

4. Team form does not vary with time. Team form can change with player injury, training cycles, etc. However, these things should remain reasonably consistent for a five-round sample.

5. The NRL Premiership ladder is a reasonable substitution for an expert opinion. Analysis of AFL Premiership results has shown that backing the higher team on the premiership ladder has a similar accuracy to expert opinions (Woodcock, 2016).

The 16 teams in the 2016 NRL competition are listed (with abbreviations) in Table 1.

Table 1: List of the 16 teams in the 2016 NRL competition

BB	Brisbane Broncos	**NK**	Newcastle Knights
CB	Canterbury Bulldogs	**NW**	New Zealand Warriors
CR	Canberra Raiders	**PE**	Parramatta Eels
CS	Cronulla Sharks	**PP**	Penrith Panthers
GT	Gold Coast Titans	**SD**	St George Dragons
ME	Manly Sea Eagles	**SR**	Sydney Roosters
MS	Melbourne Storm	**SS**	South Sydney
NC	North Qld Cowboys	**WT**	West Tigers

Communicate [3–4]

coherent and concise organisation of the response

The main body of the report clearly mathematises the concepts formulated in the introduction.

2 Results and discussion

The NRL Premiership ladder in Appendix 2a represents the expert's ranking model after the 5th round.

A comparison of the predictions against the actual results for the next 3 rounds is summarised in Table 2 with successful predictions highlighted). The mean percent correct was 66.7%.

Table 2: Expert's predictions vs.. Actual results for Rounds 6–8

Data sourced from Wikipedia 2021, '2016 NRL Season'. https://en.wikipedia.org/wiki/2016_NRL_season#Regular_season.

Communicate [3–4]

coherent and concise organisation of the response

Clear connection between discussion and labelled tables. The table concisely summarises the main results with the use of the raw data located in the appendix.

Round 6 Games	Expert's prediction	Round 7 Games	Expert's prediction	Round 8 Games	Expert's prediction
BB v SD	BB	ME v PE	PE	BB v SR	BB
SS v SR	SS	NC v SS	NC	CB v GT	CB
PE v CR	CR	GT v SD	GT	CR v WT	CR
NW v ME	NW	CB v NW	CB	NC v PE	NC
PP v NC	NC	BB v NK	BB	CS v PP	CS
CS v GT	CS	CR v CS	CR	NK v ME	ME
NK v WT	WT	WT v MS	MS	SD v SR	SD
MS v CB	MS	SR v PP	PP	MS v NW	MS
% correct	37.5%	% correct	62.5%	% correct	100.0%
Mean % correct				**66.7%**	

Evaluate and verify [4–5]

evaluation of the reasonableness of solutions by considering the assumptions and observations

In Table 2, it is evident that the accuracy rate of the predictions improved as the rounds progressed, with 100% correct in Round 8. As these predictions were only based on the data from Round 5, this appears to support assumption 1.

However, the success of the last round of predictions could also be explained through the fact that the majority of games in this round involved teams in the top half of the ladder pitted against teams in the bottom half.

Model 1a

The 'win/loss' data from Rounds 1–5 of the competition was modelled as a dominance matrix, **D** in Figure 1. The following values were used: win = 1, draw = 0.5 and loss/no game played = 0. For full details, see Appendix 3.

Figure 1: Dominance matrix, D for Model 1a

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Formulate [3–4]

accurate translation of aspects of the problem by identifying mathematical concepts and techniques

The variables used in the third-order dominance matrix calculations were defined as:

- $\mathbf{R}_n$ = nth order ranking vector
- $\mathbf{1}$ = column matrix with all values of 1

The model used to calculate the ranking vector was $\mathbf{R}_3 = \mathbf{D1} + 0.5\,\mathbf{D}^2\mathbf{1} + 0.3\,\mathbf{D}^3\mathbf{1}$.

Solve [6–7]

application of mathematical concepts and techniques relevant to the task
accurate use of complex procedures to reach a valid solution

The solution involves a combination of parts that are interconnected.

The weighting parameters of 0.5 and 0.3 were chosen as the victories in the first order were considered more significant than those of the second order, which in turn were more significant than those of the third order.

The third-order ranking vector, $\mathbf{R}_3$, was determined using spreadsheet calculations and this result, as well as the subsequent ranking of the teams, is given in Figure 2.

The parameter values could be easily manipulated on the spreadsheet to ensure that the ranking vector clearly provided for the individual ranking of each team.

Figure 2: Third-order ranking vector, R_3 for Model 1a

	R_3 values	Team	Ranked R_3 values	Team	Ranked Position 1- 16	Team
	15.35	BB	15.35	BB	1	BB
	10.25	CB	11.95	CS	2	CS
	10.5875	CR	11.8	PE	3	PE
	11.95	CS	11.725	MS	4	MS
	9.3	GT	11.2	PP	5	PP
	5.9	ME	10.5875	CR	6	CR
	11.725	MS	10.25	CB	7	CB
$\mathbf{R_3} = \mathbf{D}\,\mathbf{1} + 0.5\,\mathbf{D}^2\,\mathbf{1} + 0.3\,\mathbf{D}^3\,\mathbf{1} =$	9.4	NC	9.4	NC	8	NC
	2.1625	NK	9.3	GT	9	GT
	2.775	NW	7.35	SD	10	SD
	11.8	PE	5.9	ME	11	ME
	11.2	PP	5.675	SS	12	SS
	7.35	SD	5.05	WT	13	WT
	0	SR	2.775	NW	14	NW
	5.675	SS	2.1625	NK	15	NK
	5.05	WT	0	SR	16	SR

Solve [6–7]

accurate and appropriate use of technology

Recognition that the use of a spreadsheet promotes the efficient input, calculation, refinement and display of large amounts of data. Weighting parameters can be easily modified.

The resulting rankings and relevant sections of the spreadsheet formulas used to perform the calculations to determine $\mathbf{R_3}$ are shown in Figure 3.

