Mathematical Methods subject report

2023 cohort

January 2024

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

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Contents

Introduction 1
Audience and use 1
Report preparation 2
Subject highlights 2

Subject data summary 3
Subject completion 3
Units 1 and 2 results 3
Units 3 and 4 internal assessment (IA) results 3
Total marks for IA 3
IA1 marks 4
IA2 marks 5
IA3 marks 6
External assessment (EA) marks 7
Final subject results 7
Final marks for IA and EA 7
Grade boundaries 8
Distribution of standards 8

Internal assessment 9
Endorsement 9
Confirmation 9

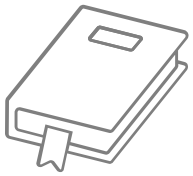
Internal assessment 1 (IA1) 10
Problem-solving and modelling task — extended response (20%) 10
Assessment design 10
Assessment decisions 12

Internal assessment 2 (IA2) 17
Examination — short response (15%) 17
Assessment design 17
Assessment decisions 19

Internal assessment 3 (IA3) 24
Examination — short response (15%) 24
Assessment design 24
Assessment decisions 26

External assessment 30
Examination — short response (50%) 30
Assessment design 30
Assessment decisions 30

Introduction



Throughout 2023, schools and the Queensland Curriculum and Assessment Authority (QCAA) continued to improve outcomes for students in the Queensland Certificate of Education (QCE) system. These efforts were consolidated by the cumulative experience in teaching, learning and assessment of the current General and General (Extension) senior syllabuses, and school engagement in QCAA endorsement and confirmation processes and external assessment marking. The current evaluation of the QCE system will further enhance understanding of the summative assessment cycle and will inform future QCAA subject reports.

The annual subject reports seek to identify strengths and opportunities for improvement of internal and external assessment processes for all Queensland schools. The 2023 subject report is the culmination of the partnership between schools and the QCAA. It addresses school-based assessment design and judgments, and student responses to external assessment for this subject. In acknowledging effective practices and areas for refinement, it offers schools timely and evidence-based guidance to further develop student learning and assessment experiences for 2024.

The report also includes information about:

- how schools have applied syllabus objectives in the design and marking of internal assessments
- how syllabus objectives have been applied in the marking of external assessments
- patterns of student achievement.

The report promotes continuous improvement by:

- identifying effective practices in the design and marking of valid, accessible and reliable assessments
- recommending where and how to enhance the design and marking of valid, accessible and reliable assessment instruments
- providing examples that demonstrate best practice.

Schools are encouraged to reflect on the effective practices identified for each assessment, consider the recommendations to strengthen assessment design and explore the authentic student work samples provided.

Audience and use

This report should be read by school leaders, subject leaders and teachers to:

- inform teaching and learning and assessment preparation
- assist in assessment design practice
- assist in making assessment decisions
- help prepare students for internal and external assessment.

The report is publicly available to promote transparency and accountability. Students, parents, community members and other education stakeholders can use it to learn about the assessment practices and outcomes for senior subjects.

Report preparation

The report includes analyses of data and other information from endorsement, confirmation and external assessment processes. It also includes advice from the chief confirmer, chief endorser and chief marker, developed in consultation with and support from QCAA subject matter experts.

Subject highlights

425
schools offered Mathematical Methods

74.66%
of students completed 4 units

96.05%
of students received a C or higher

Subject data summary

Subject completion

The following data includes students who completed the General subject.

Note: All data is correct as at January 2024. Where percentages are provided, these are rounded to two decimal places and, therefore, may not add up to 100%.

Number of schools that offered Mathematical Methods: 425.

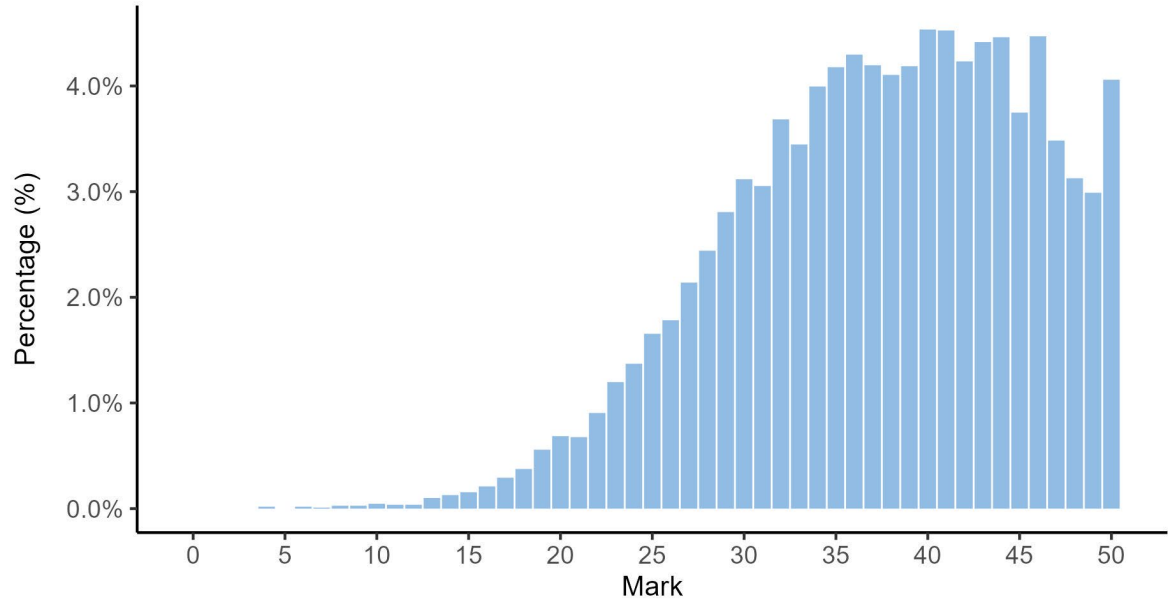
Completion of units	Unit 1	Unit 2	Units 3 and 4
Number of students completed	14,615	12,822	10,912

Units 1 and 2 results

Number of students	Satisfactory	Unsatisfactory
Unit 1	13,477	1,138
Unit 2	10,876	1,946

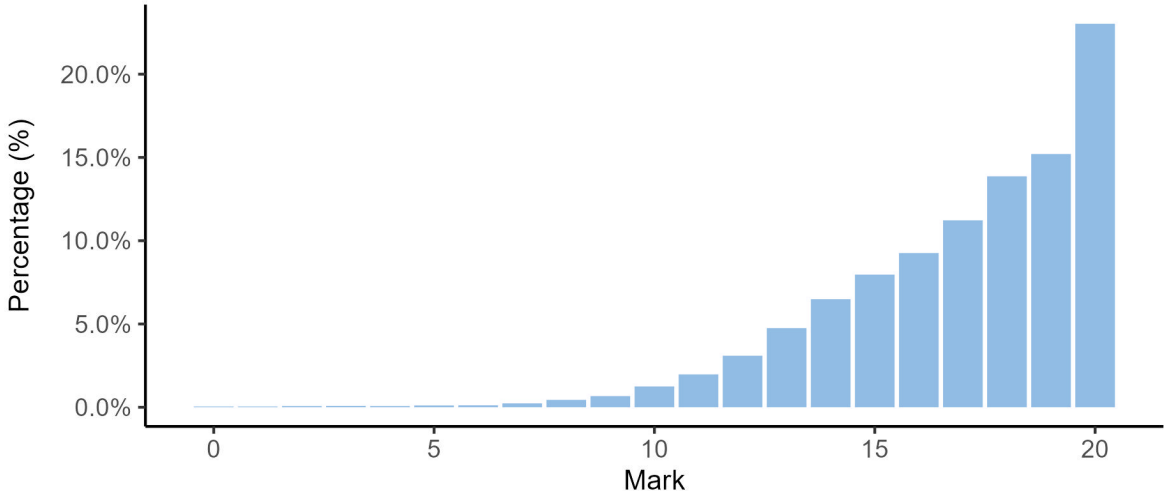
Units 3 and 4 internal assessment (IA) results

