Mathematical Methods marking guide

External assessment 2025

Paper 1: Combination response — Technology-free (55 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government
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Purpose

This document consists of a marking guide and a sample response.

The marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

The sample response demonstrates the qualities of a high-level response.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Allow FT mark/s — refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Multiple choice

Question	Response
1	B
2	A
3	D
4	B
5	D
6	D
7	C
8	C
9	A
10	C

Short response

Q	Sample response	The response:
11a)	$\int \frac{1}{x} dx$ $= \ln(x) + c$	• correctly applies the integration rule **[1 mark]**
11b)	$\int e^{4x} dx$ $= \frac{1}{4}e^{4x} + c$	• correctly integrates the exponential term **[1 mark]**
11c)	$\int x\left(x^3 + 8\right) dx$ $= \int \left(x^4 + 8x\right) dx$	• correctly expands the brackets **[1 mark]**
	$= \frac{x^5}{5} + 4x^2 + c$	• integrates the power of 4 term **[1 mark]** • integrates the linear term **[1 mark]**

Q	Sample response	The response:
12	$n = 25$ $\hat{p} = \frac{5}{25} = \frac{1}{5}$	• correctly determines the sample proportion **[1 mark]**
	$\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $= \frac{1}{5} - \sqrt{\frac{\frac{1}{5}\left(1-\frac{1}{5}\right)}{25}} \quad = \frac{1}{5} + \sqrt{\frac{\frac{1}{5}\left(1-\frac{1}{5}\right)}{25}}$	• substitutes the sample size, sample proportion and *z*-value into at least one end of approximate confidence interval formula **[1 mark]**
	$= \frac{1}{5} - \sqrt{\frac{4}{25^2}} \quad = \frac{1}{5} + \sqrt{\frac{4}{25^2}}$ $= \frac{1}{5} - \frac{\sqrt{4}}{25} \quad = \frac{1}{5} + \frac{\sqrt{4}}{25}$ $= \frac{1}{5} - \frac{2}{25} \quad = \frac{1}{5} + \frac{2}{25}$	• determines the approximate margin of error **[1 mark]**
	$= \frac{5}{25} - \frac{2}{25} \quad = \frac{5}{25} + \frac{2}{25}$ $= \frac{3}{25} \quad = \frac{7}{25}$ Confidence interval $\left(\frac{3}{25}, \frac{7}{25}\right)$	• determines the lower and upper end of the approximate confidence interval **[1 mark]**

Q	Sample response	The response:
13	Confirm that the $f''(x)=0$ Substitute $x=3$ into $f''(x)$ $f''(3)=2\times3-6$ $=0$	• correctly demonstrates that the value of the second derivative is equal to zero at $x=3$ **[1 mark]**
	To verify, there must be a change of sign of the second derivative about a point of inflection. $f''(x)$ is a linear function, so only one change of sign. Substitute x-values either side of $x=3$. $f''(2)=-2<0$ - concave down $f''(4)=+2>0$ - concave up	• correctly establishes that $f''(x)$ is negative to the left of the point **[1 mark]** • correctly establishes that $f''(x)$ is positive to the right of the point **[1 mark]**
	As the sign of the second derivative changes around $x=3$, the function has the point of inflection at that point.	• correctly concludes the point $x=3$ is an inflection point **[1 mark]**

Q	Sample response	The response:
14	$2\cos(\theta)-1=0$ $2\cos(\theta)=1$ $\cos(\theta)=\frac{1}{2}$	• correctly rearranges the equation to make $\cos(\theta)$ the subject **[1 mark]**
	$\therefore$ the first quadrant angle is $60°$	• determines the first quadrant (acute) angle in degrees **[1 mark]**
	Cosine is positive in quadrants 1 and 4. $\theta=60°, 300°$	• determines the fourth quadrant angle in degrees **[1 mark]**
	$0\le\theta\le 720°$ $\theta=60°, (360-60)°, (60+360)°, (300+360)°$ $\theta=60°, 300°, 420°, 660°$	• determines the two remaining angles in the $360°\le\theta\le 720°$ domain **[1 mark]**

