Mathematical Methods 2025 v1.2

IA2: Sample marking scheme

July 2025

This sample has been compiled by the QCAA to model one possible approach to allocating marks in an examination. It matches the examination mark allocations as specified in the syllabus (~ 60% simple familiar, ~ 20% complex familiar and ~ 20% complex unfamiliar) and ensures that a balance of the objectives are assessed.

Assessment objectives

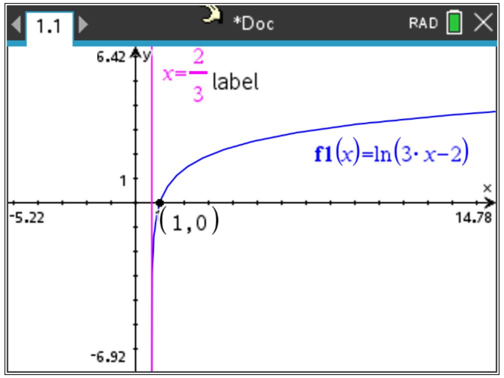
This assessment instrument is used to determine student achievement in the following objectives:

1. Recall mathematical knowledge.
2. Use mathematical knowledge.
3. Communicate mathematical knowledge.
4. Evaluate the reasonableness of solutions.
5. Justify procedures and decisions.
6. Solve mathematical problems.

Marking scheme

Q	Sample response	The response	Notes
1	$e^x - 2 = 0$ or $e^x - 3 = 0$	• correctly apply the null factor law [1 mark]	Half marks apply. One for each correct factor.
	$e^x = 2$ or $e^x = 3$ $x = \ln(2)$ or $x = \ln(3)$	• determines exact values for x **[1 mark]**	Allow FT mark for errors in prior working. Half marks apply. One for each value of x.
2a	$f(x) = e^x + \sin(2x)$ $f'(x) = e^x + 2\cos(2x)$	• correctly determines the derivative of e^x **[1 mark]** • correctly derivatives $\sin(u)$ **[1 mark]** • correctly applied chain rule **[1 mark]**	
2b	$f(x) = y = e^{\sin(x)}$ $y = e^u, \quad u = \sin(x)$	• correctly recognises that the chain rule should be used **[1 mark]**	This mark may be implied by subsequent working.
	$\frac{dy}{du} = e^u$, $\frac{du}{dx} = \cos(x)$	• correctly identifies $\frac{d}{dx}\sin(x) = \cos(x)$ and $\frac{d}{du}e^u = e^u$ **[1 mark]**	Half marks apply. One for each correct derivative. This mark may be implied by subsequent working.
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $f'(x) = \cos(x)e^{\sin(x)}$	• determines $f'(x)$ **[1 mark]**	Allow FT mark for errors in prior working.

Q	Sample response	The response	Notes
2c	$f(x) = y = \cos^3(x)$ $y = u^3, \quad u = \cos(x)$	• correctly recognises that the chain rule should be used **[1 mark]**	This mark may be implied by subsequent working.
	$\frac{dy}{du} = 3u^2, \frac{du}{dx} = -\sin(x)$	• correctly identifies that $\frac{d}{du}u^3 = 3u^2$ and $\frac{d}{dx}\cos(x) = -\sin(x)$ **[1 mark]**	Half marks apply. One for each correct derivative. This mark may be implied by subsequent working.
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $f'(x) = 3\cos^2(x) \times -\sin(x)$ $= -3\cos^2(x)\sin(x)$	• determines $f'(x)$ **[1 mark]**	Allow FT mark for errors in prior working. It is not necessary to simplify the solution to be awarded this mark.
2d	$f(x) = x + x\ln(x)$ $x\ln(x) = uv$	• correctly recognises that the product rule should be used to differentiate $x\ln(x)$ **[1 mark]**	This mark may be implied by subsequent working.
	$u = x \qquad v = \ln(x)$ $\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{1}{x}$	• correctly identifies $\frac{d}{dx}x = 1$ and $\frac{d}{dx}\ln(x) = \frac{1}{x}$ **[1 mark]**	Half marks apply. One for each correct derivative. This mark may be implied by subsequent working.
	$\frac{d}{dx}x\ln(x) = x \times \frac{1}{x} + \ln(x)$ $= 1 + \ln(x)$	• determines $\frac{d}{dx}x\ln(x)$ **[1 mark]**	Allow FT marks for errors in prior working.
	$f'(x) = 1 + 1 + \ln(x)$ $= 2 + \ln(x)$	• determines $f'(x)$ **[1 mark]**	Solution must be simplified to be awarded 1 mark. Half mark if solution is not simplified.