Total marks for IA

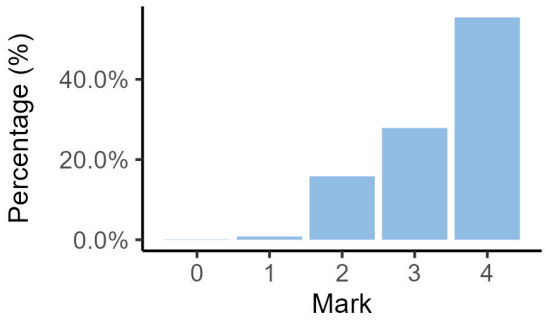


IA1 marks

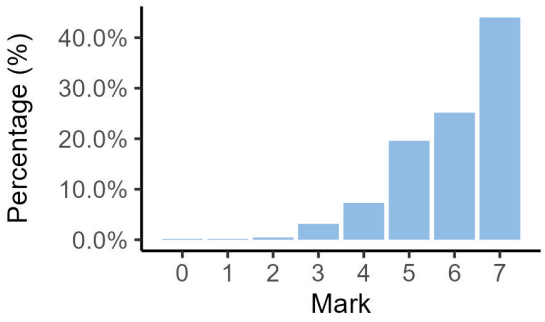
IA1 total



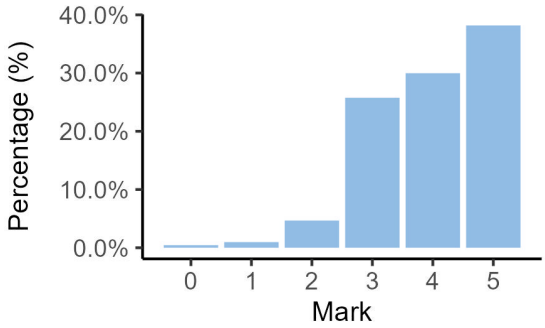
IA1 Criterion: Formulate



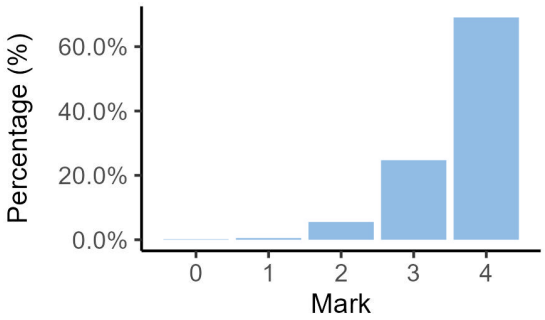
IA1 Criterion: Solve



IA1 Criterion: Evaluate and verify

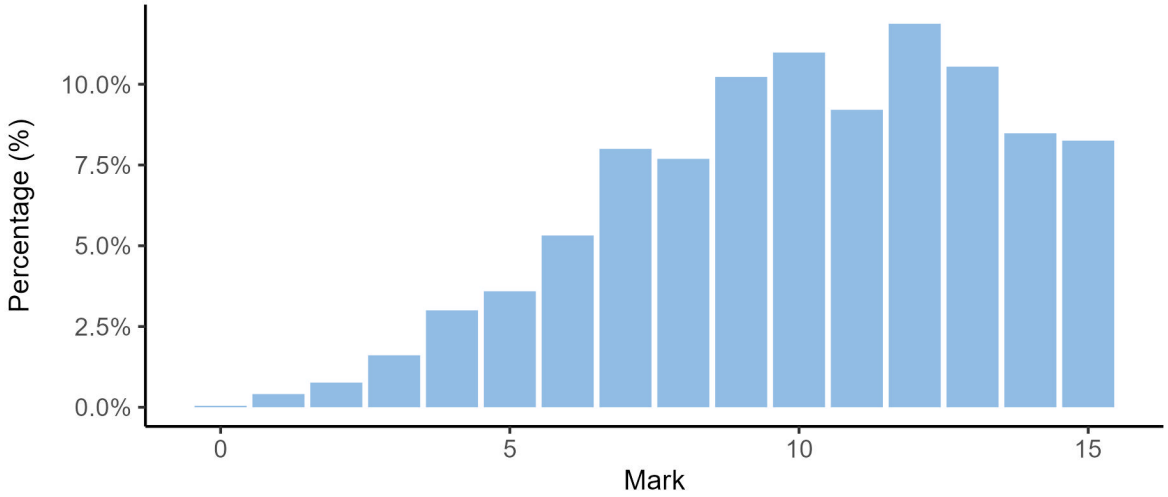


IA1 Criterion: Communicate

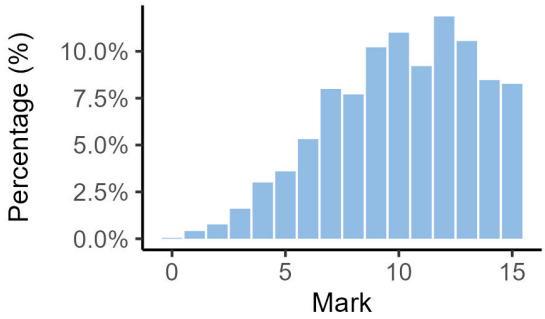


IA2 marks

IA2 total

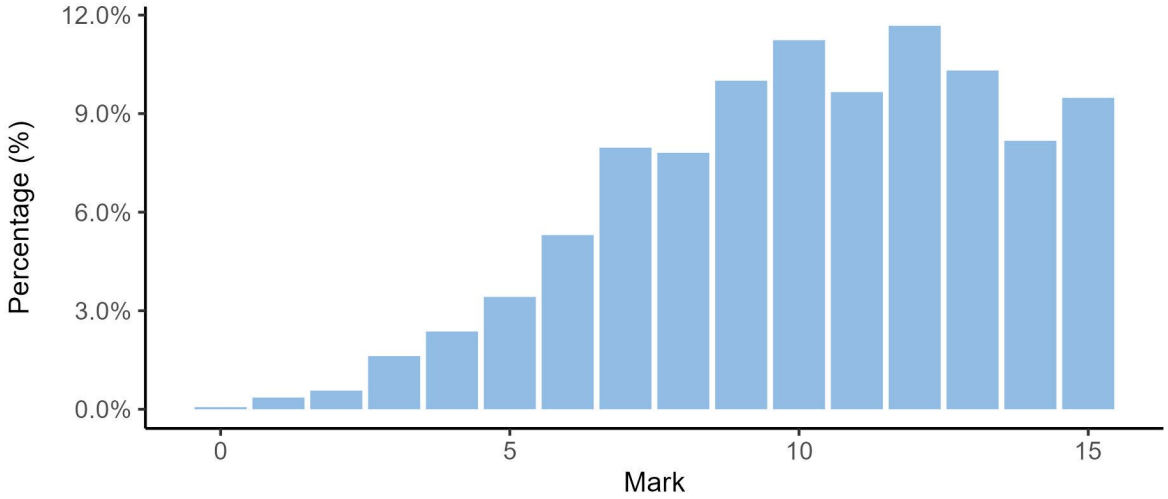


IA2 Criterion: Foundational knowledge and problem-solving

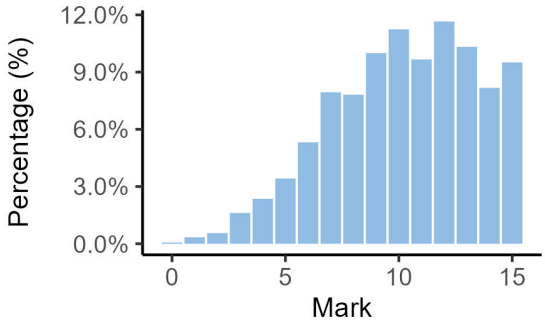


IA3 marks

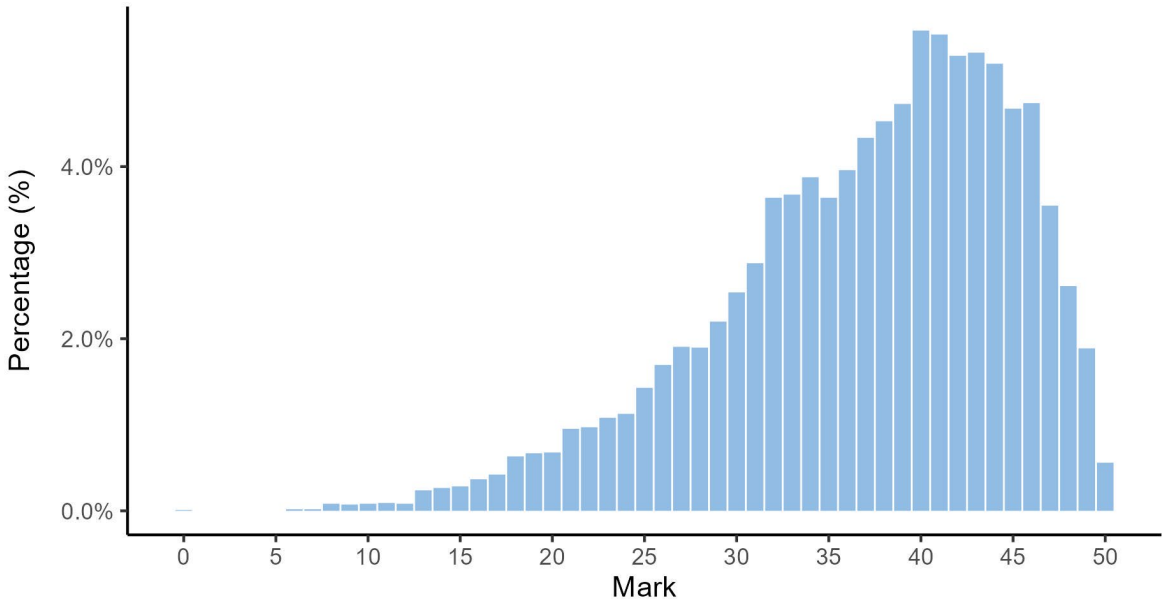
IA3 total



IA3 Criterion: Foundational knowledge and problem-solving

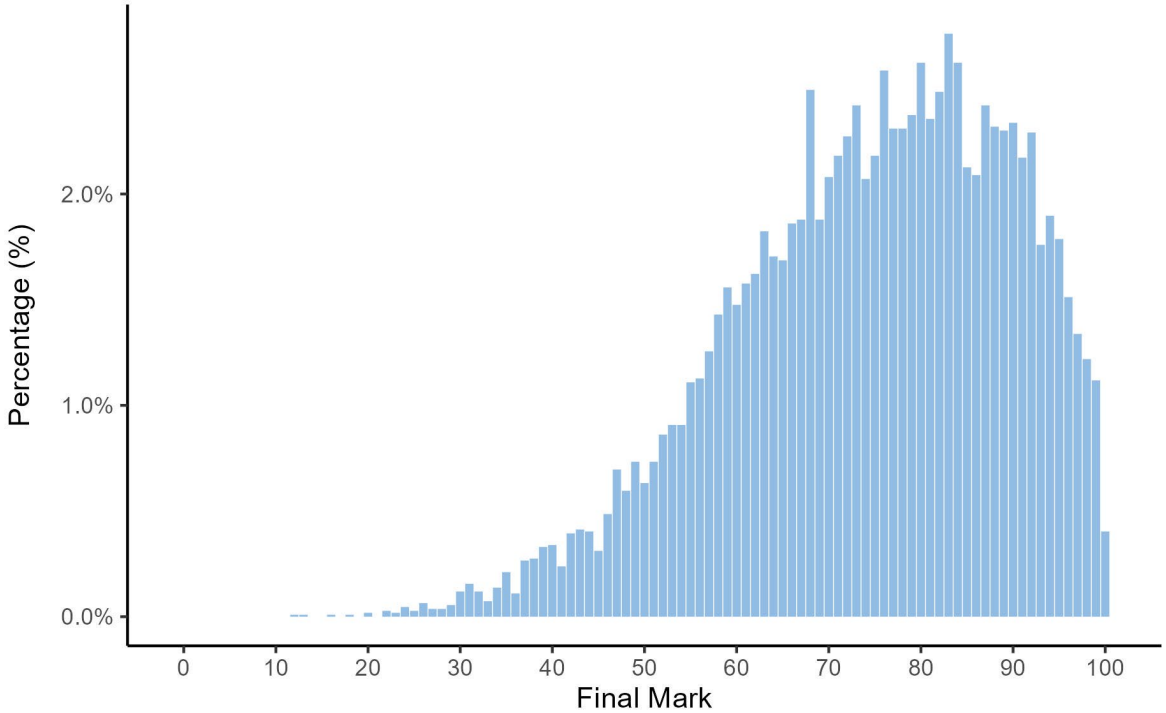


External assessment (EA) marks



Final subject results

Final marks for IA and EA



Grade boundaries

The grade boundaries are determined using a process to compare results on a numeric scale to the reporting standards.