Q	Sample response	The response:
15	$\left(\log_2(x)\right)^2 = \log_2\left(x^2\right) + 8$ $\left(\log_2(x)\right)^2 - \log_2\left(x^2\right) - 8 = 0$ $\left(\log_2(x)\right)^2 - 2\log_2(x) - 8 = 0$	• correctly applies an appropriate logarithmic law to convert the x^2 term to a term in x **[1 mark]**
	Let $y = \log_2 x$ $y^2 - 2y - 8 = 0$ $(y-4)(y+2) = 0$	• realises the need to solve a quadratic equation **[1 mark]**
	$\therefore (y-4) = 0$ or $(y+2) = 0$ $y = 4$ $y = -2$ $\log_2 x = 4$ $\log_2 x = -2$ $x = 2^4$ $x = 2^{-2}$ $x = 16$ $x = \frac{1}{4}$	• determines a solution **[1 mark]** • determines a second solution **[1 mark]**

Q	Sample response	The response:
	Substitute $x = 16$ into the original equation to evaluate its reasonableness. $\text{LHS} = \left(\log_2(16)\right)^2$	• substitutes the chosen solution into the equation **[1 mark]**
	$= \left(\log_2\left(2^4\right)\right)^2$ $= \left(4\log_2(2)\right)^2$ $= (4)^2$ $= 16$ $\text{RHS} = \log_2\left(16^2\right) + 8$ $= \log_2\left(2^8\right) + 8$ $= 8\log_2(2) + 8$ $= 8 + 8$ $= 16$	
	Therefore, $x = 16$ is a reasonable solution.	• evaluates the reasonableness of one possible solution **[1 mark]**

Q	Sample response	The response:
16	Total energy produced (units): $P(t)=\int_6^{18}\left(2.5\sin\left(\frac{\pi}{12}(t-6)\right)\right)dt$ $P(t)=\left[-\frac{30}{\pi}\cos\left(\frac{\pi}{12}(t-6)\right)\right]_6^{18}$	• correctly integrates the energy produced term **[1 mark]**
	$P(t)=-\frac{30}{\pi}\cos\left(\frac{\pi}{12}\times 12\right)+\frac{30}{\pi}\cos\left(\frac{\pi}{12}\times 0\right)$ $=-\frac{30}{\pi}\cos(\pi)+\frac{30}{\pi}\cos(0)$ $=\frac{30}{\pi}+\frac{30}{\pi}$ $=\frac{60}{\pi}$	• determines the energy produced between sunrise and sunset **[1 mark]**
	Total energy consumed (units): $C(t)=12\times\frac{4}{\pi}$	• correctly determines a method to find the energy consumed between sunrise and sunset **[1 mark]**

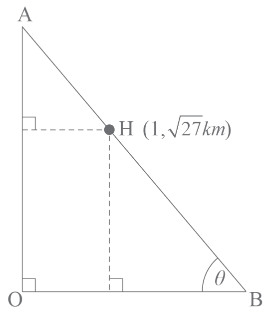
Q	Sample response	The response:
	$= \frac{48}{\pi}$	• determines the energy consumed during daylight hours **[1 mark]**
	Total energy change: $P(t) - C(t)$ $= \frac{60}{\pi} - \frac{48}{\pi}$ $= \frac{12}{\pi}$	• determines the total energy change between sunrise and sunset **[1 mark]**

Q	Sample response	The response:
17	Let X be a binomial random variable representing the number of casts that caught a fish. p corresponds to a probability of a catch (success). Blue pond: $X \sim B\left(3, \frac{2}{3}\right)$ Red pond: $X \sim B\left(3, \frac{1}{3}\right)$ In the blue pond, 30 points will require three successes in three casts: $P(X = 3)$ $= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ $= \left(\frac{2}{3}\right)^3$	• correctly identifies a method to determine at least 30 points in three casts for the blue pond **[1 mark]**
	$= \frac{8}{27}$	• determines the probability of scoring at least 30 points in the blue pond **[1 mark]**

