Mathematical Methods marking guide and response

External assessment 2024

Paper 1: Technology-free (55 marks)

Paper 2: Technology-active (55 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Purpose

This document consists of a marking guide and a sample response.

The marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

The sample response:

- demonstrates the qualities of a high-level response
- has been annotated using the marking guide.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Allow FT mark/s — refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Paper 1

Multiple choice

Question	Response
1	D
2	A
3	B
4	A
5	C
6	D
7	B
8	C
9	B
10	C

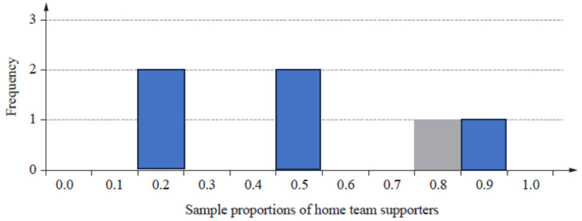
Short response

Q	Sample response	The response:
11a)	$y = x^3 - 3x^2$ $\frac{dy}{dx} = 3x^2 - 6x$ $\frac{d^2x}{dy^2} = 6x - 6$	• correctly determines the first derivative **[1 mark]** • determines the second derivative **[1 mark]**
11b)	$\frac{d^2y}{dx^2} = 6x - 6$ When $x = -1$ $\frac{d^2y}{dx^2} = 6 \times -1 - 6$ $= -12$	• determines the value of the second derivative **[1 mark]**
11c)	$\frac{d^2y}{dx^2} = 0$ $6x - 6 = 0$ $6x = 6$ $x = 1$ Substitute into the graph equation: $y = (1)^3 - 3(1)^2$ $y = -2$ Coordinates are $(1, -2)$.	• equates the second derivative to zero **[1 mark]** • determines the x-coordinate of the point **[1 mark]** • determines the y-coordinate of the point **[1 mark]**

Q	Sample response	The response:
12a)	$P(3\,\text{bushwalking days})$ $=\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$ $=\frac{1}{27}$	• correctly determines the probability **[1 mark]**
12b) (i)	$P(X=2)$ $=\binom{3}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^1$ $=3\times\frac{4}{9}\times\frac{1}{3}=\frac{4}{9}$	• correctly determines the required binomial probability expression **[1 mark]** • determines the probability as a fully simplified fraction **[1 mark]**
12b) (ii)	$P(X<3)$ $=1-P(X=3)$ $=1-\binom{3}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^0$ $=1-\frac{8}{27}$ $=\frac{19}{27}$	• correctly determines the required method **[1 mark]** • determines the required binomial expression for the calculation **[1 mark]** • determines the probability as a fully simplified fraction **[1 mark]**

Q	Sample response	The response:
13a)	$F(x)=\int\left(4e^{2x}+\sin(2x)\right)dx$ $=\frac{4e^{2x}}{2}+-\frac{1}{2}\cos(2x)+c$ $=2e^{2x}-\frac{1}{2}\cos(2x)+c$ $F(0)=5$ $5=2-\frac{1}{2}+c$ $c=3\frac{1}{2}$ $F(x)=2e^{2x}-\frac{1}{2}\cos(2x)+3\frac{1}{2}$	• correctly integrates the exponential term **[1 mark]** • correctly integrates the trigonometric term **[1 mark]** • determines the constant of integration **[1 mark]**

Q	Sample response	The response:
13b)	$\frac{dy}{dx}=\left(\frac{3x^7-2x}{x^4}\right)^2$ $=\left(\frac{3x^7}{x^4}-\frac{2x}{x^4}\right)^2$ $=\left(3x^3-\frac{2}{x^3}\right)^2$ $=\left(3x^3\right)^2-2\times3x^3\times\frac{2}{x^3}+\left(\frac{2}{x^3}\right)^2$	• correctly expands the squared bracket **[1 mark]**
	$=9x^6-12+\frac{4}{x^6}$ $=9x^6-12+4x^{-6}$	• simplifies the expanded squared bracket into separate terms **[1 mark]**
	$y=\int\left(9x^6-12+4x^{-6}\right)dx$ $y=\frac{9}{7}x^7-12x-\frac{4}{5}x^{-5}+c$	• determines y in terms of x by integrating all terms **[1 mark]**