Standard	A	B	C	D	E
Marks achieved	100–85	84–68	67–45	44–21	20–0

Distribution of standards

The number of students who achieved each standard across the state is as follows.

Standard	A	B	C	D	E
Number of students	3,174	4,364	2,943	425	6

Internal assessment

The following information and advice relate to the assessment design and assessment decisions for each IA in Units 3 and 4. These instruments have undergone quality assurance processes informed by the attributes of quality assessment (validity, accessibility and reliability).

Endorsement

Endorsement is the quality assurance process based on the attributes of validity and accessibility. These attributes are categorised further as priorities for assessment, and each priority can be further broken down into assessment practices.

Data presented in the Assessment design section identifies the reasons why IA instruments were not endorsed at Application 1, by the priority for assessments. An IA may have been identified more than once for a priority for assessment, e.g. it may have demonstrated a misalignment to both the subject matter and the assessment objective/s.

Refer to *QCE and QCIA policy and procedures handbook v5.0*, Section 9.6.

Percentage of instruments endorsed in Application 1

Number of instruments submitted	IA1	IA2	IA3
Total number of instruments	423	423	420
Percentage endorsed in Application 1	61%	26%	39%

Confirmation

Confirmation is the quality assurance process based on the attribute of reliability. The QCAA uses provisional criterion marks determined by teachers to identify the samples of student responses that schools are required to submit for confirmation.

Confirmation samples are representative of the school's decisions about the quality of student work in relation to the instrument-specific marking guide (ISMG), and are used to make decisions about the cohort's results.

Refer to *QCE and QCIA policy and procedures handbook v5.0*, Section 9.7.

The following table includes the percentage agreement between the provisional marks and confirmed marks by assessment instrument. The Assessment decisions section of this report for each assessment instrument identifies the agreement trends between provisional and confirmed marks by criterion.

Number of samples reviewed and percentage agreement

IA	Number of schools	Number of samples requested	Number of additional samples requested	Percentage agreement with provisional marks
1	412	3,010	106	85.68%
2	411	2,370	0	98.78%
3	411	2,356	0	98.78%

Internal assessment 1 (IA1)

Problem-solving and modelling task — extended response (20%)

This assessment focuses on the interpretation, analysis and evaluation of ideas and information. It is an independent task responding to a particular situation or stimuli. While students may undertake some research in the writing of the problem-solving and modelling task, it is not the focus of this technique. This assessment occurs over an extended and defined period of time. Students will use class time and their own time to develop a response.

The problem-solving and modelling task must use subject matter from one or both of the following topics in Unit 3:

- Topic 2: Further differentiation and applications 2
- Topic 3: Integrals.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	87
Authentication	39
Authenticity	20
Item construction	22
Scope and scale	34

*Each priority might contain up to four assessment practices.

Total number of submissions: 423.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided realistic and stimulating contexts to students, that highlighted a real-life application of mathematics, e.g. designing a skate park, designing a sculpture, modelling the way in which the rate of medication in the bloodstream is changing, or designing a section of BMX bike track
- provided stimulus material relevant to the task (e.g. diagrams, sample datasets and pictures) that did not require students to undertake extensive research and did not limit the task complexity

- provided opportunities for students to use technology beyond simple computations and word-processing, e.g. encouraging the use of free geometry software or applications to synthesise models
- provided opportunities to use an independent approach to address all the stages of problem-solving and mathematical modelling, e.g. a task that required students to create their own designs (logo, bike track or roller-coaster are examples) and used unique functions for each student.

Practices to strengthen

It is recommended that assessment instruments:

- avoid task instructions or contexts that require students to undertake extensive research
- assess subject matter within the scope of the syllabus, e.g. although volumes of revolution is an application of integration, students studying Specialist Mathematics may receive an unfair advantage because it is subject matter that is specific to Specialist Mathematics. Therefore, volumes of revolution should not be a required procedure in a Mathematical Methods assessment instrument
- include checkpoints that reflect the school's assessment policy and clearly indicate when and how teachers provide feedback on one draft.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	7
Language	32
Layout	6
Transparency	11

*Each priority might contain up to four assessment practices.

Total number of submissions: 423.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- provided clear and concise instructions on what was expected from students in completing the assessment instrument
- featured a clear layout, with appropriate page breaks, where text and other items (e.g. pictures, MathType, tables and graphs) appeared aligned and in their entirety on the page.

Practices to strengthen

It is recommended that assessment instruments:

- feature clear, concise language and avoid the use of jargon that may impede understanding of the subject matter or task requirements

- are reviewed before submission to check for typographical, grammatical, punctuation and spelling errors. Using the 'Print preview' function within the Endorsement application (app) to check the layout of the assessment instrument and ensure all images that may be used as stimulus or problem-solving models do not break across pages
- contain a scenario or context with minimal distractors, e.g. in a task that required students to design a waterslide, providing the history of waterslides within the context would be a distractor.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Formulate	92.96%	6.31%	0.24%	0.49%
2	Solve	93.45%	5.1%	1.46%	0%
3	Evaluate and verify	93.69%	5.1%	1.21%	0%
4	Communicate	98.3%	0.97%	0.73%	0%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- for the Communicate criterion, judgments about the
  - 'coherent and concise organisation of the response' were accurately identified where responses organised the reports into appropriate sections (i.e. introduction, body and conclusion) with enough information to be able to read the report independently of the task sheet
  - correct use of technical and procedural vocabulary and conventions were identified through teacher annotations indicating where the student had used correct vocabulary and conventions
- for the Solve criterion, judgments about the
  - 'application of mathematical concepts and techniques relevant to the task' were supported by teacher annotations that identified where evidence of choices were made, or when mastery or insight was shown in the application of concepts and techniques, i.e. discerning, which is characteristic of the top performance-level descriptor
  - 'use of complex procedures to reach a valid solution', when the response clearly identified where many different and interconnected processes were combined together in the procedure.

Samples of effective practices

The following excerpts demonstrate both the:

- documentation of appropriate assumptions and relevant observations in the Formulate criterion
- use of teacher annotations to identify judgments made in the Evaluate and verify criterion.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Excerpt 1

OBSERVATIONS		ASSUMPTIONS	
O1	Data points reveal an initial rapid increase until a maximum, and then a less steep drop-off of drug concentration. If the concentration grows quickly and declines slowly, this is favourable, as the drug will be effective for a longer period of time (above 40%). Within a Surge function, this is an expected trait of its curve, used to "accurately represent the way a drug interacts in the bloodstream" (University of South Florida, 2021).	A1	It is assumed that the second dose will not cause the patient to overdose, and assumed the drug cannot be overdosed on. This is an important consideration, as below 40% concentration, a second dose must be administered. However, Penicillin remains in the body, and it must be certain that adding a second full dose to the remaining concentration will not harm the patient. This assumption is support by the NHS – "accidentally taking 1 extra antibiotic dose is unlikely to cause serious harm" (NHS, 2022).
O2	It is observed that accurate drug concentration data and rates of absorption is critical in drug administration, as "incorrect dosages or frequency put the patient at high risk" (BMJ, 2002). Thus, Surge function use must be accurate, with minimal uncertainty. This has implications when creating an accurate and reliable solution, and considering error.	A2	It is assumed that the data used to develop the Surge function is from one patient only. This is relevant to conclusions drawn from the Surge function and its critical elements, as Penicillin's effects "vary between people or a person on different occasions" (OOHC Toolbox, 2020).
O3	It is observed that Penicillin MM is only effective at ≥ 40% concentration. This influences the drug absorption graph (Graph 1), as every hour where the concentration of Penicillin is below this value, the drug can be considered ineffective and its value disregarded.	A3	It must be assumed that the patient does not have underlying health abnormalities or undertake any action that may disrupt Penicillin absorption (also taking other medication), as "antibiotics may interact with other drugs and be less effective, like blood thinners and antacids" (Cleveland Clinic, 2016).
O4	The Surge function does not reach zero after the initial origin starting point – a $y = 0$ asymptote appears. It is an exponential function with an index, therefore can never equal 0 (University of Minnesota, 2004). Therefore, the Surge function displays the Penicillin concentration getting very close to 0, but never theoretically reaches it.	A4	It is assumed that bloodstream Penicillin concentration levels after 1 dose are safe, and that 1 dosage is the optimum amount within the bloodstream regarding both effectiveness and safety. This is assumed, as 1 dose was the original amount recommended to this patient by a doctor, and "health care providers are the most knowledgeable when it comes to prescriptions" (fda.gov, 2016).