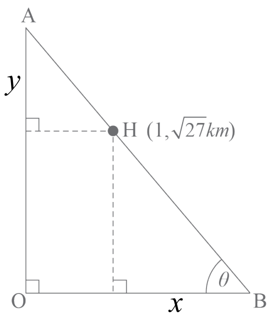
Q	Sample response	The response:
	In the red pond, 30 points will require at least two successes in three casts: i.e. $X \geq 2$ $P(X \geq 2)$ $= P(X=2)+P(X=3)$ $=\binom{3}{2}p^2q^1+\binom{3}{3}p^3q^0$	• correctly identifies a method to determine at least 30 points in three casts for the red pond **[1 mark]**
	$=3\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)+1\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0$	• substitutes appropriate values into the chosen method for the red pond **[1 mark]**
	$=\frac{6}{27}+\frac{1}{27}$ $=\frac{7}{27}$	• determines the probability of scoring at least 30 points in the red pond **[1 mark]**
	In the blue pond, the required probability is $\frac{8}{27}$ In red pond, the required probability is $\frac{7}{27}$ $\frac{8}{27}-\frac{7}{27}=\frac{1}{27}$ The claim is correct. Probability of winning if casting in the blue pond is $\frac{1}{27}$ more than the probability of winning using the red pond.	• evaluates the claim **[1 mark]**

Q	Sample response	The response:
18	Finding velocity of the first object: $v_1(t) = \frac{d}{dt}(\frac{1}{3}t^3 - \frac{1}{2}t^2 + kt)$ $v_1(t) = t^2 - t + k$	• correctly determines the equation for the first object's velocity **[1 mark]**
	Find the rule for the velocity for the second object: $v_2(t) = \int 4dt$ $v_2(t) = 4t + c$	• correctly determines the equation for the second object's velocity including the constant c **[1 mark]**
	Given condition for $v_1(1) = v_2(1)$ $1^2 - 1 + k = 4 + c$ $c = k - 4$	• determine the relationship between constants *k* and *c* **[1 mark]**
	Find the time when the velocities are the same. $v_1(t) = v_2(t)$ $t^2 - t + k = 4t + k - 4$ $t^2 - 5t + 4 = 0$ $(t-1)(t-4) = 0$ $\therefore t = 1, 4$ Need the time different to the given 1 s, so $t = 4$.	• determines the second time when the two velocities are the same **[1 mark]**

Q	Sample response	The response:
	As objects travel in the one direction between given times, the distance travelled is equal to the displacement. Find the difference between the objects' travelling distances. $\text{displacement} = \int \text{velocity}\, dt$ $d_2 - d_1 = \int_1^4 (v_2 - v_1)\, dt$ $= \int_1^4 ((4t-2)-(t^2-t+2))\, dt$	• uses a suitable method for determining the difference between distances the objects have travelled in the given time interval **[1 mark]**
	$= \int_1^4 (-t^2+5t-4)\, dt$ $= \left[\frac{-1}{3}t^3 + \frac{5}{2}t^2 - 4t\right]_1^4$ $= \frac{-1}{3}\times(4)^3 + \frac{5}{2}\times(4)^2 - 4\times 4 - (\frac{-1}{3} + \frac{5}{2} - 4)$ $= \frac{-63}{3} + 28 - \frac{5}{2}$ $= -21 + 28 - 2.5$ $= 4.5$	• determines the difference between distances of the two objects **[1 mark]**

Q	Sample response	The response:
19	**Method 1** Let the new fence be AB and the old homestead gate be H  $AH = \frac{1}{\cos(\theta)} \quad HB = \frac{\sqrt{27}}{\sin(\theta)}$ $AB = AH + HB$ $= \frac{1}{\cos(\theta)} + \frac{\sqrt{27}}{\sin(\theta)}$	• correctly determines an expression for the total length of the new fence in terms of the angle θ **[1 mark]**
	Differentiating length AB wrt θ: $AB = (\cos(\theta))^{-1} + \sqrt{27}(\sin(\theta))^{-1}$ $\frac{d(AB)}{d\theta} = \frac{\sin(\theta)}{(\cos(\theta))^2} - \frac{\sqrt{27}\cos(\theta)}{(\sin(\theta))^2}$	• determines the derivative of the length expression of the new fence **[1 mark]**

