Mathematical Methods subject report

2022 cohort

May 2023

Queensland Government

QCAA
Queensland Curriculum & Assessment Authority

For all Queensland schools

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Queensland Curriculum & Assessment Authority
PO Box 307 Spring Hill QLD 4004 Australia

Phone: (07) 3864 0299
Email: office@qcaa.qld.edu.au
Website: www.qcaa.qld.edu.au

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Introduction

Throughout 2022, schools and the QCAA worked together to further consolidate the new Queensland Certificate of Education (QCE) system. The familiar challenges of flood disruption and pandemic restrictions were managed, and the system continued to mature regardless.

We have now accumulated three years of assessment information, and our growing experience of the new system is helping us to deliver more authentic learning experiences for students. An independent evaluation will commence in 2023 so that we can better understand how well the system is achieving its goals and, as required, make strategic improvements. The subject reports are a good example of what is available for the evaluators to use in their research.

This report analyses the summative assessment cycle for the past year — from endorsing internal assessment instruments to confirming internal assessment marks, and marking external assessment. It also gives readers information about:

- how schools have applied syllabus objectives in the design and marking of internal assessments
- how syllabus objectives have been applied in the marking of external assessments
- patterns of student achievement.

The report promotes continuous improvement by:

- identifying effective practices in the design and marking of valid, accessible and reliable assessments
- recommending where and how to enhance the design and marking of valid, accessible and reliable assessment instruments
- providing examples, including those that demonstrate best practice.

Schools are encouraged to reflect on the effective practices identified for each assessment, consider the recommendations to strengthen assessment design and explore the authentic student work samples provided.

Audience and use

This report should be read by school leaders, subject leaders and teachers to:

- inform teaching and learning and assessment preparation
- assist in assessment design practice
- assist in making assessment decisions
- help prepare students for external assessment.

The report is publicly available to promote transparency and accountability. Students, parents, community members and other education stakeholders can use it to learn about the assessment practices and outcomes for General subjects (including alternative sequences (AS) and Senior External Examination (SEE) subjects, where relevant) and General (Extension) subjects.

Report preparation

The report includes analyses of data and other information from endorsement, confirmation and external assessment processes. It also includes advice from the chief confirmer, chief endorser and chief marker, developed in consultation with and support from QCAA subject matter experts.

Subject data summary

Subject completion

The following data includes students who completed the General subject or AS.

Note: All data is correct as at 31 January 2023. Where percentages are provided, these are rounded to two decimal places and, therefore, may not add up to 100%.

Number of schools that offered the subject: 427.

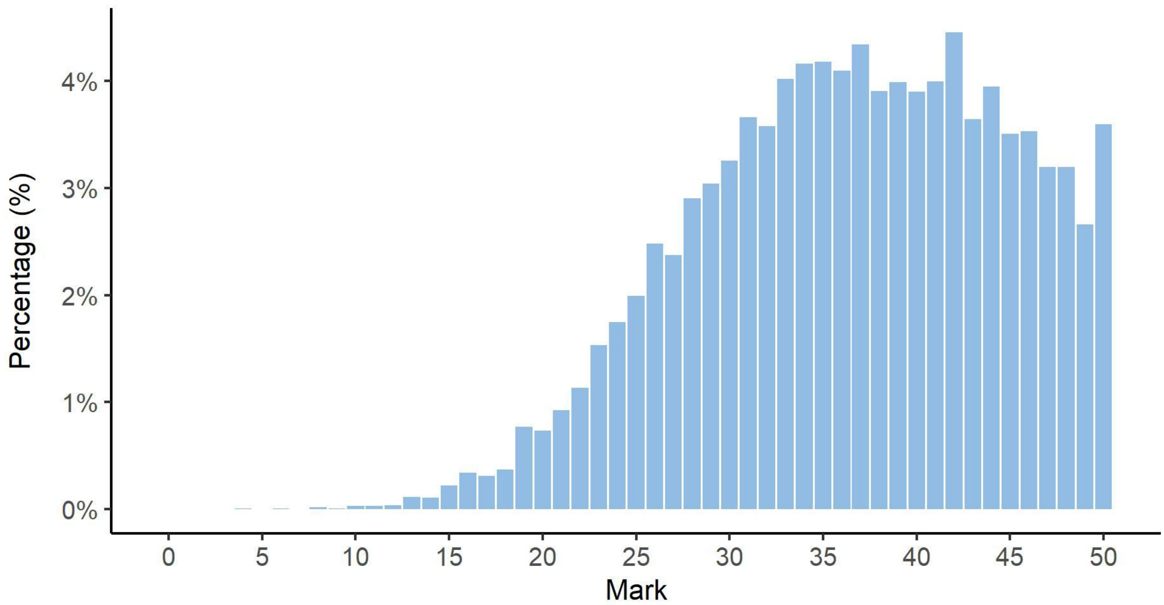
Completion of units	Unit 1	Unit 2	Units 3 and 4
Number of students completed	13 913	12 090	10 195