Good

Good.

V. good documentation of observations + assumptions.

Excerpt 2

$$f(x) = \frac{6.79 \times 10^7}{1 + e^{-0.058(x-1973.4)}} + 6.74 \times 10^6$$

$$f(x) = 6.79 \times 10^7 \left(1 + e^{-0.058(x-1973.4)}\right)^{-1} + 6.74 \times 10^6 \quad \checkmark$$

$$u = 1 + e^{-0.058(x-1973.4)} \qquad u' = -0.064e^{-0.058(x-1973.4)}$$

$$f'(x) = -1 \times 6.79 \times 10^7 \times u' \times (u)^{-1-1} + 0$$

$$f'(x) = -1 \times 6.42 \times 10^7 \times -0.058e^{-0.058(x-1973.4)} \times \left(1 + e^{-0.058(x-1973.4)}\right)^{-2}$$

$$f'(x) = 3723600e^{-0.058(x-1973.4)}\left(1 + e^{-0.058(x-1973.4)}\right)^{-2} \quad \checkmark$$

By analysing the logistic function above, the -0.058 represents the logistic growth rate or rate of change of the curve. The 1973.4 indicates the sigmoid midpoint (inflection point). ✓ Justify
As such, at $f'(1973.4)$ the maximum growth rate is expected. This is because mathematically, when $x = 1973.4$,

6

the $e^{-0.058(x-1973.4)}$ becomes e^0 *or* 1. Therefore, when multiplying $3723600 \times e^{-0.058(x-1973.4)} \times \left(1 + e^{-0.058(x-1973.4)}\right)^{-2}$, or $3723600 \times 1 \times (1+1)^{-2}$ it produces the largest value. ✓ Justify Therefore, as a value approach $x = 1973.4$, the rate of change increases and vice-versa. Rounding to nearest year, it can be assumed that during 1973, the population growth was the largest. The data is between 1880 and 2022. Since, 1880 is furthest value from 1973.4, it is expected that this year would have the lowest rate of change. Therefore, the maximum rate of change is:

$$f'(1973) = 3{,}723{,}600e^{-0.058(x-1973.4)}\left(1 + e^{-0.058(x-1973.4)}\right)^{-2} \quad \checkmark$$

$$f'(1973) = 930{,}774.75$$

$$f'(1973) \approx 930{,}775 \quad \checkmark \text{ valid solution}$$

Very well justified!!

The minimum rate of change:

$$f'(1880) = 3{,}723{,}600e^{-0.058(x-1973.4)}\left(1 + e^{-0.058(x-1973.4)}\right)^{-2} \quad \checkmark$$

$$f'(1880) = 16{,}385.34$$

$$f'(1880) \approx 16{,}385 \quad \checkmark \text{ valid solution}$$

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- when making judgments for the Formulate criterion and the Evaluate and verify criterion, the difference between documentation and statements of assumptions, observations and/or strengths and limitations is clearly identified. Further, responses that exhibit the top performance-level descriptor of
  - ‘documentation of appropriate assumptions’ can be seen to link each assumption to the task and/or model/solution and explain the necessity of making the assumption, or a possible effect of not making it
  - ‘documentation of relevant observations’ support each observation with evidence. This could include explaining where the observations come from, the reliability of the observations or features of the observations relevant to the model/solution
  - ‘documentation of relevant strengths and limitations of the solutions and/or model’ explain the supporting elements of both the strengths and limitations, e.g. the strength of a model that encompasses all data points provided could be how it can effectively be used to

interpolate to reach conclusions. Conversely, a limitation may be explained by identifying how a model is only effective within the given domain and extrapolation beyond this domain would yield unreliable results

- when making judgments for the Evaluate and verify criterion, the 'evaluation of reasonableness of solutions' must include consideration of results, observations and assumptions. Specifically, consideration of
  - results could include a check of validity of the model/solution and/or demonstration of how the solution appropriately addresses the needs of the task. Evidence of this may be found in other parts of the report and should be appropriately annotated by teachers to highlight this
  - assumptions and observations should explore the effect of these on the solution, an evaluation of the significance of the assumption/observation and/or whether the same solution could have been reached without this.

Additional advice

- When responses exceed the specified conditions for word length and/or page count, teachers must annotate the response to indicate how the school's assessment policy has been applied. Judgments should only be made on 2000 words/10 pages. See the *QCE and QCIA policy and procedures handbook v5.0* (Section 8.2.6) for advice on managing response length. Communication with the QCAA is recommended if further advice is required about managing responses that exceed the allowable length.

Internal assessment 2 (IA2)

Examination — short response (15%)

This examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 3 topics. Student responses must be completed individually, under supervised conditions and in a set timeframe.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	298
Authentication	0
Authenticity	0
Item construction	16
Scope and scale	35

*Each priority might contain up to four assessment practices.

Total number of submissions: 423.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided questions that representatively sampled subject matter for all Unit 3 topics
- explicitly provided opportunities for students to demonstrate all assessment objectives, including Assessment objective 4: evaluate the reasonableness of solutions, and allocated marks in the marking scheme for assessment of this objective, e.g. allocated a mark for evaluating whether an obtained result meets the requirements as stated in the context of the question
- featured a balance of items requiring both technology-free and technology-active responses
- comprised an appropriate number of questions that matched the degree-of-difficulty specifications as defined in the syllabus (Section 4.6.2) and allowed students to respond within the syllabus time conditions, i.e. 120 minutes plus 5 minutes perusal.

Practices to strengthen

It is recommended that assessment instruments:

- require students to demonstrate knowledge and understanding of Unit 3 subject matter and do not solely assess subject matter from Units 1, 2 or 4, e.g. simplifying $\log_3 27$ is part of Unit 2 and calculating $\frac{d^2y}{dx^2}$ is part of Unit 4

- provide a correct marking scheme that clearly indicates how marks will be allocated, including the allocation of marks to reflect Assessment objective 4: evaluate the reasonableness of solutions
- correctly align complex unfamiliar questions to the degree-of-difficulty specification (Syllabus section 4.6.2) where relationships and interactions have a number of elements, and all the information to solve the problem is not immediately identifiable, e.g. explicitly providing a function to derive or integrate or explicitly requesting students to 'calculate a' may identify the required procedure to solve a complex unfamiliar question
- provide simple familiar and complex familiar questions that appropriately match the degree-of-difficulty specification (Syllabus section 4.6.2), e.g. integrating a simple velocity function to determine displacement with given initial conditions may be considered simple familiar.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	9
Language	76
Layout	18
Transparency	29

*Each priority might contain up to four assessment practices.

Total number of submissions: 423.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- featured a clear layout, with appropriate page breaks, so that questions appeared in their entirety on one page
- provided sufficient space for student responses
- used consistent formatting, including the use of MathType (or appropriate equation software), throughout the instrument
- modelled correct mathematical conventions, including the use of correct mathematical notation such as including dx when requiring an integral, e.g. $\int e^x \, dx$.

Practices to strengthen

It is recommended that assessment instruments:

- feature questions of an appropriate length so students are not disadvantaged by having to read excessive or unnecessary information, as this may affect a student's ability to solve the problem within the allocated time
- are reviewed using the 'Print preview' function prior to submission to check for layout, typographical, grammatical, punctuation and spelling errors.

Additional advice

- It is recommended that schools develop complex unfamiliar questions that are significantly different in context from the task provided to students in the school's IA1 problem-solving and modelling task to avoid compromising the unfamiliarity of the questions.
- It is recommended that schools watch the Maths moments videos (Writing examinations), found under Resources in the Syllabuses app on the QCAA Portal.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	98.78%	0.49%	0.49%	0.24%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- detailed marking schemes were submitted for confirmation that clearly indicated where marks were allocated within a question. This included allocation of part marks
- total marks achieved by a student, along with the resulting percentage and alignment to the percentage cut-offs on the ISMG, were clearly indicated on the student response. This included where schools used total marks to calculate precise percentages before applying the ISMG, e.g. 80.001% would be awarded 13 out of 15 rather than 12, being > 80%.