Figure 3: Formulas used to determine third-order rankings for Model 1a

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	U	Y	Z
29		0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0		1		=MMULT(C29:R44,U29:U44)
30		0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0		1		=MMULT(C29:R44,U29:U44)
31		0	1	0	0	0	0	0	0	0.5	0	0	1	0	1	0	0		1		=MMULT(C29:R44,U29:U44)
32		0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1		1		=MMULT(C29:R44,U29:U44)
33		0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1		1		=MMULT(C29:R44,U29:U44)
34		0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0		1		=MMULT(C29:R44,U29:U44)
35		0	0	0	0	1	0	0	0	1	1	0	0	1	0	0	0		1		=MMULT(C29:R44,U29:U44)
36	**D =**	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	**1 =**	1	**D 1 =**	=MMULT(C29:R44,U29:U44)
37		0	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0		1		=MMULT(C29:R44,U29:U44)
38		0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0		1		=MMULT(C29:R44,U29:U44)
39		0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1		1		=MMULT(C29:R44,U29:U44)
40		1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0		1		=MMULT(C29:R44,U29:U44)
41		0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0		1		=MMULT(C29:R44,U29:U44)
42		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		1		=MMULT(C29:R44,U29:U44)
43		0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0		1		=MMULT(C29:R44,U29:U44)
44		0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0		1		=MMULT(C29:R44,U29:U44)

	B	C	D	E	F
133		R_3 values	Team	Weighting	
134		=Z29:Z44+F135*X70:X85+F136*X113:X128	BB	parameters	
135		=Z29:Z44+F135*X70:X85+F136*X113:X128	CB	a =	0.5
136		=Z29:Z44+F135*X70:X85+F136*X113:X128	CR	b =	0.3
137		=Z29:Z44+F135*X70:X85+F136*X113:X128	CS		
138		=Z29:Z44+F135*X70:X85+F136*X113:X128	GT		
139		=Z29:Z44+F135*X70:X85+F136*X113:X128	ME		
140		=Z29:Z44+F135*X70:X85+F136*X113:X128	MS		
141	$\mathbf{R_3} = \mathbf{D}\,\mathbf{1} + 0.5\,\mathbf{D}^2\,\mathbf{1} + 0.3\,\mathbf{D}^3\,\mathbf{1} =$	=Z29:Z44+F135*X70:X85+F136*X113:X128	NC		
142		=Z29:Z44+F135*X70:X85+F136*X113:X128	NK		
143		=Z29:Z44+F135*X70:X85+F136*X113:X128	NW		
144		=Z29:Z44+F135*X70:X85+F136*X113:X128	PE		
145		=Z29:Z44+F135*X70:X85+F136*X113:X128	PP		
146		=Z29:Z44+F135*X70:X85+F136*X113:X128	SD		
147		=Z29:Z44+F135*X70:X85+F136*X113:X128	SR		
148		=Z29:Z44+F135*X70:X85+F136*X113:X128	SS		
149		=Z29:Z44+F135*X70:X85+F136*X113:X128	WT		

A comparison of the predictions against the actual results for the next 3 rounds using Model 1a is summarised in Table 3.

Table 3: Model 1a Predictions vs. Actual results for Rounds 6–8

Data sourced from Wikipedia 2021, '2016 NRL Season', https://en.wikipedia.org/wiki/2016_NRL_season#Regular_season

Round 6 Games	Model 1(a) prediction	Round 7 Games	Model 1(a) prediction	Round 8 Games	Model 1(a) prediction
BB v SD	BB	ME v PE	PE	BB v SR	BB
SS v SR	SS	NC v SS	NC	CB v GT	CB
PE v CR	PE	GT v SD	GT	CR v WT	CR
NW v ME	ME	CB v NW	CB	NC v PE	NC
PP v NC	PP	BB v NK	BB	CS v PP	CS
CS v GT	CS	CR v CS	CS	NK v ME	ME
NK v WT	WT	WT v MS	MS	SD v SR	SD
MS v CB	MS	SR v PP	PP	MS v NW	MS
% correct	50%	% correct	75%	% correct	87.5%
Mean % correct				**70.8%**	

Evaluate and verify [4–5]

evaluation of the reasonableness of solutions by considering the results, assumptions and observations

The mean accuracy rate of 70.8% was only marginally better than that gained by the expert. Also, the expert gained 100% correct predictions in one round while the best achieved by the dominance matrix model was 87.5%. Therefore Model 1(a) is not considered to produce a valid solution, as the model did not provide significantly better success. The evaluation is that the data for 5 rounds of competition did not provide sufficient data to establish a trend so assumption 1 needed to be reviewed.

Model 1b

Formulate [3–4]

documentation of appropriate assumptions

To refine the model, assumption 1 was amended such that the first 8 rounds are adequate to demonstrate the form of each team allowing accurate predictions to be made. This allowed each team to have played over half of the opposition teams, providing data involving a greater cross-section of form. The refined dominance matrix D is shown in Appendix 4 with the updated $\mathbf{R}_3$ shown below.

Figure 4: Third-order ranking vector, R_3 for Model 1b

Solve [6–7]

accurate and appropriate use of technology

The spreadsheet from Model 1a could be easily adapted for Model 1b calculations.

$\mathbf{R}_3 = \mathbf{D}\,\mathbf{1} + 0.5\,\mathbf{D}^2\,\mathbf{1} + 0.3\,\mathbf{D}^3\,\mathbf{1} =$

R_3 values	Team
43.675	BB
31.15	CB
23.6125	CR
37.125	CS
13.9	GT
20.925	ME
27.525	MS
39.95	NC
7.4875	NK
14.625	NW
37.125	PE
24.5	PP
18.15	SD
4.45	SR
11.875	SS
11.2	WT

Ranked R_3 values	Team
43.675	BB
39.95	NC
37.125	CS
37.125	PE
31.15	CB
27.525	MS
24.5	PP
23.6125	CR
20.925	ME
18.15	SD
14.625	NW
13.9	GT
11.875	SS
11.2	WT
7.4875	NK
4.45	SR

Ranked Position 1- 16	Team
1	BB
2	NC
3	CS
4	PE
5	CB
6	MS
7	PP
8	CR
9	ME
10	SD
11	NW
12	GT
13	SS
14	WT
15	NK
16	SR

Communicate [3–4]

correct use of appropriate technical vocabulary, procedural vocabulary, and conventions

The spreadsheet formulas used to perform the calculations to determine the updated vector $\mathbf{R}_3$ are shown in Appendix 5. The expert's model was also amended using the NRL ladder after Round 8 (see Appendix 2b). Table 4 shows a comparison of the predictions and their accuracy.