Q	Sample response	The response:
14a)	Sample proportion 0.2, frequency = 2 Sample proportion 0.5, frequency = 2 Sample proportion 0.9, frequency = 1 	• correctly sketches the sample proportion 0.2 **[1 mark]** • correctly sketches the sample proportion 0.5 **[1 mark]** • correctly sketches the sample proportion 0.9 **[1 mark]**
14b)	One way: For larger samples, the distribution of $\hat{p}$ would be expected to become more symmetrical about the population proportion of 0.6 (it is not currently symmetrical). Second way: For larger samples, there would be less variability about the mean of 0.6, i.e. the graph would appear to look narrower. There should be less high or low sample proportions.	• correctly identifies one change to the distribution of $\hat{p}$ as it more closely approximates normality due to the larger samples – a more symmetrical graph **[1 mark]** • correctly identifies a second change to the distribution of $\hat{p}$ as it more closely approximates normality due to the larger samples – there would be less variability and the appearance of the graph would look narrower or 'skinnier' **[1 mark]**

Q	Sample response	The response:
15a)	The confidence interval corresponds to $(\hat{p}-\mathrm{E}, \hat{p}+\mathrm{E})$, where E is the margin of error about $\hat{p}$. $\hat{p}+\mathrm{E}=\frac{7}{10}$ $\hat{p}-\mathrm{E}=\frac{3}{10}$ subtracting: $\therefore 2\mathrm{E}=\frac{7}{10}-\frac{3}{10}=\frac{4}{10}$ $\therefore \mathrm{E}=\frac{2}{10}=\frac{1}{5}$	• correctly determines the margin of error **[1 mark]**
15b)	$\hat{p}+\mathrm{E}=\frac{7}{10}$ $\hat{p}+\frac{2}{10}=\frac{7}{10}$ $\hat{p}=\frac{5}{10}=\frac{1}{2}$ Upper CI value $=\hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $\frac{7}{10}=\frac{1}{2}+2\sqrt{\frac{\frac{1}{2}\left(1-\frac{1}{2}\right)}{n}}$	• determines the value of the sample proportion $\hat{p}$ **[1 mark]** • substitutes $\hat{p}$ and z-score into the confidence interval formula **[1 mark]**

Q	Sample response	The response:
	$\frac{1}{5}=2\sqrt{\frac{\frac{1}{2}\left(1-\frac{1}{2}\right)}{n}}$ $\frac{1}{5}=2\sqrt{\frac{\frac{1}{4}}{n}}$ $\frac{1}{5}=2\sqrt{\frac{1}{4n}}$ $\frac{1}{10}=\sqrt{\frac{1}{4n}}$ $\frac{1}{100}=\frac{1}{4n}$ $100=4n$ $n=25$ 25 people were surveyed.	• determines the number of people surveyed **[1 mark]**

Q	Sample response	The response:
16	Find the equation of the graph $y = \log_a(x+b)$. There is a vertical asymptote at $x = -4$ This is equivalent to a horizontal translation of -4 for the function $y = \log_a(x)$ $y = \log_a(x-(-4))$ $y = \log_a(x+4)$ $\therefore b = 4$ $\quad y = \log_a(x+4)$	• correctly determines the value of b in the equation **[1 mark]**
	Substitute (4, 3) $3 = \log_a(4+4)$ $3 = \log_a 8$ $a^3 = 8$ $\therefore a = 2$ $\quad y = \log_2(x+4)$	• determines the value of a in the equation **[1 mark]**
	Find the y-intercept Substitute $x = 0$ $y = \log_2(0+4)$ $y = \log_2 4$ $y = \log_2 2^2$ $y = 2\log_2 2$ $y = 2$	

Q	Sample response	The response:
	At point P: $y_p = 2 \times 2 = 4$ $4 = \log_2\left(x_p + 4\right)$ $x_p + 4 = 2^4$ $x_p = 12$	• determines the *y*-coordinate of point P **[1 mark]** • determines the *x*-coordinate of point P **[1 mark]**

Q	Sample response	The response:
17	**Method 1** 7 times the members is 7000. Let m be the time when 7000 members is reached. The required change in members is 6000. $6000 = \int_0^m 3e^{0.5t}\,dt$ $= \left[3\times\frac{1}{0.5}e^{0.5t}\right]_0^m$ $= \left[6e^{0.5t}\right]_0^m$ $= \left[6e^{0.5m} - 6e^0\right]$ $= \left[6e^{0.5m} - 6\right]$ $6000 = 6e^{0.5m} - 6$ $6006 = 6e^{0.5m}$ $\frac{6006}{6} = e^{0.5m}$ $0.5m = \ln 1001$ $m = 2\ln 1001$ days	• correctly uses the initial conditions to determine the increase **[1 mark]** • correctly determines the integral **[1 mark]** • determines the number of days required **[1 mark]**