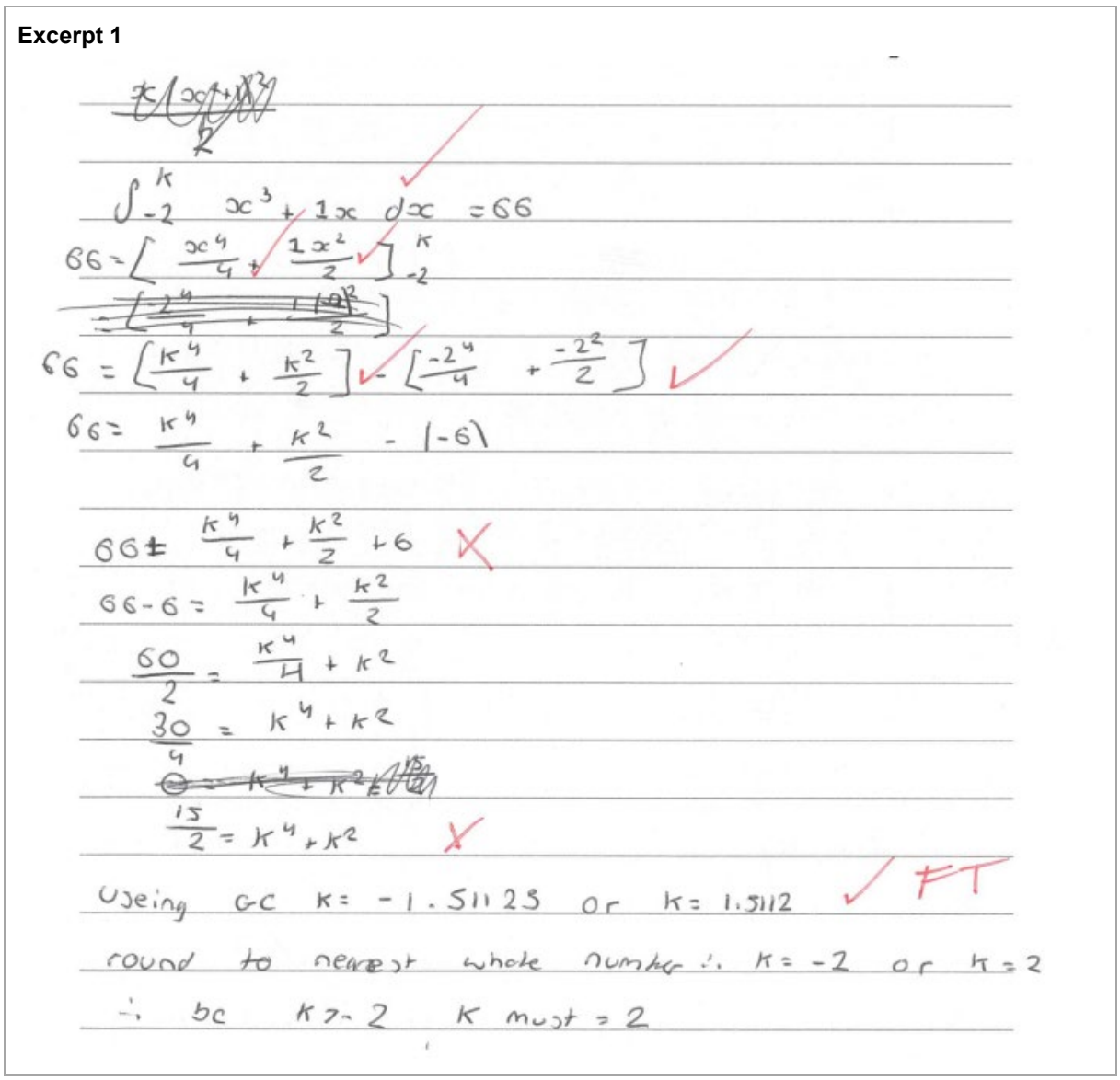
Samples of effective practices

The following excerpts demonstrate the clear teacher annotation of marks allocated including:

- follow-through marks after student error
- an annotation regarding the omission of the reasonableness of a solution in a response
- an ISMG that clearly shows the marks allocated, the percentage and the alignment to the percentage cut-off.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Excerpt 1


$\int_{-2}^{k} x^3 + 1x \, dx = 66$
$66 = \left[\frac{x^4}{4} + \frac{1x^2}{2}\right]_{-2}^{k}$
$66 = \left[\frac{k^4}{4} + \frac{k^2}{2}\right] - \left[\frac{-2^4}{4} + \frac{-2^2}{2}\right]$
$66 = \frac{k^4}{4} + \frac{k^2}{2} - (-6)$
$66 \pm \frac{k^4}{4} + \frac{k^2}{2} + 6$
$66 - 6 = \frac{k^4}{4} + \frac{k^2}{2}$
$\frac{60}{2} = \frac{k^4}{4} + k^2$
$\frac{30}{4} = k^4 + k^2$
$\frac{15}{2} = k^4 + k^2$
Useing GC $k = -1.5112$ or $k = 1.5112$ FT
round to nearest whole number $\therefore k = -2$ or $k = 2$
$\therefore$ bc $k > -2$ k must = 2

Excerpt 2

$f'(x) = 3x^2 + k$

at turning point, $f'(x) = 0$.

$\therefore f'(-2) = 0$

$3(-2)^2 + k = 0$

$12 + k = 0$

$k = -12$ ✓½

$\therefore f'(x) = 3x^2 - 12$.

$f(x) = \int f'(x)$

$= \int 3x^2 - 12\,dx$

$f(x) = \frac{3}{3}x^3 - 12x + c$

$= x^3 - 12x + c$ ✓½

at $x = -2$, $y = 6$.

6 ✓½ $= (-2)^3 - 12(-2) + c$

$6 = -8 + 24 + c$

$c = -10$. ✓½

T.P $(-2, 6)$

$x = -2 \quad y = 6$.

$\therefore f(x)$

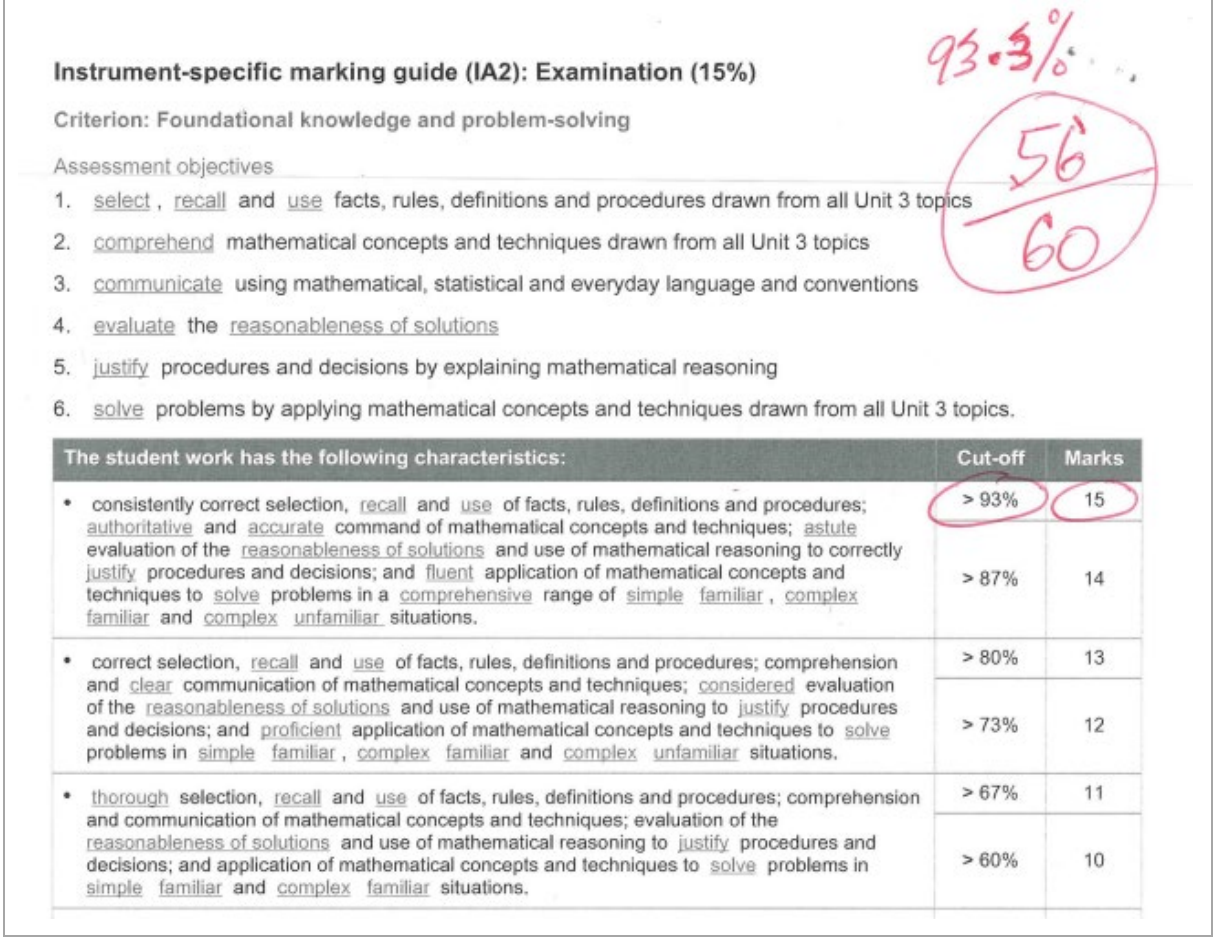
$= x^3 - 12x - 10$ ✓½

Reasonableness ?

X.

2½

Excerpt 3



Instrument-specific marking guide (IA2): Examination (15%)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- responses are annotated clearly by teachers, indicating where marks were awarded, e.g. in situations where follow-through marks were given, how these were distributed must be clearly indicated on the response
- when a student has responded in a different way to that expected in the marking scheme, the student solution is annotated to show how marks were awarded based on the merit of the response.

Additional advice

- If a comparable assessment instrument is administered to a sampled student, then the school must indicate this in Student Management on the individual student's learning account and in the Confirmation app. To assist with this, comparable assessments should be developed in the Endorsement app to ensure the correct examination and its matching marking scheme are available for the confirmation review (see the *QCE and QCIA policy and procedures handbook v5.0,* Section 7.4). For further information, see the quick-step guide *Upload samples* in the Help section of the Confirmation app.
- Schools are required to submit samples of student assessment responses for review during confirmation. Submissions should align to the relevant confirmation submission information (*QCE and QCIA policy and procedures handbook v5.0*, Section 9.7.1). The *Confirmation*

submission information for Mathematical Methods is available under Resources in the Syllabuses app on the QCAA Portal. Before submission of responses for confirmation, schools are advised to check that all scanning of student work has been completed without error. This includes ensuring that

- no pages are missing from the response
- all pages are visible and easy to read
- the submitted response matches the student selected.

Internal assessment 3 (IA3)

Examination — short response (15%)

This examination assesses the application of a range of cognitions to a number of items, drawn from Unit 4 Topics 1–5. Student responses must be completed individually, under supervised conditions and in a set timeframe.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	213
Authentication	0
Authenticity	1
Item construction	16
Scope and scale	53

*Each priority might contain up to four assessment practices.