Table 4: Dominance matrix model, Expert's predictions vs. Actual results for Rounds 6–8

Data sourced from Wikipedia 2021, '2016 NRL Season', https://en.wikipedia.org/wiki/2016_NRL_season#Regular_season.

Round 9 Games	Expert's prediction	Model 1(b) prediction	Round 10 Games	Expert's prediction	Model 1(b) prediction winner	Round 11 Games	Expert's prediction	Model 1(b) prediction winner
SS v WT	SS	SS	SD v CR	CR	CR	SS v SD	SD	SD
PE v CB	PE	PE	PE v SS	PE	PE	NC v BB	BB	BB
PP v CR	CR	PP	PP v NW	PP	PP	WT v NK	WT	WT
SR v NK	NK	NK	MS v NC	NC	NC	NW v CR	CR	CR
ME v NC	NC	NC	ME v BB	BB	BB	CS v ME	CS	CS
NW v SD	SD	SD	NK v CS	CS	CS	PP v GT	GT	PP
GT V MS	MS	MS	WT v CB	CB	CB	CB v SR	CB	CB
CS V BB	BB	BB	GT v SR	GT	GT	PE V MS	MS	PE
% correct	37.5 %	50.0 %	% correct	62.5 %	62.5%	% correct	75.0 %	50.0%
Mean % correct			**Expert's Model**		**58.3%**	**Model 1(b)**		**54.2%**

Evaluate and verify [4–5]

evaluation of the reasonableness of solutions by considering the results and assumptions

The mean success rate for the predictions was 54.2%, lower than the accuracy achieved using the expert's model as well as the original model. These results indicate that the refined model did not improve the solution so further revisions to some of the original assumptions needed to occur.

Model 2

Formulate [3–4]

documentation of appropriate assumptions
accurate translation of aspects of the problem by identifying mathematical concepts and techniques

New model introduced.

While maintaining the assumption that 8 rounds are adequate to provide sufficient data, Assumption 3 was amended such that the home and away venue does affect the results as evidenced by sports commentators' discussions. An additional assumption was that the actual scores from each match provide significant data in making accurate predictions, which is reasonable if an equitable draw is provided. A new model to rank teams was developed, based on a statistical analysis of this additional data. The analysis considered the home ground advantage for each team, using the mean points margin at home and away as defined below.

$$\text{Mean points margin at home} = \frac{\text{Total points margin at home games}}{\text{Number of home games}}$$

Communicate [3–4]

correct use of appropriate technical vocabulary, procedural vocabulary, and conventions

Solve [6–7]

accurate and appropriate use of technology

The use of a spreadsheet promotes the efficient input, calculation, refinement and display of large amounts of data for this statistical analysis.

$$\text{Mean points margin away} = \frac{\text{Total points margin at away games}}{\text{Number of away games}}$$

To calculate this set of statistics for each team, the winning margin from each game across the 8 rounds was determined, e.g. in Round 1, PE (home team) was defeated by BB (away team) by 4–17, giving a point's margin of 13. As shown in the home team points margin matrix of Table 5a, PE received –13 while in the away team points margin matrix of Table 5b, BB received 13. These winning margins were then used to calculate the mean points margin in home and away games.

Table 5a: Home games winning points margin summary

Data sourced from Wikipedia 2021, '2016 NRL Season', https://en.wikipedia.org/wiki/2016_NRL_season#Regular_season

Home Ground Advantage Statistical Analysis											
	Home Team Margins										
	Round Number								Number of Home games	Total margin in Home games	Average margin in Home games
Team	1	2	3	4	5	6	7	8			
BB		15		1		26	53	22	5	117	23.4
CB			-14		-14		-4	1	4	-31	-7.8
CR	8	1		-4			-24	52	5	33	6.6
CS		28		8		5		2	4	43	10.8
GT	18		12		-8		-5		4	17	4.3
ME	-22		10		-4		-12		4	-28	-7.0
MS	2	18			4	-6		42	5	60	12.0
NC	6		40		36		26	16	5	124	24.8
NK			0			2		-14	3	-12	-4.0
NW			-7	32		-16			3	9	3.0
PE	-13	4			-2	30			4	19	4.8
PP		-2	1			-5			3	-6	-2.0
SD			2	2				2	3	6	2.0
SR	-32			-2	-6		-4		4	-44	-11.0
SS		42		-30		-7			3	5	1.7
WT	8	14		-8	8		-1		5	21	4.2

Table 5b: Away games winning points margin summary

Data sourced from Wikipedia 2021, '2016 NRL Season', https://en.wikipedia.org/wiki/2016_NRL_season#Regular_season

Away Ground Advantage Statistical Analysis											
	Away Team Margins										
	Round Number								Number of Away games	Total margin in Away	Average margin in Away
Team	1	2	3	4	5	6	7	8			
BB	13		-1		8				3	20	6.7
CB	22	2		30		6			4	60	15.0
CR			0		14	-30			3	-16	-5.3
CS	-6		-10		-8		24		4	0	0.0
GT		-18		4		-5		-1	4	-20	-5.0
ME		-14		2		16		14	4	18	4.5
MS			7	-8			1		3	0	0.0
NC		-4		-1		5			3	0	0.0
NK	-18	-42		-32	-4		-53		5	-149	-29.8
NW	-8	-15			6		4	-42	5	-55	-11.0
PE			14	8			12	-16	4	18	4.5
PP	-8			-2	2		4	-2	5	-6	-1.2
SD	-2	-28			-36	-26	5		5	-87	-17.4
SR		-1	-40			7		-2	4	-36	-9.0
SS	32		-2		4		-26	-22	5	-14	-2.8
WT			-12			-2		-52	3	-66	-22.0

An example of the average point's margin in home and away games for BB is given below (see results highlighted in Tables 5a and 5b).

$$\text{Average points margin scored at home for BB} = \frac{117}{5} = 23.4$$

$$\text{Average points margin scored away for BB} = \frac{20}{3} = 6.7$$

Solve [6–7]

accurate and appropriate use of technology

An excerpt of the spreadsheet formulas used for Model 2 is shown below in Figure 5. See Appendixes 6 and 7 for the full calculations/formulas.