Q	Sample response	The response:
	Method 2	
	$f'(t) = 3e^{0.5t}$	
	$f(t) = \int 3e^{0.5t}\,dt$	
	$= 6e^{0.5t} + c$	• correctly determines the integral **[1 mark]**
	Currently they have 1000 members:	
	$1000 = 6e^{0.5\times 0} + c$	
	$1000 = 6 + c$	
	$c = 994$	
	$\therefore f(t) = 6e^{0.5t} + 994$	• correctly uses the initial conditions to determine c **[1 mark]**
	Let m be the day when 7000 members is reached.	
	$7000 = 6e^{0.5m} + 994$	
	$6006 = 6e^{0.5m}$	
	$\frac{6006}{6} = e^{0.5m}$	
	$1001 = e^{0.5m}$	
	$0.5m = \ln 1001$	
	$m = 2\ln 1001$ days	• determines the number of days required **[1 mark]**

Q	Sample response	The response:
18	Let h be the height of the rectangular prism. $BC=\sqrt{4^2+h^2}$ $BC=\sqrt{16+h^2}=a$ $AB=\sqrt{4^2+3^2}$ $AB=\sqrt{25}$ $AB=5=c$ $AC=\sqrt{3^2+h^2}$ $AC=\sqrt{9+h^2}=b$ Use the cosine rule $b^2=a^2+c^2-2ac\cos B$ $\left(\sqrt{9+h^2}\right)^2=$ $\left(\sqrt{16+h^2}\right)^2+5^2-2\times\sqrt{16+h^2}\times5\times\cos 60$	• correctly expresses BC in terms of 'height' **[1 mark]** • correctly expresses AC in terms of 'height' **[1 mark]**

Q	Sample response	The response:
	$9+h^2=16+h^2+25-10\times\sqrt{16+h^2}\times\frac{1}{2}$ $\frac{32}{5}=\sqrt{16+h^2}$ $h^2=\left(\frac{32}{5}\right)^2-16$ $h=\sqrt{\left(\frac{32}{5}\right)^2-16}$ units $=\sqrt{(6.4)^2-16}$ $>\sqrt{6^2-16}$ $=\sqrt{36-16}$ $h=\sqrt{20}$ Since $\sqrt{16}=4$ $\sqrt{20}>\sqrt{16}>4$ The height of the container $\sqrt{20}$ is greater than the 4 m required, therefore the container does meet the person's requirements.	• determines an expression for the 'height' of the container **[1 mark]** • decides if the container meets the requirements **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
19	The periodic motion can be modelled by a sine function. Choose a general sine function: $g(t) = A\sin(B(x+C)) + D$ $A = 5$ $T = \frac{2\pi}{B}$ $B = \frac{2\pi}{12}$ $= \frac{\pi}{6}$ $C = 0$ (no phase shift in this context) $D = 275$ (Halfway between 270 and 280) $\therefore g(t) = 5\sin\left(\frac{\pi}{6}t\right) + 275$ $g'(t) = \frac{\pi}{6} \times 5\cos\left(\frac{\pi}{6}t\right)$	• correctly determines the sine function that models the front edge of the glacier position **[1 mark]** • determines the velocity of the front edge of the glacier **[1 mark]**

Q	Sample response	The response:
	$g''(x) = -\left(\frac{\pi}{6}\right)^2 \times 5\sin(0.5t - 1.6)$ $g''(x) = -5\left(\frac{\pi}{6}\right)^2 \sin\left(\frac{\pi}{6}t\right)$ Equate acceleration to $\frac{5\pi^2\sqrt{3}}{72}$ or $-\frac{5\pi^2\sqrt{3}}{72}$ (for the absolute value) $-5\left(\frac{\pi}{6}\right)^2 \sin\left(\frac{\pi}{6}t\right) = \frac{5\pi^2\sqrt{3}}{72}$ $-5\frac{\pi^2}{36}\sin\left(\frac{\pi}{6}t\right) = \frac{5\pi^2\sqrt{3}}{72}$ Solve for time, t $\quad -\sin\left(\frac{\pi}{6}t\right) = \frac{\sqrt{3}}{2}$ $\sin\left(\frac{\pi}{6}t\right) = -\frac{\sqrt{3}}{2}$ $\frac{\pi}{6}t = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$ $t = 8$ or 10 months	• determines the acceleration of the front edge of the glacier (second derivative) **[1 mark]**