Total number of submissions: 420.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- featured a balance of technology-free and technology-active questions. In instruments that allowed access to technology for the entire paper, appropriate cues were provided to direct students to use an algebraic procedure, e.g. ‘Use calculus methods to determine …’
- included questions that explicitly provided opportunities to address all assessment objectives, including Assessment objective 4: evaluate the reasonableness of solutions, e.g. allocating a mark for evaluating whether an obtained result met the requirements as stated in the context of the question
- provided a correct marking scheme with a clear indication of how marks would be awarded in each question, reflecting the intended learning across the topics of Unit 4.

Practices to strengthen

It is recommended that assessment instruments:

- provide a balance of questions from all topics in Unit 4
- correctly align complex unfamiliar questions to the degree-of-difficulty specification in the syllabus (Section 5.8.1), where relationships and interactions have a number of elements, and

questions are not scaffolded so all the information to solve the problem is not immediately identifiable, e.g. avoiding cues in the problem such as ‘Use the second derivative test to …’

- include a correct marking scheme that shows where marks are allocated for the use of the first and second derivative when solving optimisation problems.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	6
Language	56
Layout	10
Transparency	19

*Each priority might contain up to four assessment practices.

Total number of submissions: 420.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- featured a clear layout, with appropriate page breaks, so that questions appeared in their entirety on one page
- provided sufficient space for student responses, where appropriate
- used consistent formatting, including the use of MathType (or other equation software), throughout the instrument
- modelled correct language conventions, including the use of correct grammar, punctuation and mathematical notation.

Practices to strengthen

It is recommended that assessment instruments:

- use the language of the assessment objectives, e.g. ‘evaluate the reasonableness of solutions’ instead of ‘comment on the reasonableness’
- are reviewed using the ‘Print preview’ button, prior to submission, to check for typographical, grammatical, punctuation and spelling errors.

Additional advice

- When constructing complex unfamiliar opportunities for optimisation problems, it is recommended that schools ensure they allocate marks for using the second derivative to check the nature of local maximums and minimums so that the question addresses the intended learning of Topic 1 in Unit 4. Schools that verify the nature turning points of a polynomial or power function using the first derivative test only are using subject matter assessed in Unit 2, which should not be a focus in this instrument.

- It is recommended that schools watch the Maths moments videos (Writing examinations), found under Resources in the Syllabuses app on the QCAA Portal.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	98.78%	0.73%	0.49%	0%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- the marking scheme indicated the use of full marks or half marks for questions and was applied consistently across the instrument and student samples. This was evident when the allocation of marks on a student's examination corresponded with the marking scheme, e.g. a question worth 3 marks had the appropriate number of ticks
- the greater than percentage cut-offs indicated on the ISMG were correctly applied to the calculated percentages and the correct provisional marks awarded, e.g. a total mark of 36/60 is exactly 60%. This would be awarded 9/15, which is within the cut-off of > 53% rather than 10/15.

Samples of effective practices

The following excerpts demonstrate the use of extra pages that are clearly indicated by the student, teacher annotations to support the allocated marks, and the clear use of ticks that can be tallied to determine the total marks allocated.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Excerpt 1

ball 40m is at least 0.95.

Binomial.

$\hat{p} = 0.8$ ✓ $z = 1.96$

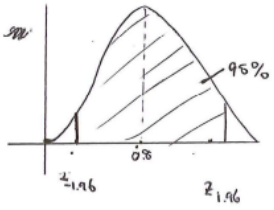
$\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1$ ×

$0.8 - 1.96\sqrt{\frac{0.8 \times 0.2}{n}} = 1$

$-1.96\sqrt{\frac{0.8 \times 0.2}{n}} = 0.2$

$\sqrt{\frac{0.8 \times 0.2}{n}} = \frac{0.2}{-1.96}$

n = undefined



$\mu = np$

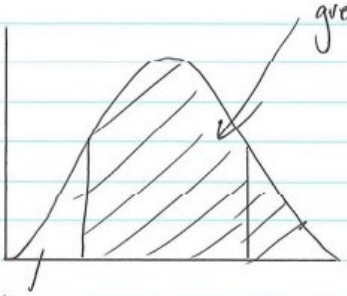
SD =

$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$

$P(X = 1) = \binom{n}{1} p^1 (1-p)^{n-1}$

$= \binom{n}{1} (0.8)(0.2)^{n-1}$

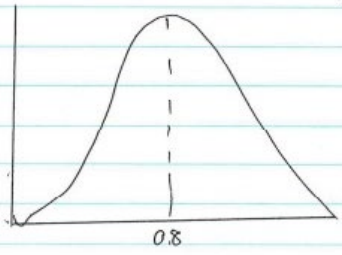
See extra paper



×

$\hat{p} = 0.8$ $q = 0.2$ $z = 1.96$

kick ball over 80% of time



$np \geq 1$

$n \times 0.8 \geq 1$

$n \geq 1.25$

∴ at least 2 people are required for the one of them to kick over 40m

$np - z\sqrt{np(1-p)} = 1$

$0.8n - 1.96\sqrt{n \times 0.8 \times 0.2} = 1$ ×

From GC Solve N

$n = 2.93$

∴ at least 3 people will be required to ensure 1 of them will kick over 40m 95% of the time

Excerpt 2

$\hat{p}_1 = \hat{p}_2$ as the proportion of male are the same.

$\therefore \hat{p} = \hat{p}_1 = \hat{p}_2$ ✓

$\left(\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n_2}}\right) = \left(\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1}}\right)$ ∵ 99% CI for both

$z_2 = \frac{1}{4} z_1$

$4z_2 = z_1$

$z_2\sqrt{\frac{\hat{p}(1-\hat{p})}{n_2}} = z_1\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1}}$

$4z_1\sqrt{\frac{\hat{p}(1-\hat{p})}{n_2}} = 4z_1\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1}}$ ✓ sets up

$4\sqrt{\frac{\hat{p}(1-\hat{p})}{n_2}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1}}$ ✓ working

$16\frac{\hat{p}(1-\hat{p})}{n_2} = \frac{\hat{p}(1-\hat{p})}{n_1}$

$\frac{16}{n_2} = \frac{1}{n_1}$

$16 = \frac{n_2}{n_1}$ ✓

Excerpt 3

a) stationary points when $\frac{dy}{dx} = 0$

$f(x) = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2 = 0$ ✓✓

$3x(2-x) = 0$

$3x = 0 \quad 2-x = 0$

$x = 0$ ✓ $\quad x = 2$ ✓

sub into original function to find y

$f(0) = 3(0)^2 - (0)^3$

$= 0$

$f(2) = 3(2)^2 - (2)^3$

$= 12 - 8$

$= 4$

SP: (0,0) ✓ and (2,4) ✓

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- teachers annotate the student response by identifying
  - follow-through marks — in partially correct responses where an error occurs and annotations allow confirmers to see clearly where follow-through marks are awarded for subsequent correct working
  - implied marks — where steps in the marking guide are not explicitly stated but can be implied through further working
  - alternative solutions — where marks are awarded for a valid method of solution that is significantly different from the marking guide.

Additional advice

- If a comparable assessment instrument is administered to a sampled student, then the school must indicate this in Student Management on the individual student's learning account and in the Confirmation app. To assist with this, comparable assessments should be developed in the Endorsement app to ensure the correct examination and its matching marking scheme are available for the confirmation review (*QCE and QCIA policy and procedures handbook v5.0,* Section 7.4). For further information, see the quick-step guide *Upload samples* in the Help section of the Confirmation app.
- Teachers should use the mark boxes to identify the marks awarded for a question in a consistent and easily identifiable location. This option is available as a check box when printing the instrument from the Endorsement app.
- When additional pages are used to respond to a question, students should be instructed to annotate work indicating that additional pages are used and ensure that the additional pages have questions clearly labelled. Teachers should annotate the additional pages with ticks or comments to indicate that these pages have been considered when awarding the marks for the question.
- Teachers should annotate the ISMG with the total marks awarded, the total marks possible and the calculated percentage (*QCE and QCIA policy and procedures handbook v5.0,* Section 9.7.1 — 'Marking ISMGs'). This is done by accurately determining the percentage obtained, using precise values without rounding, to ensure errors in allocating percentage cut-offs do not occur.

External assessment

External assessment (EA) is developed and marked by the QCAA. The external assessment for a subject is common to all schools and administered under the same conditions, at the same time, on the same day.

Examination — short response (50%)

Assessment design

The assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the syllabus.
The examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (45 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (45 marks).

The examination assessed subject matter from Units 3 and 4. Questions were derived from the contexts of:

- The logarithmic function 2
- Further differentiation and applications 2 and 3
- Integrals
- Trigonometric functions 2
- Discrete random variables 2
- Continuous random variables and the normal distribution
- Interval estimates for proportions.

The assessment required students to respond to multiple choice and short response questions.

Assessment decisions

Assessment decisions are made by markers by matching student responses to the external assessment marking guide (EAMG). The external assessment papers and the EAMG are published in the year after they are administered.

Multiple choice question responses

There were 10 multiple choice questions in Paper 1.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Mathematical Methods (General): Paper 1

Question	A	B	C	D
1	5.88	16.96	**69.54**	7.23
2	2.92	10.76	1.18	**84.9**
3	**67.14**	19.44	9.11	3.76
4	10.15	19.66	**61.84**	7.94
5	13.62	3.13	15.7	**67.09**
6	**59.87**	16.1	15.89	7.36
7	8.01	13.51	16.63	**61.23**
8	17.88	**64.13**	7.69	9.57
9	21.33	**47.11**	21.02	9.82
10	6.03	13.6	**60.3**	19.1

There were 10 multiple choice questions in Paper 2.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Mathematical Methods (General): Paper 2

Question	A	B	C	D
1	7.21	**83.19**	3.42	5.85
2	3.57	1.73	**89.67**	4.79
3	**62.26**	9.79	22.93	3.62
4	3.33	6.64	8.51	**81.1**
5	3.89	1.73	**89.86**	4.23
6	12.18	23.54	10.53	**51.85**
7	**77.42**	11.48	7.18	3.15
8	18.27	14.97	12.68	**53.31**
9	**40.38**	32.4	12.1	13.78
10	5.93	**67.57**	16.09	9.63

Effective practices

Overall, students responded well to:

- opportunities that used the definite integral to calculate areas under or between curves, with and without technology, and opportunities where integration was recognised as the reverse of differentiation
- contexts that required understanding of binomial random variables and parameters to calculate probabilities associated with the binomial distribution

- the opportunity to use technology to determine an approximate confidence interval estimate for population proportion and interpret the reasonableness of a given claim based on this interval
- opportunities that used trigonometric rules (e.g. sine rule, cosine rule and area formula) to solve problems that involved straight side lengths, angles between sides and enclosed areas.