Figure 5: Formulas used to determine statistical rankings for Model 2

	A	C	D	E	F	G	H	I	J	K	N	O
1	Home Ground Advantage Statistical Analysis											
2											Home Team Margins	
3	Team	Round Number								Number of home games	Total margin in Home games	Average margin in Home games
4		1	2	3	4	5	6	7	8			
5	BB		15		1		26	53	22	=COUNTA(W5:AD5)	=SUM(C5:J5)	=N5/K5
6	CB			-14		-14		-4	1	=COUNTA(W6:AD6)	=SUM(C6:J6)	=N6/K6
7	CR	8	1		-4			-24	52	=COUNTA(W7:AD7)	=SUM(C7:J7)	=N7/K7
8	CS		28		8		5		2	=COUNTA(W8:AD8)	=SUM(C8:J8)	=N8/K8

Communicate [3–4]

coherent and concise organisation of the response

Teams were then ranked based on their average points winning margin for both home and away games as shown in Table 6. These values represent each team's 'level of home and away advantage'. Subsequently, winners from each match were predicted by determining who had the highest home and away ranking from the opposing home team and away team.

Table 6: Rank order of teams using the average home and away games points winning margins

Team	Average winning margin in home games
NC	24.8
BB	23.4
MS	12.0
CS	10.8
CR	6.6
PE	4.8
GT	4.3
WT	4.2
NW	3.0
SD	2.0
SS	1.7
PP	–2.0
NK	–4.0
ME	–7.0
CB	–7.8
SR	–11.0

Team	Average winning margin in away games
CB	15.0
BB	6.7
ME	4.5
PE	4.5
CS	0.0
MS	0.0
NC	0.0
PP	–1.2
SS	–2.8
GT	–5.0
CR	–5.3
SR	–9.0
NW	–11.0
SD	–17.4
WT	–22.0
NK	–29.8

Solve [6–7]

accurate and appropriate use of technology

Formulate [3–4]

accurate documentation of relevant observations

Solve [6–7]

accurate use of complex procedures to reach a valid solution

The solution involves a combination of parts that are interconnected.

Solve [6–7]

discerning application of mathematical concepts and techniques relevant to the task

Shows good judgment to make thoughtful and astute choices.

Evaluate and verify [4–5]

justification of decisions made using mathematical reasoning

Student shows consideration of measures of success in the prediction process.

Based on the average margin in home games, the upper end of Table 6 indicated that NC, BB, MS, CS and CR (shaded blue) have at least one converted try (6 points) advantage on average over their opposing team when they play at home. However, at the lower end, ME, CB and SR (shaded red) 'concede' at least one converted try when they play at home.

Conversely, based on the average margin in away games, CB and BB (shaded blue) hold an advantage when they play away, while SR, NW, SD, WT, NK (shaded red) perform poorly.

To make predictions, Table 6 was used to compare the ranking of a home team's advantage against an away team's advantage. For example, in Round 9, when PP (home) played CR (away), PP's average home margin of –2.0 points was compared against CR's average away margin of –5.3. Since PP's margin was larger, PP was subsequently predicted as the likely winner.

Predictions and success against the actual results for Rounds 9–11 are summarised in Table 7. Accurate predictions are shaded in grey. See Appendix 8 for more details.

Table 7: Average home and away games points margin ranking predictions vs. Actual results for Rounds 6–8

Data sourced from Wikipedia 2021, '2016 NRL Season', https://en.wikipedia.org/wiki/2016_NRL_season#Regular_season

Home v Away Team	Predicted winner using average home/away margin	Home v Away Team	Predicted winner using average home/away margin	Home v Away Team	Predicted winner using average home/away margin
Round 9 Games		Round 10 Games		Round 11 Games	
SS v WT	SS	SD v CR	SD	SS v SD	SS
PE v CB	CB	PE v SS	SS	NC v BB	NC
PP v CR	PP	PP v NW	PP	WT v NK	WT
SR v NK	SR	MS v NC	MS	NW v CR	NW
ME v NC	NC	ME v BB	BB	CS v ME	CS
NW v SD	NW	NK v CS	NK	PP v GT	PP
GT V MS	GT	WT v CB	CB	CB v SR	SR
CS V BB	CS	GT v SR	GT	PE V MS	MS
Percentage correct	62.5%	Percentage correct	100%	Percentage correct	62.5%
Mean percentage correct				**75.0%**	

As shown in the table, the mean success rate of the predictions increased to 75%, representing improvement over both the previous model (54.2%) and the expert's model (58.3%).

Communicate [3–4]

coherent and concise organisation of the response, appropriate to the genre

Conclusion provides an overview of the significance of the information presented in the previous sections.

Evaluate and verify [4–5]

documentation of relevant strengths and limitations of the solution and model

Communicate [3–4]

correct use of appropriate technical vocabulary, procedural vocabulary, and conventions to develop the response

3 Conclusion

For this task, two variations of dominance matrix models and a statistical-based matrix model were used to rank the 16 teams in the 2016 NRL competition. While the models were based on similar samples of games, different principles were used. The NRL Premiership ladder was used as the basis for simulating a comparative expert's model.

Overall, the models produced mean prediction accuracies better than 50% and exhibited a level of agreement with respect to each other.

The strength of the models include: all models compare favourably to results (better results than randomly selecting a team, which would give a 50% probability), Models 1 and 1a (using dominance matrices) take into account indirect results, rather than just head-to-head results and Model 2 was formulated by reviewing some of the original assumptions, introducing factors that are commonly recognised to affect sporting outcomes (points margin/home advantage).

A limitation is that it was not possible to use any of the models to obtain a 100% accuracy rate. However, common sense dictates that this success rate could never be achievable due to the degree of unpredictability in results. A further limitation was that Model 2 did not consider indirect results.

The home ground advantage model resulted in a 'game high' prediction success rate of 75%. This was better than the expert's corresponding result of 58.3%. Research into the accuracy of New Zealand NRL sports analyst David Scott revealed that he had attained less than a 60% accuracy rate on average over the past few years (Stats Chat 2017) . Similar research by Quora (2013) into a number of US sports betting agencies indicates that success rates of 60% or less are quite typical.