Q	Sample response	The response	Notes
3a	Asymptote $x=\frac{2}{3}$	• correctly identifies the asymptote **[1 mark]**	This mark may be awarded if it is correctly labelled on the graph.
	x-intercept $(1,0)$	• correctly identifies the x-intercept **[1 mark]**	This mark may be awarded if it is correctly labelled on the graph.
		• determines the general shape of the curve **[1 mark]**	Allow FT mark for errors in prior working. This mark is awarded if the curve has the correct general shape and is consistent with the asymptote and x-intercept identified by the student.
		• correctly follows graphing conventions **[1 mark]**	Graphing conventions include appropriate labels, axes etc.
3b	$f(x)=\ln(3x-2)$ $f'(x)=\frac{3}{3x-2}$	• correctly applies the derivative of the ln function **[1 mark]** • correctly applies the chain rule **[1 mark]**	
3c	$g(x)=-4\cos(0.5x)+2$ $g'(x)=2\sin(0.5x)$	• correctly recognises $\frac{d}{du}\cos(u)=-\sin(u)$ **[1 mark]** • correctly applies the chain rule **[1 mark]**	
3d	Using G.C. Gradients are the same at $x=1.43$ and $x=6.1$	• determines that there are points with the same gradient for $1\le x\le 8$ **[1 mark]**	Allow FT marks for errors in prior working.
	The claim is reasonable as there are 2 points with the same gradients.	• makes a statement about the reasonableness of the claim **[1 mark]**	Evidence must be provided for this mark to be awarded. The student statement must be consistent with their evidence.

Q	Sample response	The response	Notes
4a	Given $N = 4200$ when $t = 8$, $n(t) = Ae^{0.55t}, t \geq 0$ $4200 = Ae^{0.55\times8}$	• correctly substitutes for $N = 4200$ and $t = 8$ **[1 mark]**	This mark may be implied by subsequent working.
	$A = \frac{4200}{e^{0.55\times8}}$ $= 51.5648$	• determines value for A **[1 mark]**	Allow FT marks for errors in prior working.
4b	Determine $n(12)$ $n(12) = 51.5648e^{0.55\times12}$ $= 37905$	• determines $n(12)$ **[1 mark]**	
4c	$n(t) = 51.5684e^{0.55t}$ $n'(t) = 51.5684 \times 0.55e^{0.55t}$ $= 28.36262e^{0.55t}$	• determines $n'(t)$ **[1 mark]**	
4d	$250000 = 28.36262e^{0.55t}$	• substitutes $n'(t) = 250000$ **[1 mark]**	This mark may be implied by subsequent working.
	Using G.C. $t = 16.5168$ This will occur 16.5 years after they were introduced.	• solves for t including appropriate units **[1 mark]**	Half marks apply. Award if units are not correct or not included.
4e	$1000000 = 51.5648e^{0.55\times t}$ $t = 17.95$	• determines the time taken for population to reach 1,000,000 **[1 mark]**	Award this mark for other valid method of determining reasonableness.
	The claim is reasonable as the population will reach 1,000,000 before 20 years.	• determines that the claim is reasonable **[1 mark]**	This mark can only be awarded if prior working supports conclusion.

Q	Sample response	The response	Notes
5	Given $f(x)=\frac{\ln(2x)}{x}, x>0$ Stationary point occurs when $f'(x)=0$	• correctly identifies that the stationary point occurs when $f'(x)=0$ **[1 mark]**	This mark may be implied by subsequent working.
	$f(x)=\frac{u}{v}$ $u=\ln(2x)$ $\quad v=x$ $u'=\frac{1}{x}$ $\quad v'=1$	• correctly recognises that the quotient rule should be used **[1 mark]**	This mark may be implied by subsequent working. This mark may be awarded for using correctly using the product rule.
	$f'(x)=\frac{vu'-uv'}{v^2}$ $=\frac{x\times\frac{1}{x}-\ln(2x)}{x^2}$ $=\frac{1-\ln(2x)}{x^2}$	• determines $f'(x)$ **[1 mark]**	Allow FT marks for errors in prior working.
	$\frac{1-\ln(2x)}{x^2}=0$ $1-\ln(2x)=0$ $\ln(2x)=1$	• solves $f'(x)=0$ in terms of $\ln(2x)$ **[1 mark]**	This mark may be implied by subsequent working.
	$2x=e \Rightarrow x=\frac{e}{2}$	• determines x-coordinate for the stationary point. **[1 mark]**	
	$f\left(\frac{e}{2}\right)=\frac{\ln(e)}{\frac{e}{2}}=\frac{2}{e}$ Stationary point occurs at $\left(\frac{e}{2},\frac{2}{e}\right)$	• determines y-coordinate for the stationary point. **[1 mark]**	It is not necessary to express the stationary point in coordinate form.