Q	Sample response	The response:
	For minimum length: $\frac{d(AB)}{d\theta}=0$ $(\sin(\theta))^3-\sqrt{27}(\cos(\theta))^3=0$ $(\tan(\theta))^3=\sqrt{27}$ $(\tan(\theta))^3=\left(\sqrt{3}\right)^3$ $\tan(\theta)=\sqrt{3}$ $\theta=60°$	• determines the angle θ corresponding to the minimum length **[1 mark]**
	$AB=AH+HB$ $=\frac{1}{\cos(60)}+\frac{\sqrt{27}}{\sin(60)}$ $=2+\frac{\sqrt{27}}{1}\times\frac{2}{\sqrt{3}}$ $=8$ The shortest fence would be 8 km long.	• determines the minimum length of the new fence **[1 mark]** • shows logical organisation **[1 mark]**

Q	Sample response	The response:
19	**Method 2** Let the new fence be AB and the old homestead gate be H  $\tan\theta = \frac{y}{1} = \frac{\sqrt{27}}{x}$ $OB = 1 + x$ $OA = \sqrt{27} + y = \sqrt{27} + \frac{\sqrt{27}}{x}$ $AB = \sqrt{(1+x)^2 + \left(\sqrt{27} + \frac{\sqrt{27}}{x}\right)^2}$	• correctly determines an expression for the total length of the new fence in terms of a single variable **[1 mark]**

Q	Sample response	The response:
	Differentiating length AB wrt x : $AB = \left((1+x)^2 + \left(\sqrt{27} + \frac{\sqrt{27}}{x} \right)^2 \right)^{\frac{1}{2}}$ $\frac{d(AB)}{dx} = \frac{1}{2}\left((1+x)^2 + \left(\sqrt{27} + \frac{\sqrt{27}}{x} \right)^2 \right)^{\frac{-1}{2}}$ $\times \left(2(1+x) + 2\left(\sqrt{27} + \frac{\sqrt{27}}{x} \right)\left(\frac{-\sqrt{27}}{x^2} \right) \right)$ $= \frac{\left((1+x) - \frac{\sqrt{27}}{x^2}\left(\sqrt{27} + \frac{\sqrt{27}}{x} \right) \right)}{\sqrt{(1+x)^2 + \left(\sqrt{27} + \frac{\sqrt{27}}{x} \right)^2}}$	• determines the derivative of the length expression of the new fence **[1 mark]**

Q	Sample response	The response:
	For minimum length: $\frac{d(AB)}{dx}=0$ $\frac{\left((1+x)-\frac{\sqrt{27}}{x^2}\left(\sqrt{27}+\frac{\sqrt{27}}{x}\right)\right)}{\sqrt{(1+x)^2+\left(\sqrt{27}+\frac{\sqrt{27}}{x}\right)^2}}=0$ $(1+x)-\frac{\sqrt{27}}{x^2}\left(\sqrt{27}+\frac{\sqrt{27}}{x}\right)=0$ $1+x-\frac{27}{x^2}-\frac{27}{x^3}=0$ $x^4+x^3-27x-27=0$ $x^4-27x+x^3-27=0$ $x(x^3-27)+1(x^3-27)=0$ $(x+1)(x^3-27)=0$ $\therefore x=-1$ (invalid) or $x=3$ $\therefore x=3$	• determines the value of the variable corresponding to the minimum length **[1 mark]**
	$AB=\sqrt{(1+x)^2+\left(\sqrt{27}+\frac{\sqrt{27}}{x}\right)^2}$ $=\sqrt{(1+3)^2+\left(\sqrt{27}+\frac{\sqrt{27}}{3}\right)^2}$ $=8$ The shortest fence would be 8 km long.	• determines the minimum length of the new fence **[1 mark]** • shows logical organisation **[1 mark]**