Samples of effective practices

Short response

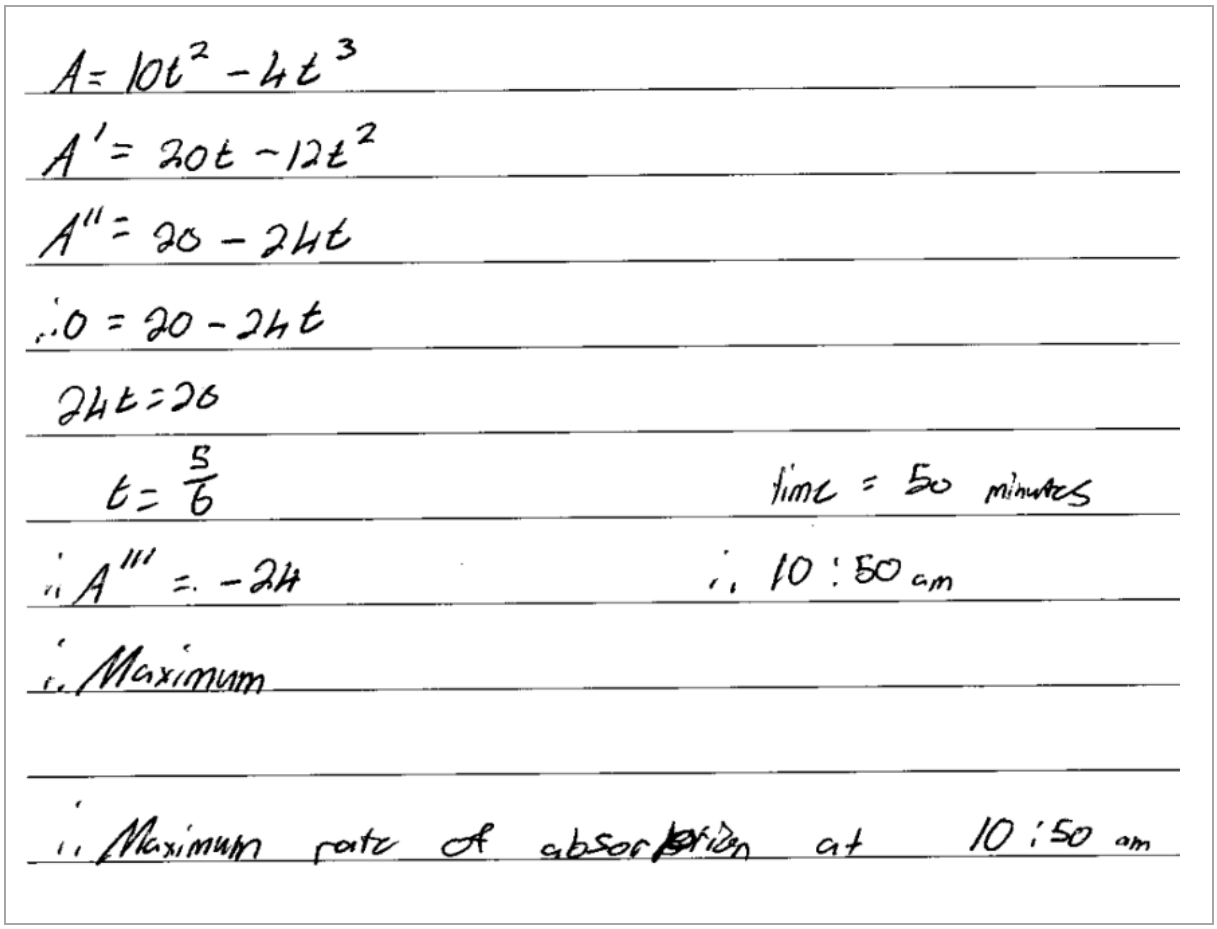
The following excerpt is from Question 17 in Paper 1. It required students to use differential calculus techniques to determine the time when the maximum rate of absorption was reached and to verify that a maximum rate occurred at a given time.

Effective student responses:

- understood that the first derivative was the quantity to be maximised
- recognised that the second derivative needed to be equated to zero to determine the required time
- performed a derivative test to verify that the rate was a maximum.

This excerpt has been included:

- to demonstrate the use of multiple derivatives to solve the question
- as it provides evidence of using a derivative test to verify a maximum rate
- to demonstrate the use of a third derivative as required in this context.



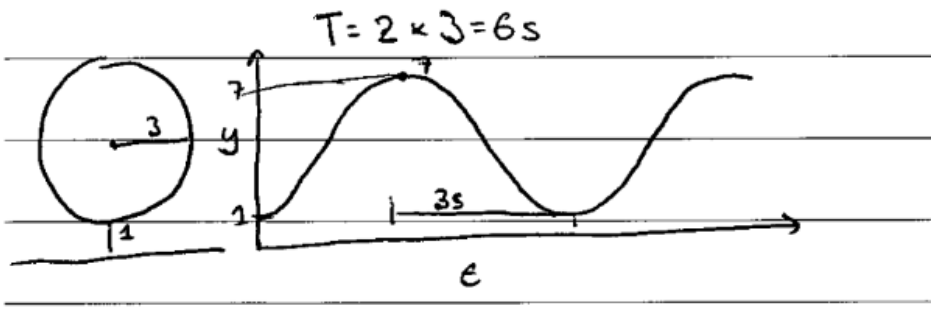
The following excerpt is from Question 18 in Paper 1. It required students to model the repetitive vertical position of a carriage on a Ferris wheel with a trigonometric function. Students were then required to use calculus techniques to evaluate the reasonableness of a claim regarding the vertical acceleration.

Effective student responses:

- recognised that the periodic nature of the motion could be modelled with a trigonometric equation
- determined a trigonometric equation to model the vertical position of the carriage
- used the second derivative to establish an expression for the vertical acceleration.

This excerpt has been included:

- to show the development of a trigonometric model using the scenario information
- as it provides evidence of how the maximum acceleration was determined
- to show how the reasonableness of the claim was evaluated using approximated numerical values from the developed model compared to the value claimed.



Model vertical motion with $y = A\cos(Bx) + D$

$r = 3m \therefore A = 3$

$D = 1 + 3 = 4m$

$B = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$

$y = -3\cos\left(\frac{\pi}{3}x\right) + 4$

$\frac{dy}{dx} = \pi \sin\left(\frac{\pi}{3}x\right)$

$\frac{d^2y}{dx^2} = \frac{\pi^2}{3}\cos\left(\frac{\pi}{3}x\right)$

Maximum occurs at $x = 0$ (cosine)

$\frac{d^2y}{dx^2} = \frac{\pi^2}{3}\cos(0)$

$= \frac{\pi^2}{3}\ m/s^2$

Claim states that $\frac{\pi^2}{3} > 9.8 \times \frac{1}{2}$

$\frac{\pi^2}{3} \approx 3 \quad 3 > 4.9$ incorrect

$\therefore$ The claim is not reasonable

The following excerpt is from Question 18 in Paper 2. It required students to use mathematical modelling to determine an equation for a parabolic sheet of glass and hence determine the area and mass of a modified parabolic window.

Effective student responses:

- used the dimensions of the window provided to define coordinates for use in representing the window shape in the form of a quadratic function
- determined the area beneath the quadratic function either with or without technology
- recognised a missing section to be triangular in shape and subsequently used the cosine rule to determine an angle between sides and calculated the triangular area

- determined the remaining glass area and hence the mass of the window
- provided evidence of logical organisation in the sequential development of the response.

This excerpt has been included:

- to demonstrate the introduction of suitable coordinates
- as it provides evidence of the development of a quadratic model
- to demonstrate the logical organisation of the key steps in the response, including an attempted procedure that was not fit for purpose and how this was clearly removed and replaced by a more appropriate method.

Consider parabola

let leftmost edge be at (0,0), highest point be (4,12), rightmost edge at (8,0). → x-intercepts at (0,12) & (8,0).

x-intercept form $y = a(x-h)(x-k)$

$\therefore y = a(x-0)(x-8)$

substitute (4,12) $\therefore 12 = a(4)(-4)$

$\therefore a = \frac{-12}{16} = -\frac{3}{4}$

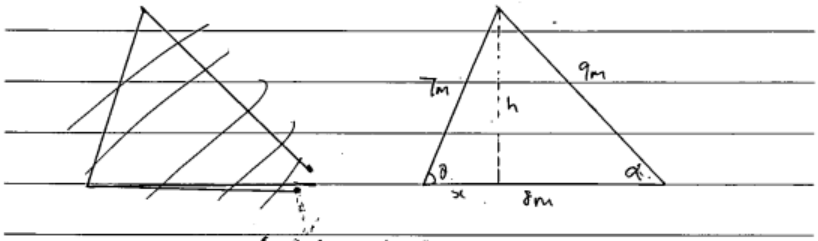
$\therefore y = -\frac{3}{4}x(x-8)$

Find area contained by parabola and x-axis.