In conclusion, a 'true' ranking of teams guaranteeing a 100% accuracy rate is essentially an impossible task. So at 75% accuracy, Model 2 is considered to have produced a valid solution. Therefore, based on the analysis of this investigation, it is reasonable to claim that a mathematics student can predict a set of sporting results more accurately than 'so-called experts'.

Mathematical modelling that considers other factors, such as the timing of games, turnaround times/travel between games and consistency of player availability could further enhance success in predictions.

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4 Appendixes

Appendix 1

2016 NRL season results for rounds 1–11

Data sourced from Wikipedia 2021, '2016 NRL Season',
https://en.wikipedia.org/wiki/2016_NRL_season#Regular_season

Round 1				
3-Mar-16	Parramatta Eels	Brisbane Broncos	4	17
4-Mar-16	Manly Sea Eagles	Canterbury Bulldogs	6	28
5-Mar-16	Canberra Raiders	Penrith Panthers	30	22
5-Mar-16	Wests Tigers	New Zealand Warriors	34	26
5-Mar-16	North QLD Cowboys	Cronulla Sharks	20	14
6-Mar-16	Sydney Roosters	South Sydney Rabbitohs	10	42
6-Mar-16	Gold Coast Titans	Newcastle Knights	30	12
7-Mar-16	Melbourne Storm	St George Dragons	18	16
Round 2				
10-Mar-16	Penrith Panthers	Canterbury Bulldogs	16	18
11-Mar-16	Brisbane Broncos	New Zealand Warriors	25	10
12-Mar-16	Canberra Raiders	Sydney Roosters	21	20
12-Mar-16	South Sydney Rabbitohs	Newcastle Knights	48	6
12-Mar-16	Parramatta Eels	North QLD Cowboys	20	16
13-Mar-16	Cronulla Sharks	St George Dragons	30	2
13-Mar-16	Melbourne Storm	Gold Coast Titans	34	16
14-Mar-16	Wests Tigers	Manly Sea Eagles	36	22
Round 3				
17-Mar-16	North QLD Cowboys	Sydney Roosters	40	0
18-Mar-16	Canterbury Bulldogs	Parramatta Eels	6	20
19-Mar-16	Newcastle Knights	Canberra Raiders	24	24
19-Mar-16	Penrith Panthers	Brisbane Broncos	23	22
19-Mar-16	Gold Coast Titans	Wests Tigers	30	18
20-Mar-16	New Zealand Warriors	Melbourne Storm	14	21
20-Mar-16	St George Dragons	South Sydney Rabbitohs	8	6
21-Mar-16	Manly Sea Eagles	Cronulla Sharks	22	12
Round 4				
25-Mar-16	South Sydney Rabbitohs	Canterbury Bulldogs	12	42
25-Mar-16	Brisbane Broncos	North QLD Cowboys	21	20
26-Mar-16	Canberra Raiders	Gold Coast Titans	20	24
26-Mar-16	Sydney Roosters	Manly Sea Eagles	20	22
27-Mar-16	St George Dragons	Penrith Panthers	14	12
28-Mar-16	New Zealand Warriors	Newcastle Knights	40	18
28-Mar-16	Wests Tigers	Parramatta Eels	0	8
28-Mar-16	Cronulla Sharks	Melbourne Storm	14	6
Round 5				
31-Mar-16	Manly Sea Eagles	South Sydney Rabbitohs	12	16
01-Apr-16	Gold Coast Titans	Brisbane Broncos	16	24

02-Apr-16	Melbourne Storm	Newcastle Knights	18	14
02-Apr-16	Wests Tigers	Cronulla Sharks	26	34
02-Apr-16	North QLD Cowboys	St George Dragons	36	0
03-Apr-16	Sydney Roosters	New Zealand Warriors	28	32
03-Apr-16	Parramatta Eels	Penrith Panthers	18	20
04-Apr-16	Canterbury Bulldogs	Canberra Raiders	8	22
Round 6				
07-Apr-16	Brisbane Broncos	St George Dragons	26	0
08-Apr-16	South Sydney Rabbitohs	Sydney Roosters	10	17
09-Apr-16	Parramatta Eels	Canberra Raiders	36	6
09-Apr-16	New Zealand Warriors	Manly Sea Eagles	18	34
09-Apr-16	Penrith Panthers	North QLD Cowboys	18	23
10-Apr-16	Cronulla Sharks	Gold Coast Titans	25	20
10-Apr-16	Newcastle Knights	Wests Tigers	18	16
11-Apr-16	Melbourne Storm	Canterbury Bulldogs	12	18
Round 7				
14-Apr-16	Manly Sea Eagles	Parramatta Eels	10	22
15-Apr-16	North QLD Cowboys	South Sydney Rabbitohs	44	18
16-Apr-16	Gold Coast Titans	St George Dragons	14	19
16-Apr-16	Canterbury Bulldogs	New Zealand Warriors	20	24
16-Apr-16	Brisbane Broncos	Newcastle Knights	53	0
17-Apr-16	Canberra Raiders	Cronulla Sharks	16	40
17-Apr-16	Wests Tigers	Melbourne Storm	18	19
18-Apr-16	Sydney Roosters	Penrith Panthers	16	20
Round 8				
22-Apr-16	Brisbane Broncos	South Sydney Rabbitohs	30	8
23-Apr-16	Canterbury Bulldogs	Gold Coast Titans	21	20
23-Apr-16	Canberra Raiders	Wests Tigers	60	6
23-Apr-16	North QLD Cowboys	Parramatta Eels	32	16
24-Apr-16	Cronulla Sharks	Penrith Panthers	20	18
25-Apr-16	Newcastle Knights	Manly Sea Eagles	10	26
25-Apr-16	St George Dragons	Sydney Roosters	20	18
25-Apr-16	Melbourne Storm	New Zealand Warriors	42	0
Round 9				
28-Apr-16	South Sydney Rabbitohs	Wests Tigers	22	30
29-Apr-16	Parramatta Eels	Canterbury Bulldogs	20	12
30-Apr-16	Penrith Panthers	Canberra Raiders	19	18
30-Apr-16	Sydney Roosters	Newcastle Knights	38	0
30-Apr-16	Manly Sea Eagles	North QLD Cowboys	18	34
01-May-16	New Zealand Warriors	St George Dragons	26	10
01-May-16	Gold Coast Titans	Melbourne Storm	0	38
01-May-16	Cronulla Sharks	Brisbane Broncos	30	28
Round 10				
12-May-16	St George Dragons	Canberra Raiders	16	12
13-May-16	Parramatta Eels	South Sydney Rabbitohs	20	22
14-May-16	Penrith Panthers	New Zealand Warriors	30	18