Q	Sample response	The response:
	Given the sine model used, the acceleration will be greater between these times, i.e. between 8 and 10 = 2 month period of time $-5\left(\frac{\pi}{6}\right)^2\sin\left(\frac{\pi}{6}t\right)=-\frac{5\pi^2\sqrt{3}}{72}$ $-5\frac{\pi^2}{36}\sin\left(\frac{\pi}{6}t\right)=-\frac{5\pi^2\sqrt{3}}{72}$ $\sin\left(\frac{\pi}{6}t\right)=\frac{\sqrt{3}}{2}$ $\frac{\pi}{6}t=\frac{\pi}{3}$ or $\frac{2\pi}{3}$ $t=2$ or 4 months Given the sine model used, the acceleration will be greater between these times, i.e. between 2 and 4 = 2 month period of time	• determines a period of time in a year where the acceleration is greater than the value given **[1 mark]**
	The acceleration will be greater than the given absolute value for a total of 2 + 2 = 4 months in a year.	• determines the sum of the two possible time periods when acceleration is greater than the absolute value given **[1 mark]**
	Evaluation of the reasonableness of the claim: 4 months is less than the stated 'between 7 and 8 months' mentioned in the claim. Therefore, the claim is not reasonable.	• decides if the claim is reasonable **[1 mark]**

Marking guide

Paper 2

Multiple choice

Question	Response
1	C
2	A
3	C
4	B
5	D
6	A
7	B
8	D
9	C
10	B

Short response

Q	Sample response	The response:
11	$w=\frac{b-a}{n}$ $=\frac{3-0}{6}$ $=\frac{1}{2}$ $A\approx\frac{1}{2}w\left[f(x_0)+2f(x_1)+2f(x_2)+\ldots+f(x_n)\right]$ $=\frac{1}{2}\times\frac{1}{2}\times\left[36+2\times25+2\times16+2\times9+2\times4+2\times1+0\right]$ $=36.5\text{ units}^2$	• correctly determines the rectangle width **[1 mark]** • correctly states the trapezoidal rule **[1 mark]** • substitutes appropriate values into the trapezoidal rule **[1 mark]** • determines the approximate value of the definite integral **[1 mark]**

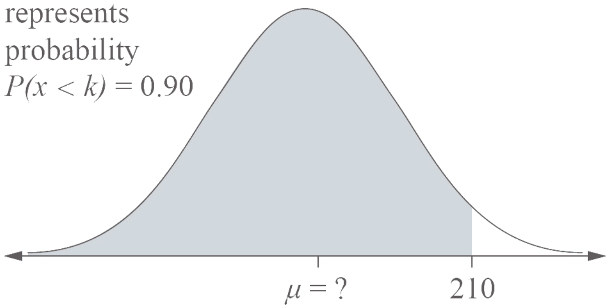
Q	Sample response	The response:
12a)	At location P: $M_P = \log_{10}\left(\frac{I_P}{I_0}\right)$ At location Q: $M_Q = \log_{10}\left(\frac{I_Q}{I_0}\right)$ Subtracting the magnitudes: $M_P - M_Q = \log_{10}\left(\frac{I_P}{I_0}\right) - \log_{10}\left(\frac{I_Q}{I_0}\right)$	• correctly determines an equation involving the difference in magnitudes **[1 mark]**

Q	Sample response	The response:
12b)	$= \log_{10}\left(\dfrac{\frac{I_P}{I_0}}{\frac{I_Q}{I_0}}\right)$ $\therefore M_P - M_Q = \log_{10}\left(\dfrac{I_P}{I_Q}\right)$ Changing to index form: $\dfrac{I_P}{I_Q} = 10^{M_P - M_Q}$ $\dfrac{I_P}{I_Q} = 10^{1.7}$ $\dfrac{I_P}{I_Q} = 50.1187$ $I_P = 50.1187 I_Q$ Therefore, the intensity at P is 50.1187 times that of the intensity at Q.	• determines a simplified logarithmic equation **[1 mark]** • states an equation with intensities made the subject, i.e. the log equation is converted to an index equation **[1 mark]** • substitutes the given magnitude values appropriately **[1 mark]** • determines how more intense the earthquake is at location P compared with location Q **[1 mark]**

