Mathematical Methods marking guide

External assessment 2025

Paper 2: Combination response — Technology-active (55 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Purpose

This document consists of a marking guide and a sample response.

The marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

The sample response demonstrates the qualities of a high-level response.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Allow FT mark/s — refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Multiple choice

Question	Response
1	A
2	C
3	A
4	A
5	D
6	D
7	C
8	C
9	B
10	B

Short response

Q	Sample response	The response:
11a)	Integrating: $f(x) = \sin(x) - 4 \times -\cos(2x+\pi) \times \frac{1}{2} + c$ $= \sin(x) + 2\cos(2x+\pi) + c$	• correctly integrates the cosine term in $f'(x)$ **[1 mark]** • correctly integrates the sine term in $f'(x)$ **[1 mark]**
	$f(\pi) = 2$ $2 = \sin(\pi) + 2\cos(3\pi) + c$	• substitutes π and 2 appropriately into the equation for $f(x)$ **[1 mark]**
	$2 = 0 - 2 + c$ $c = 4$ $f(x) = \sin(x) + 2\cos(2x+\pi) + 4$	• determines the constant of integration in the function $f(x)$ **[1 mark]**
11b)	Use a GDC to graph $f(x)$ between 2 and 4: The solution of $f(x) = 4.8$ for $2 \le x \le 4$ $x = 2.3364$	• determines the solution in the stated domain **[1 mark]**
11c)	Use a GDC: The minimum value of $f(x)$: $y = 1.9375$	• determines the minimum $f(x)$ value **[1 mark]**

Q	Sample response	The response:
12a)	$t=0,\ P=100$ $P_0=100$	• correctly identifies the value of P_0 in the model **[1 mark]**
	$P(t)=100e^{kt}$ $P(3)=120$ $120=100e^{3k}$ Use a GDC to solve for k $k=\frac{1}{3}\ln(1.2)$ $k=0.06077$ $\therefore P(t)=100e^{0.06077t}$	• determines the value of k **[1 mark]**
12b)	$P'(t)=100e^{0.06077t}\times 0.06077$ $=6.077e^{0.06077t}$	• determines the derivative function **[1 mark]**
12c)	Use a GDC to solve for t using: $10=P'(t)$ $\therefore t=8.1945$ weeks	• determines the time when the rate of population growth reached 10 cockroaches per week **[1 mark]**

Q	Sample response	The response:
12d)	$I(t) = c\ln(t+8)+172$ $I'(t) = \dfrac{c}{t+8}$	• correctly differentiates the pest control function **[1 mark]**
	$I'(3) = 5$ $\dfrac{c}{3+8} = 5$ $\dfrac{c}{11} = 5$ $c = 55$	• determines the value of *c* **[1 mark]**
12e)	$I(t) = 55\ln(t+8)+172$ $t = 0$ $I(0) = 55\ln(8)+172$ $= 286.3693$ 286 cockroaches when the treatment began	• determines the population when the treatment began **[1 mark]**

Q	Sample response	The response:
13a)	Total students in the sample: $15+19+20+27+5=86$	• correctly determines the total number of students in the sample **[1 mark]**
	Total students having 7+ hours sleep: $20+27+5=52$	• correctly determines the total number of students having seven hours of sleep or more **[1 mark]**
	Sample proportion: $\frac{52}{86}=0.605$	• determines the sample proportion of students having seven hours of sleep or more **[1 mark]**
13b)	Use a GDC Use Interval function or equivalent method (0.537, 0.672)	• determines the 80% approximate confidence interval **[1 mark]**

Q	Sample response	The response:
14a)	Use a GDC to solve for t: $36\,000 = 18\,000\sin\left(\frac{\pi}{6}t + 6\right) + 22\,000$ $t = 2.2428$ months	• correctly determines when tourists first reach 36 000 as a decimal **[1 mark]**
14b)	$N(t) = 18\,000\sin\left(\frac{\pi}{6}t + 6\right) + 22\,000$ $N'(t) = 18\,000 \times \frac{\pi}{6}\cos\left(\frac{\pi}{6}t + 6\right)$ $N'(t) = 3000\pi\cos\left(\frac{\pi}{6}t + 6\right)$	• correctly determines the equation of the first derivative **[1 mark]**
	$N''(t) = -3000\pi \times \frac{\pi}{6}\sin\left(\frac{\pi}{6}t + 6\right)$ $N''(t) = -500\pi^2\sin\left(\frac{\pi}{6}t + 6\right)$	• determines the equation of the second derivative **[1 mark]**

Q	Sample response	The response:
14c)	Use a GDC: Minimum occurs when $t = 9.5408$ months	• correctly determines the time when the number of tourists is a minimum **[1 mark]**
	Use a GDC: $N''(9.5408) = 4934.80$	• determines the value of the second derivative when the number of tourists is a minimum **[1 mark]**
14d)	The value of the second derivative is 4934.80. This is a positive value, which supports the number of tourists being a minimum.	• explains how a positive value of the second derivative is consistent with a local minimum **[1 mark]**

Q	Sample response	The response:
15	Each day is either a success or failure, i.e. X is a binomial variable $p = 0.65$ $n = 40$ $\mu = np$ $= 40 \times 0.65$ $= 26$	• correctly determines the mean **[1 mark]**
	$\sigma = \sqrt{40 \times 0.65 \times 0.35}$ $= 3.0166$	• correctly determines the standard deviation **[1 mark]**
	Use a GDC: Binomial distribution with: Lower value 26 – 3 = 23 Upper value 26 + 3 = 29 Answer: 0.7547 = 75%	• determines the required probability **[1 mark]**