14-May-16	Melbourne Storm	North QLD Cowboys	15	14
14-May-16	Manly Sea Eagles	Brisbane Broncos	6	30
15-May-16	Newcastle Knights	Cronulla Sharks	0	62
15-May-16	Wests Tigers	Canterbury Bulldogs	4	36
16-May-16	Gold Coast Titans	Sydney Roosters	26	6
Round 11				
19-May-16	South Sydney Rabbitohs	St George Dragons	34	24
20-May-16	North QLD Cowboys	Brisbane Broncos	19	18
21-May-16	Wests Tigers	Newcastle Knights	20	12
21-May-16	New Zealand Warriors	Canberra Raiders	12	38
21-May-16	Cronulla Sharks	Manly Sea Eagles	20	12
22-May-16	Penrith Panthers	Gold Coast Titans	24	28
22-May-16	Canterbury Bulldogs	Sydney Roosters	32	20
23-May-16	Parramatta Eels	Melbourne Storm	6	18

Appendix 2a:

2016 NRL points ladder after Round 5

Data sourced from Wikipedia 2021, '2016 NRL Season', https://en.wikipedia.org/wiki/2016_NRL_season#Regular_season

Ranking after round 5		
Rank	Team	Points
1	BB	8
2	MS	8
3	CR	7
4	NC	6
5	SS	6
6	CS	6
7	CB	6
8	PE	6
9	GT	6
10	NW	4
11	WT	4
12	PP	4
13	ME	4
14	SD	4
15	NK	1
16	SR	0

Appendix 2b:

2016 NRL points ladder after round 8

Data sourced from Wikipedia 2021, '2016 NRL Season', https://en.wikipedia.org/wiki/2016_NRL_season#Regular_season

Ranking after round 8		
Rank	Team	Points
1	BB	14
2	NC	12
3	MS	12
4	CS	12
5	PE	10
6	CB	10
7	CR	9
8	ME	8
9	SD	8
10	GT	6
11	SS	6
12	PP	6
13	NW	6
14	WT	4
15	NK	3
16	SR	2

Appendix 3

Competition data — Rounds 1–5 to produce Dominance matrix D used in Model 1a

	Results after 5 rounds	Brisbane Broncos (BB)	Canterbury Bulldogs (CB)	Canberra Raiders (CR)	Cronulla Sharks (CS)	Gold Coast Titans (GT)	Manly Sea Eagles (ME)	Melbourne Storm (MS)	North Qld Cowboys (NC)	Newcastle Knights (NK)	New Zealand Warriors (NW)	Parramatta Eels (PE)	Penrith Panthers (PP)	St George Dragons (SD)	Sydney Roosters (SR)	South Sydney (SS)	West Tigers (WT)		Wins
BB	Brisbane Broncos	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0		4
CB	Canterbury Bulldogs	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0		3
CR	Canberra Raiders	0	1	0	0	0	0	0	0	0.5	0	0	1	0	1	0	0		3.5
CS	Cronulla Sharks	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1		3
GT	Gold Coast Titans	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1		3
ME	Manly Sea Eagles	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0		2
MS	Melbourne Storm	0	0	0	0	1	0	0	0	1	1	0	0	1	0	0	0		4
NC	North Qld Cowboys	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0		3
NK	Newcastle Knights	0	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0		0.5
NW	New Zealand Warriors	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0		2
PE	Parramatta Eels	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1		3
PP	Penrith Panthers	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0		2
SD	St George Dragons	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0		2
SR	Sydney Roosters	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
SS	South Sydney	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0		3
WT	West Tigers	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0		2
																			40
	Losses	1	2	1.5	2	2	3	1	2	4.5	3	2	3	3	5	2	3	40	Total

$$
D = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Appendix 4

Competition data from modelling Rounds 1–8 used to develop Dominance matrix D in Model 1b

		Brisbane Broncos	Canterbury Bulldogs	Canberra Raiders	Cronulla Sharks	Gold Coast Titans	Manly Sea Eagles	Melbourne Storm	North Qld Cowboys	Newcastle Knights	New Zealand Warriors	Parramatta Eels	Penrith Panthers	St George Dragons	Sydney Roosters	South Sydney	West Tigers		Wins
BB	Brisbane Broncos	0	0	0	0	1	0	0	1	1	1	1	0	1	0	1	0		7
CB	Canterbury Bulldogs	0	0	0	0	1	1	1	0	0	0	0	1	0	0	1	0		5
CR	Canberra Raiders	0	1	0	0	0	0	0	0	0.5	0	0	1	0	1	0	1		4.5
CS	Cronulla Sharks	0	0	1	0	1	0	1	0	0	0	0	1	1	0	0	1		6
GT	Gold Coast Titans	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1		3
ME	Manly Sea Eagles	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0		4
MS	Melbourne Storm	0	0	0	0	1	0	0	0	1	2	0	0	1	0	0	1		6
NC	North Qld Cowboys	0	0	0	1	0	0	0	0	0	0	1	1	1	1	1	0		6
NK	Newcastle Knights	0	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0	1		1.5
NW	New Zealand Warriors	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0		3
PE	Parramatta Eels	0	1	1	0	0	1	0	1	0	0	0	0	0	0	0	1		5
PP	Penrith Panthers	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0		3
SD	St George Dragons	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0		4
SR	Sydney Roosters	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0		1
SS	South Sydney	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0		3
WT	West Tigers	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0		2
																			64
	Losses	1	3	3.5	2	5	4	2	2	6.5	5	3	5	4	7	5	6	64	Total