Q	Sample response	The response:
13a)	$N(0)=\dfrac{A}{2+e^{-0}}$ $3\times10^5=\dfrac{A}{2+1}$ $A=3\times3\times10^5$ $A=9\times10^5$	• correctly determines the value of A **[1 mark]**
13b)	$N(t)=\dfrac{9\times10^5}{2+e^{-t}}$ $N(6)=\dfrac{9\times10^5}{2+e^{-6}}$ $=449\ 442.9711$ $=449\ 443$ termites	• correctly determines $t=6$ **[1 mark]** • determines the number of termites **[1 mark]**
13c)	$300\ 000+130\ 000=430\ 000$ $N(t)=\dfrac{9\times10^5}{2+e^{-t}}$ $430\ 000=\dfrac{9\times10^5}{2+e^{-t}}$ Using a GDC: $t=2.3749$ months	• correctly determines the required number of termites **[1 mark]** • determines the time required **[1 mark]**

Q	Sample response	The response:
13d)	$N(t)=\frac{9\times10^5}{2+e^{-t}}$ $N(t)=\left(9\times10^5\right)\left(2+e^{-t}\right)^{-1}$ $\frac{dN}{dt}=-\left(9\times10^5\right)\left(2+e^{-t}\right)^{-2}\times-e^{-t}$ $\frac{dN}{dt}=\frac{\left(9\times10^5\right)}{e^t\left(2+e^{-t}\right)^2}$	• shows application of the chain rule **[1 mark]** • determines a formula for the number of termites expressed as a fraction **[1 mark]**
13e)	Using the formula developed in d) or a GDC: $N'(5)=1505.8744$ $=1506$ termites/month	• determines the rate of change **[1 mark]**

Q	Sample response	The response:
14a)	$H(0) = 6$ m	• correctly determines the initial height **[1 mark]**
14b)	Using a GDC: $t = 9$ days	• correctly determines the time required **[1 mark]**
14c)	Initially, the kick height was 6 metres. At the end of the course, $t = 12$, the kick height increased to 7.1139 metres. The kick height has increased by 1.1139 metres during the course.	• correctly determines the kick height at the end of the course **[1 mark]** • determines the overall kick height improvement **[1 mark]**
14d)	Using a GDC: $H'(1.5) = 0.17372$ m/day	• correctly determines the derivative value when $t = 1.5$ **[1 mark]**
14e)	Using a GDC: Graph derivative function and $y = 0.09$ Find point of intersection. $H'(t) = 0.09$ $t = 3.82549$ days	• correctly determines the time as a decimal **[1 mark]**

Q	Sample response	The response:
15	In the original country, the cut-off for extremely tall is: $\mu + 3\sigma = 180 + 3 \times 10$ $= 180 + 30$ $= 210$ cm	• correctly determines the cut-off for extremely tall in the original country **[1 mark]**
	In the destination country: Shaded area represents probability $P(x < k) = 0.90$  Using a GDC: Calculate the z-score for 90% area Use standard normal distribution with $\mu = 0$ and $\sigma = 1$ $z = 1.28155$	• correctly determines the z-score for a 90% area **[1 mark]**
	$z = \dfrac{X - \mu}{\sigma}$ $1.28155 = \dfrac{210 - \mu}{10}$ $12.8155 = 210 - \mu$ $\mu = 210 - 12.8155$ $= 197.1845$ cm	• determines the mean height of the destination country **[1 mark]**

Q	Sample response	The response:
	Cut-off for extremely tall in the destination country is: $\mu + 3\sigma = 197.1845 + 3 \times 10$ $= 227.1845$ $= 227.18$ cm	• determines the minimum height required in the destination country **[1 mark]**

<table>
<tr><th>Q</th><th>Sample response</th><th>The response:</th></tr>
<tr><td>16</td><td>Analytical procedure:
$\sigma = \sqrt{\frac{p(1-p)}{n}}$ (formula book)
$0.04 = \sqrt{\frac{p(1-p)}{120}}$
$0.0016 = \frac{p(1-p)}{120}$
$0.192 = p(1-p)$
$0.192 = p - p^2$
$p^2 - p + 0.192 = 0$

Use GDC equation solver or graph $y = 0.04$ and the square root function to find intersections:
$p = 0.25921$
$= 26\%$
or
$p = 0.7408$
$= 74\%$

Both population proportions (26% and 74%) are less than 80%.
Therefore, the bus route proposal would not be discussed at a council meeting.</td><td>• correctly substitutes the given information into the standard deviation formula for sample proportion [1 mark]

• determines a possible value for the population proportion [1 mark]

• determines a second possible population proportion value [1 mark]