Units 1 and 2 results

Number of students	Satisfactory	Unsatisfactory
Unit 1	12 799	1114
Unit 2	10 297	1793

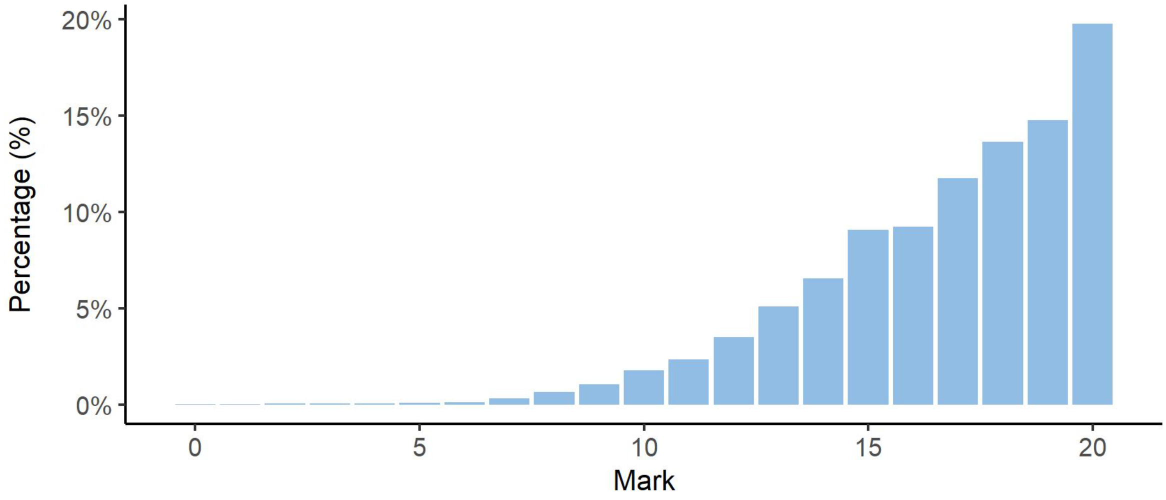
Units 3 and 4 internal assessment (IA) results