D =

0	0	0	0	1	0	0	1	1	1	1	0	1	0	1	0
0	0	0	0	1	1	1	0	0	0	0	1	0	0	1	0
0	1	0	0	0	0	0	0	0.5	0	0	1	0	1	0	1
0	0	1	0	1	0	1	0	0	0	0	1	1	0	0	1
0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0
0	0	0	0	1	0	0	0	1	2	0	0	1	0	0	1
0	0	0	1	0	0	0	0	0	0	1	1	1	1	1	0
0	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0
0	1	1	0	0	1	0	1	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0

Appendix 5

Excerpt of the formulas used to determine the third-order rankings for Model 1b

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	U	Y	Z
30		0	0	0	0	1	0	0	1	1	1	1	0	1	0	1	0		1		=MMULT(C30:R45,U30:U45)
31		0	0	0	0	1	1	1	0	0	0	0	1	0	0	1	0		1		=MMULT(C30:R45,U30:U45)
32		0	1	0	0	0	0	0	0	0.5	0	0	1	0	1	0	1		1		=MMULT(C30:R45,U30:U45)
33		0	0	1	0	1	0	1	0	0	0	0	1	1	0	0	1		1		=MMULT(C30:R45,U30:U45)
34		0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1		1		=MMULT(C30:R45,U30:U45)
35		0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0		1		=MMULT(C30:R45,U30:U45)
36		0	0	0	0	1	0	0	0	1	2	0	0	1	0	0	1		1		=MMULT(C30:R45,U30:U45)
37	**D =**	0	0	0	1	0	0	0	0	0	0	1	1	1	1	1	0	**1 =**	1	**D 1 =**	=MMULT(C30:R45,U30:U45)
38		0	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0	1		1		=MMULT(C30:R45,U30:U45)
39		0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0		1		=MMULT(C30:R45,U30:U45)
40		0	1	1	0	0	1	0	1	0	0	0	0	0	0	0	1		1		=MMULT(C30:R45,U30:U45)
41		1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0		1		=MMULT(C30:R45,U30:U45)
42		0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0		1		=MMULT(C30:R45,U30:U45)
43		0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0		1		=MMULT(C30:R45,U30:U45)
44		0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0		1		=MMULT(C30:R45,U30:U45)
45		0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0		1		=MMULT(C30:R45,U30:U45)

	B	C	D	E	F
134					
135		**R_3 values**	**Team**	**Weighting**	
136		=Z30:Z45+F137*X71:X86+F138*X114:X129	BB	**parameters**	
137		=Z30:Z45+F137*X71:X86+F138*X114:X129	CB	a =	0.5
138		=Z30:Z45+F137*X71:X86+F138*X114:X129	CR	b =	0.3
139		=Z30:Z45+F137*X71:X86+F138*X114:X129	CS		
140		=Z30:Z45+F137*X71:X86+F138*X114:X129	GT		
141		=Z30:Z45+F137*X71:X86+F138*X114:X129	ME		
142		=Z30:Z45+F137*X71:X86+F138*X114:X129	MS		
143	**R_3 = D 1 + 0.5 D^2 1 + 0.3 D^3 1 =**	=Z30:Z45+F137*X71:X86+F138*X114:X129	NC		
144		=Z30:Z45+F137*X71:X86+F138*X114:X129	NK		
145		=Z30:Z45+F137*X71:X86+F138*X114:X129	NW		
146		=Z30:Z45+F137*X71:X86+F138*X114:X129	PE		
147		=Z30:Z45+F137*X71:X86+F138*X114:X129	PP		
148		=Z30:Z45+F137*X71:X86+F138*X114:X129	SD		
149		=Z30:Z45+F137*X71:X86+F138*X114:X129	SR		
150		=Z30:Z45+F137*X71:X86+F138*X114:X129	SS		
151		=Z30:Z45+F137*X71:X86+F138*X114:X129	WT		

Appendix 6

Points scores for Rounds 1–8 used to determine points margin values for home and away games in Model 2

Data sourced from Wikipedia 2021, '2016 NRL Season',
https://en.wikipedia.org/wiki/2016_NRL_season#Regular_season

Home and Away Gound Advantage Statistical Analysis									
		Round Number							
Home Team Scores		1	2	3	4	5	6	7	8
BB	Brisbane Broncos		25		21		26	53	30
CB	Canterbury Bulldogs			6		8		20	21
CR	Canberra Raiders	30	21		20			16	60
CS	Cronulla Sharks		30		14		25		20
GT	Gold Coast Titans	30		30		16		14	
ME	Manly Sea Eagles	6		22		12		10	
MS	Melbourne Storm	18	34			18	12		42
NC	North Qld Cowboys	20		40		36		44	32
NK	Newcastle Knights			24			18		10
NW	New Zealand Warriors			14	40		18		
PE	Parramatta Eels	4	20			16	36		
PP	Penrith Panthers		16	23			18		
SD	St George Dragons			8	14				20
SR	Sydney Roosters	10			20	28		16	
SS	South Sydney		48		12		10		
WT	West Tigers	34	36		0	26		18	
	Total games at home	8	8	8	8	8	8	8	8

		Round Number							
Away Team Scores		1	2	3	4	5	6	7	8
BB	Brisbane Broncos	17		22		24			
CB	Canterbury Bulldogs	28	18		42		18		
CR	Canberra Raiders			24		22	6		
CS	Cronulla Sharks	14		12		34		40	
GT	Gold Coast Titans		16		24		20		20
ME	Manly Sea Eagles		22		22		34		26
MS	Melbourne Storm			21	6			19	
NC	North Qld Cowboys		16		20		23		
NK	Newcastle Knights	12	6		18	14		0	
NW	New Zealand Warriors	26	10			32		24	0
PE	Parramatta Eels			20	8			22	16
PP	Penrith Panthers	22			12	20		20	18
SD	St George Dragons	16	2			0	0	19	
SR	Sydney Roosters		20	0			17		18
SS	South Sydney	42		6		16		18	8
WT	West Tigers			18			16		6
	Total games away	8	8	8	8	8	8	8	8

Appendix 7

Formulas used to determine the home and away ground advantage rankings for Model 2