$\therefore A = \int_0^8 -0.75x(x-8)\,dx$

$= 64\ m^2$ (Using graphics calculator)

Consider triangle cutout



Using Sine Rule, $\frac{\sin A}{a} = \frac{\sin B}{b}$,

$\frac{\sin\theta}{9} = \frac{\sin\alpha}{7}$ ①

$\sin\theta = \frac{h}{7}$, $\sin\alpha = \frac{h}{9}$

$\therefore \frac{\frac{h}{7}}{9} = \frac{\frac{h}{9}}{7}$

Using Cosine Rule, $a^2 = b^2 + c^2 - 2bc\cos A$

$\therefore 9^2 = 7^2 + 8^2 - 2\times 7\times 8\cos\theta$

Using Graphics calculator, $\theta \approx 73.4°$.

$A_\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}\times 7\times 8\times \sin 73.4$

$\approx 26.83\ m^2$

$\therefore$ Area of unusual window $= 64 - 26.83$

$\approx 37.17\ m^2$

Mass $= 5.6 \times 37.17$

≈ 208.15 kg

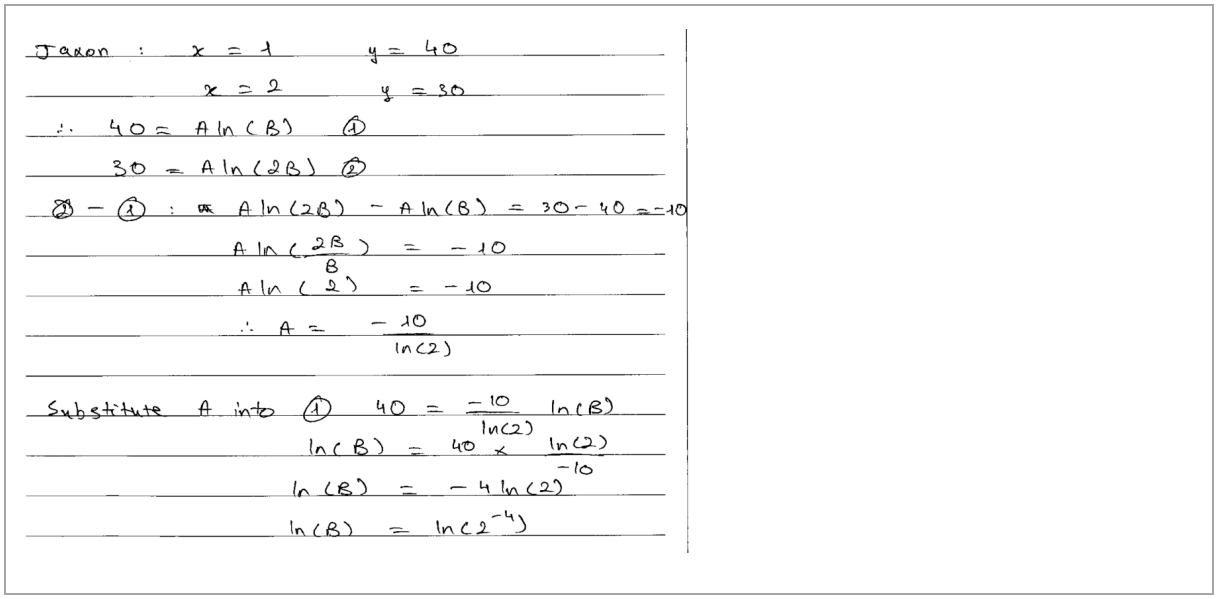
The following excerpt is from Question 19 in Paper 1. It required students to develop two logarithmic models, sum the models and use algebraic techniques to determine a simplified expression.

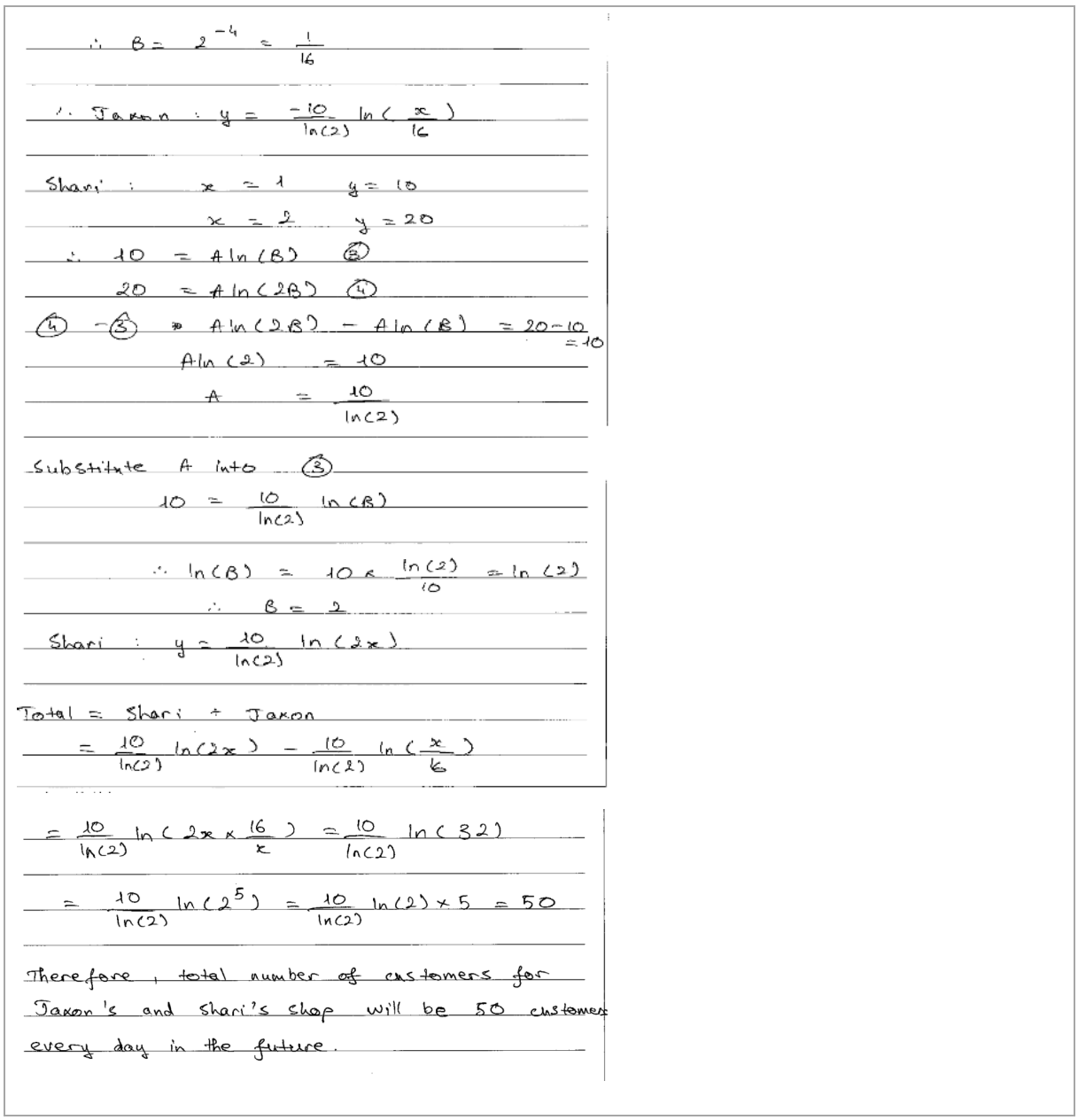
Effective student responses:

- determined the different constants for each of the models
- effectively used logarithmic laws and definitions to simplify a compound expression involving logarithms
- provided a justified decision regarding a statement presented in the question.

This excerpt has been included:

- to demonstrate a high-level algebraic response communicating key steps
- as it provides evidence of effectively using logarithmic laws and definitions to develop a simplified numerical value
- as it provides a justified decision.





Practices to strengthen

When preparing students for external assessment, it is recommended that teachers consider:

- providing more opportunities for students to use applications of the second derivative in varied contexts. In Paper 1, Question 17, many students did not realise that it was the rate of absorption that was required to be maximised. Many students performed the second derivative test on the function provided in the question instead. Being a well-rehearsed procedure, it is likely many students incorrectly assumed that this standard procedure was sufficient to solve the problem. However, more careful reading of the context would have indicated the need to find the derivative of the given function to obtain the rate formula. Then, applying the derivative test for finding a maximum in this context required the use of the third derivative
- increasing students' opportunities to explore a broad range of contexts that require the development of models to solve the problems. In Paper 1, Question 18, the context required a trigonometric model to be developed that could be used to determine the vertical acceleration of a carriage on a Ferris wheel. In Paper 1, Question 19, two logarithmic models were required

to algebraically determine a simplified expression for the sum of the models. In Paper 2, Question 18, the parabolic shape of a window needed to be modelled mathematically to enable the area and hence mass of a glass window to be calculated

- providing more opportunities for students to practise Objective 4: evaluate the reasonableness of solutions. In Paper 1, Question 18, the evaluation of the claim needed to be referenced against the calculation obtained for acceleration. In Paper 2, Question 19, the evaluation of the prediction required the comparison of a calculated distance to the suggested distance.

Additional advice

- Teachers should provide opportunities for students to develop their algebraic skills in procedures such as manipulating equations, working with powers, expanding brackets and simplifying algebraic expressions.
- Teachers should provide technology-free opportunities for students to maintain their number skills, including using the four operations with fractions and decimals and the interchangeable nature of fractions and decimals.
- Teachers should encourage students to use the perusal time to identify the number of questions in the examination and where the ‘END OF PAPER’ message is located.
- Teachers should, as per the example above, encourage students to always read questions carefully to ensure they have actually answered the question being asked, e.g. establishing clearly whether differentiation or integration is the procedure required to answer the question.
- Teachers are encouraged to share alternative methods with students where possible in more complex questions.