Q	Sample response	The response	Notes
6	$x(t)=e^{t}\sin(t)$ $\quad =uv$ $u=e^{t} \qquad v=\sin(t)$ $u'=e^{t} \qquad v'=\cos(t)$	• correctly recognises that the product rule should be used to differentiate the position **[1 mark]**	This mark may be implied by subsequent working.
	$v(t)=uv'+vu'$ $\quad =e^{t}\cos(t)+e^{t}\sin(t)$	• correctly determines the velocity function **[1 mark]**	
	$v(t)=e^{t}\left(\cos(t)+\sin(t)\right)$ $\quad =uv$ $u=e^{t} \qquad v=\cos(t)+\sin(t)$ $u'=e^{t} \qquad v'=-\sin(t)+\cos(t)$	• recognises that the product rule should be used to differentiate the velocity **[1 mark]**	Allow FT marks for errors in prior working.
	$a(t)=uv'+vu'$ $\quad =e^{t}\left(-\sin(t)+\cos(t)\right)$ $\qquad +e^{t}\left(\cos(t)+\sin(t)\right)$ $\quad =2e^{t}\cos(t)$	• determines the acceleration function **[1 mark]**	
	$a(t)=0 \Rightarrow \cos(t)=0$ $t=\frac{\pi}{2},\frac{3\pi}{2}$	• determines both times within the domain **[1 mark]**	Half marks apply. One for each time.

Q	Sample response	The response	Notes
7	$f(x)=3x^5-5x^4+kx$ $f'(x)=15x^4-20x^3+k$ $f''(x)=60x^3-60x^2$	• correctly determines $f''(x)$ **[1 mark]**	Half marks apply. One for each derivative.
	Point of inflection: $f''(x)=0$	• correctly recognises that $f''(x)=0$ is a condition for a point of inflection **[1 mark]**	This mark may be implied by subsequent working.
	$f''(x)=0$ $x=0$ or $x=1$	• determined the values of x when $f''=0$ **[1 mark]**	Allow FT marks for errors in prior working. Half marks apply. One for each value of x.
	Investigate the sign of $f''(x)$: $x=-1 \Rightarrow f''(-1)=-120<0$ $x=0.5 \Rightarrow f''(0.5)=-7.5<0$ $x=2 \Rightarrow f''(2)=240>0$ Only one point of inflection $x=1$	• determines the point of inflection **[1 mark]**	The full mark can only be awarded if there is a reasoning to support that there is only one point of inflection. Accept equivalent reasoning about the concavity of the function. Accept an alternative approach, e.g. $f''(x)<0$ for $x<1$, $f''(x)>0$ for $x>1$
	$a=1$ $f(a)-f'(a)=f(1)-f'(1)=$ $=3-5+k-(15-20+k)$ $=-2+k+5-k$ $=3$	• shows that $f(1)-f'(1)=3$ **[1 mark]**	

Q	Sample response	The response	Notes
8	$y = e^{2-x}$ $\frac{dy}{dx} = -e^{2-x}$	• correctly determines $\frac{dy}{dx}$ **[1 mark]**	
	Let the maximum area be at the point (a,b) where $a,b \geq 0$ $b = e^{2-a}$ Therefore point (a, e^{2-a}) The gradient of the line is $m = -e^{2-a}$	• correctly determines a point on the gradient and the gradient of the tangent **[1 mark]**	Half marks apply. One for the point and one for the gradient.
	Equation of tangent: $y - y_1 = m(x - x_1)$ $y - e^{2-a} = -e^{2-a}(x-a)$ $y = -e^{2-a}x + ae^{2-a} + e^{2-a}$	• determines the equation of the tangent **[1 mark]**	Allow FT marks for errors in prior working.
	y-intercept is $(0, ae^{2-a} + e^{2-a})$ x-intercept is $\left(\frac{ae^{2-a} + e^{2-a}}{e^{2-a}}, 0\right)$ $(a+1, 0)$	• determines the x and y-intercepts **[1 mark]**	Half marks apply. One for each intercept.
	Area of triangle $A = \frac{1}{2}(a+1)(ae^{2-a} + e^{2-a})$ $= \frac{1}{2}(a+1)^2 e^{2-a}$	• determines an equation for area of the triangle **[1 mark]**	
	Using technology, this is a maximum when $a = 1$. $\therefore$ maximum at $(1, e)$	• determines the point for maximum area. **[1 mark]**	Half marks apply. Award half if student solves $a = 1$ but point is not found.