• makes a justified decision regarding the proposal [1 mark]</td></tr>
</table>

Q	Sample response	The response:
17	**Method 1** $P(\text{fruit bruised}) = 0.03$ Determine the probability of a box containing bruised fruit: $P(\text{at least one bruised out of 4})$ $= 1 - P(\text{none bruised out of 4})$ $= 1 - (0.97)^4$ $= 1 - 0.88529$ $= 0.11471$	• correctly determines the probability of bruised fruit in one box **[1 mark]**
	The expected number (mean) of boxes out of 140 $E(X) = np$ $= 140 \times 0.11471$ $= 16.0594$	• determines the expected (mean) number of boxes with some bruised fruit as a decimal **[1 mark]**
	Answer: 16 boxes can be expected to contain bruised fruit.	• determines the number of boxes with some bruised fruit as a whole number **[1 mark]**

Q	Sample response	The response:
	Method 2 $P(\text{fruit not bruised}) = 1.0 - 0.03 = 0.97$ $P(4 \text{ fruit not bruised}) = (0.97)^4 = 0.88529$	• correctly determines the probability of no bruised fruit in a box **[1 mark]**
	Expected number of these in 140 boxes: $E(X) = np = 140 \times 0.88529 = 123.9406$	• determines the expected (mean) number of boxes with no bruised fruit as a decimal **[1 mark]**
	Boxes with some bruised fruit: 140-123.9406 = 16.0594 Therefore, 16 boxes contain bruised fruit.	• determines the number of boxes with some bruised fruit as a whole number **[1 mark]**

Q	Sample response	The response:
18	$v(t)=\int a(t)\,dt$ $=\int 3\sin(2t)\,dt$ $=-\frac{3}{2}\cos(2t)+c$ $v(0)=0$ $0=-\frac{3}{2}\cos(0)+c$ $0=-\frac{3}{2}\times 1+c$ $c=\frac{3}{2}$ $\therefore v(t)=-\frac{3}{2}\cos(2t)+\frac{3}{2}$ $d(t)=\int v(t)\,dt$ $=\int -\frac{3}{2}\cos(2t)+\frac{3}{2}\,dt$ $=-\frac{3}{4}\sin(2t)+\frac{3}{2}t+c$ $d(0)=0$ $c=0$ $d(t)=-\frac{3}{4}\sin(2t)+\frac{3}{2}t$	• correctly determines the velocity formula **[1 mark]** • determines the displacement formula **[1 mark]**

Q	Sample response	The response:
	Using a GDC: When $t = 3$, $d(3) = 4.70956$ m	• determines the object displacement when $t = 3$ **[1 mark]**
	The sensor can detect motion 2 metres away, therefore between d= 2.70956 and d= 6.70956	
	Find the times associated with these: Using a GC; $d = 2.70956$ $t = 1.68914$ $d = 6.70956$ $t = 4.59214$	
	The total time within sensor range is: $4.5921 - 1.6891$ $= 2.90296$ sec	• determines the time when the object is within sensor range **[1 mark]**
	Average velocity within sensor range: $\bar{v} = \dfrac{4}{2.90296}$ $= 1.378 \; m/s$	• determines the average velocity **[1 mark]**

Q	Sample response	The response:
19	Given that $p(10) = 0.0135$ $0.0135 = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{10-18}{\sigma}\right)^2}$ $0.0135 = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{-8}{\sigma}\right)^2}$ Using a GDC: Two solutions are obtained for the std deviation $\sigma = 4.0002$ and 28.4020 Reject 28.4020 as it is not a possible standard deviation because for example three standard deviations less than the mean would produce a negative speed or three above would result in an impossible speed on an e-scooter (>100 km/h). Use a GDC to determine $P\ (X \geq 23)$ with N ~(18, 4.00), $P(X \geq 23) = 0.10566$ The number of riders is: $0.10566 \times 75 = 7.9245$	• correctly substitutes the known information, $x = 10$ and $p(10) = 0.0135$, into the given normal distribution formula **[1 mark]** • determines a possible value for the standard deviation **[1 mark]** • determines the proportion of riders above 23 km/h **[1 mark]** • determines the number of riders above 23 km/h **[1 mark]**
	The total fines obtained: $7.9245 \times 143 = 1133.20$ The expected total fines is about $1133, which is less than the suggested $1500, so the suggestion is not reasonable.	• determines the expected total fines **[1 mark]** • provides appropriate statement of reasonableness **[1 mark]**