	A	C	D	E	F	G	H	I	J	K	N	O
1	Home Ground Advantage Statistical Analysis											
2											Home Team Margins	
3	Team	Round Number								Number of home games	Total margin in Home games	Average margin in Home games
4		1	2	3	4	5	6	7	8			
5	BB		15		1		26	53	22	=COUNTA(W5:AD5)	=SUM(C5:J5)	=N5/K5
6	CB			-14		-14		-4	1	=COUNTA(W6:AD6)	=SUM(C6:J6)	=N6/K6
7	CR	8	1		-4			-24	52	=COUNTA(W7:AD7)	=SUM(C7:J7)	=N7/K7
8	CS		28		8		5		2	=COUNTA(W8:AD8)	=SUM(C8:J8)	=N8/K8
9	GT	18		12		-8		-5		=COUNTA(W9:AD9)	=SUM(C9:J9)	=N9/K9
10	ME	-22		10		-4		-12		=COUNTA(W10:AD10)	=SUM(C10:J10)	=N10/K10
11	MS	2	18			4	-6		42	=COUNTA(W11:AD11)	=SUM(C11:J11)	=N11/K11
12	NC	6		40		36		26	16	=COUNTA(W12:AD12)	=SUM(C12:J12)	=N12/K12
13	NK			0			2		-14	=COUNTA(W13:AD13)	=SUM(C13:J13)	=N13/K13
14	NW			-7	32		-16			=COUNTA(W14:AD14)	=SUM(C14:J14)	=N14/K14
15	PE	-13	4			-2	30			=COUNTA(W15:AD15)	=SUM(C15:J15)	=N15/K15
16	PP		-2	1			-5			=COUNTA(W16:AD16)	=SUM(C16:J16)	=N16/K16
17	SD			2	2				2	=COUNTA(W17:AD17)	=SUM(C17:J17)	=N17/K17
18	SR	-32			-2	-6		-4		=COUNTA(W18:AD18)	=SUM(C18:J18)	=N18/K18
19	SS		42		-30		-7			=COUNTA(W19:AD19)	=SUM(C19:J19)	=N19/K19
20	WT	8	14		-8	8		-1		=COUNTA(W20:AD20)	=SUM(C20:J20)	=N20/K20
22											Away Team Margins	
23	Team	Round Number								Number of home games	Total margin in Away games	Average margin in Away games
24		1	2	3	4	5	6	7	8			
25	BB	13		-1		8				=COUNTA(W25:AD25)	=SUM(C25:J25)	=N25/K25
26	CB	22	2		30		6			=COUNTA(W26:AD26)	=SUM(C26:J26)	=N26/K26
27	CR			0		14	-30			=COUNTA(W27:AD27)	=SUM(C27:J27)	=N27/K27
28	CS	-6		-10		-8		24		=COUNTA(W28:AD28)	=SUM(C28:J28)	=N28/K28
29	GT		-18		4		-5		-1	=COUNTA(W29:AD29)	=SUM(C29:J29)	=N29/K29
30	ME		-14		2		16		14	=COUNTA(W30:AD30)	=SUM(C30:J30)	=N30/K30
31	MS			7	-8			1		=COUNTA(W31:AD31)	=SUM(C31:J31)	=N31/K31
32	NC		-4		-1		5			=COUNTA(W32:AD32)	=SUM(C32:J32)	=N32/K32
33	NK	-18	-42		-32	-4		-53		=COUNTA(W33:AD33)	=SUM(C33:J33)	=N33/K33
34	NW	-8	-15			6		4	-42	=COUNTA(W34:AD34)	=SUM(C34:J34)	=N34/K34
35	PE			14	8			12	-16	=COUNTA(W35:AD35)	=SUM(C35:J35)	=N35/K35
36	PP	-8			-2	2		4	-2	=COUNTA(W36:AD36)	=SUM(C36:J36)	=N36/K36
37	SD	-2	-28			-36	-26	5		=COUNTA(W37:AD37)	=SUM(C37:J37)	=N37/K37
38	SR		-1	-40			7		-2	=COUNTA(W38:AD38)	=SUM(C38:J38)	=N38/K38
39	SS	32		-2		4		-26	-22	=COUNTA(W39:AD39)	=SUM(C39:J39)	=N39/K39
40	WT			-12			-2		-52	=COUNTA(W40:AD40)	=SUM(C40:J40)	=N40/K40

Appendix 8

Detailed view of home and away margin rankings comparisons in Model 2 used to predict the winning teams

Round 9 Games Home v Away Team	Home Team % Wins	Away Team % Wins	Predicted winner using greater average	Actual Winner	Correct Prediction using home team (Yes/No)
Round 9 Games					
SS v WT	-7.8	-22.0	SS	WT	No
PE v CB	-7.0	15.0	CB	PE	No
PP v CR	1.7	-5.3	PP	PP	Yes
SR v NK	3.0	-29.8	SR	SR	Yes
ME v NC	-4.0	0.0	NC	NC	Yes
NW v SD	2.0	-17.4	NW	NW	Yes
GT V MS	4.2	0.0	GT	MS	No
CS V BB	12.0	6.7	CS	CS	Yes
Percentage correct		62.5%			
Round 10 Games					
SD v CR	23.4	-5.3	SD	SD	Yes
PE v SS	-7.0	-2.8	SS	SS	Yes
PP v NW	1.7	-11.0	PP	PP	Yes
MS v NC	4.3	0.0	MS	MS	Yes
ME v BB	-4.0	6.7	BB	BB	Yes
NK v CS	10.8	0.0	NK	CS	Yes
WT v CB	4.8	15.0	CB	CB	Yes
GT v SR	4.2	-9.0	GT	GT	Yes
Percentage correct		100.0%			
Round 11 Games					
SS v SD	-7.8	-17.4	SS	SS	Yes
NC v BB	24.8	6.7	NC	NC	Yes
WT v NK	4.8	-29.8	WT	WT	Yes
NW v CR	2.0	-5.3	NW	CR	No

CS v ME	12.0	4.5	CS	CS	Yes
PP v GT	1.7	-5.0	PP	GT	No
CB v SR	-11.0	-9.0	SR	CB	No
PE V MS	-7.0	0.0	MS	MS	Yes
Percentage correct		62.5%			
Mean percentage correct		**75.0%**			