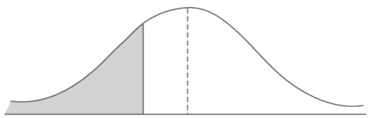
Q	Sample response	The response:
16	**Method 1: Time is calculated** $t=7$ $M=150$ $150=100b^7$ $b=(1.5)^{\frac{1}{7}}$ $\therefore M=100\times(1.5)^{\frac{1}{7}t}$	• correctly determines the constant b **[1 mark]**
	$M=5\times M(0)$ $=5\times100$ $=500$	• correctly determines five times the initial number of fish **[1 mark]**
	$M=500$ $500=100(1.5)^{\frac{1}{7}t}$ Use a GCD to solve the equation $t=27.7855$ days	• determines when the pond contains five times the initial number of fish **[1 mark]**
	Use a GDC to evaluate the first derivative: Rate = 28.9618 fish per day	• determines the rate of growth **[1 mark]**

Q	Sample response	The response:
16	**Method 2: Time is not calculated** $t=7$ $M=150$ $150=100b^7$ $b=\left(\frac{150}{100}\right)^{\frac{1}{7}}$ $b=\sqrt[7]{1.5}$ $\therefore M=100\times(1.5)^{t/7}$	• correctly determines the constant b **[1 mark]**
	$M=5\times M(0)$ $=5\times 100$ $=500$	• correctly determines five times the initial number of fish **[1 mark]**
	$M=100b^t$ $=100e^{\ln b^t}$ $=100e^{t\ln b}$	• correctly expresses the model in terms of the exponential function **[1 mark]**
	$M'=100\ln b\, e^{t\ln b}$ $=M\ln b$ $=500\ln b$ $=28.9618$ fish per day	• determines the rate of growth when the pond contains five times the initial number **[1 mark]**

Q	Sample response	The response:
17	This is a probability density function so: $1=\int_0^{20} k e^{-\frac{7}{50}t} dt$	• correctly equates the probability density function to 1 over the appropriate domain **[1 mark]**
	Use a GDC: $1=k\int_0^{20} e^{-\frac{7}{50}t} dt$ $k=0.1491$	• correctly determines the value of k **[1 mark]**
	From the formula book: $E(X)=\int_{-\infty}^{\infty} x\,p(x)\,dx$ $=\int_0^{20} t\times k\times e^{-\frac{7}{50}t} dt$	• substitutes domain and developed function $p(t)$ into the formula for the mean of a continuous random variable **[1 mark]**
	$E(X)=5.85$ ≈ 6 s	• determines the mean time **[1 mark]**

Q	Sample response	The response:
18	Set up the 95% confidence interval: $\left(\hat{p}-z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$ $\left(0.648,\ \hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$ Use a GDC: with a 95% central area, $z=-1.960$ and 1.960	• correctly determines the *z*-score for a 95% confidence interval **[1 mark]**
	The lower cut-off of the 75% CI is given by: $0.648=\hat{p}-1.960\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{500}}$	• determines an equation involving the lower cut-off of the CI formula **[1 mark]**
	Solving for $\hat{p}$ using a GDC $0.648=\hat{p}-1.960\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{500}}$ $\hat{p}=0.6886$	• determines the sample proportion for the survey results **[1 mark]**

Q	Sample response	The response:
	Substituting $\hat{p}$ into the upper CI formula: $= \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $= 0.6886 + 1.960\sqrt{\frac{0.6886(1-0.6886)}{500}}$ $= 0.729$ The 95% CI is therefore: (0.648, 0.729)	• determines the upper end of the 95% confidence interval **[1 mark]**
	The survey suggests we can be 95% confident the population proportion is between 0.648 and 0.729, i.e. between 64.8% and 72.9%. The claim of 75% for the population proportion is outside of the 95% confidence interval in the survey, so the claim is not reasonable.	• determines if the claim is reasonable **[1 mark]**

Q	Sample response	The response:
19	Let: A = species A B = species B Calculate $P(\text{A}<18)$  $\mu = 20$ $\sigma = 2$ Use a GDC: Normal distribution Lower = 0; Upper = 18 $\mu = 20$ $\sigma = 2$ $\therefore P(\text{A} < 18) = 0.1587$	• correctly determines the proportion of species A beetles with horn length shorter than 18 mm **[1 mark]**
	Total proportion of beetles with horns shorter than 18 mm in the region: $P(<18) = P(\text{A and} < 18) + P(\text{B and} < 18)$	• correctly determines a method to find the proportion of the total population with horn length shorter than 18 mm **[1 mark]**
	$= 0.7 \times 0.1587 + 0.3 \times 0.146$ $= 0.15489$	• determines the proportion of all beetles with horn length shorter than 18 mm **[1 mark]**

Q	Sample response	The response:
	Probability the captured beetle is species A, given the horn is shorter than 18 mm. This is a conditional probability: $P(\text{A} \mid <18) =$ $= \dfrac{P(\text{A} \cap <18)}{P(<18)}$	• uses conditional probability to solve the problem **[1 mark]**
	$= \dfrac{0.7 \times 0.1587}{0.15489}$ $= 0.717$ $= 71.7\%$	• determines the probability the captured beetle is from species A **[1 mark]**
		• shows logical organisation **[1 mark]**