Total marks for IA

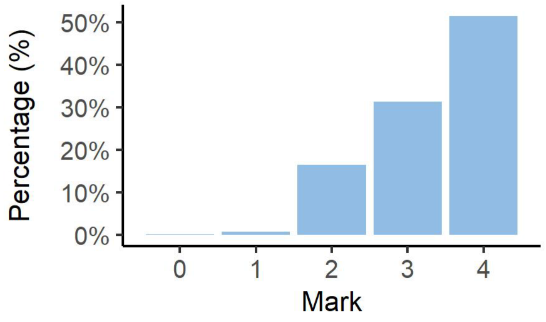


IA1 marks

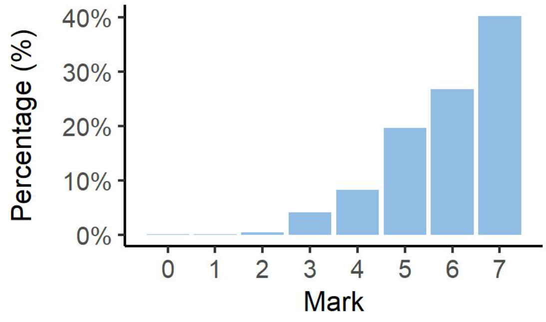
IA1 total



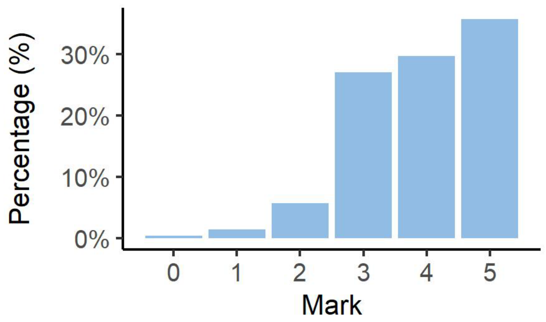
IA1 Criterion: Formulate



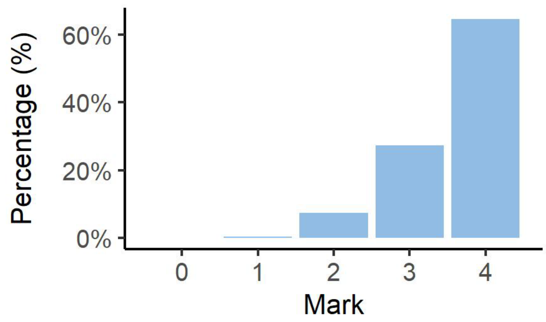
IA1 Criterion: Solve



IA1 Criterion: Evaluate and verify

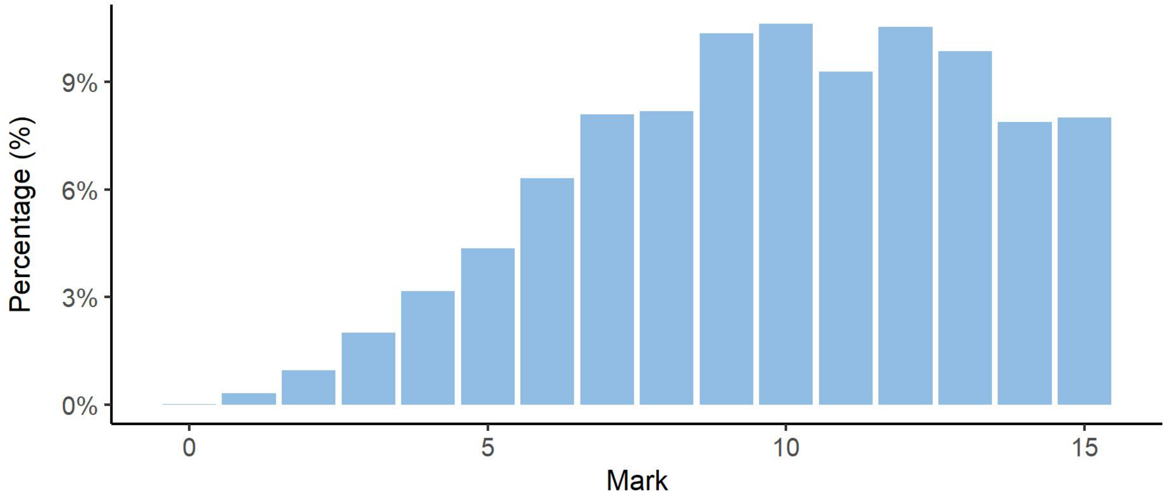


IA1 Criterion: Communicate

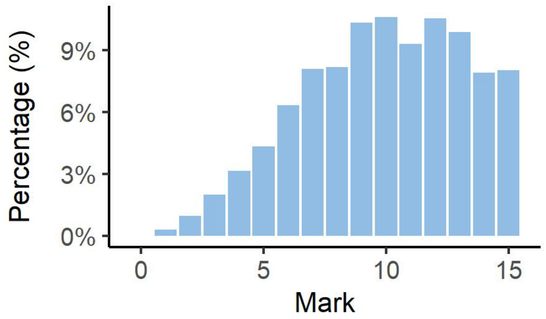


IA2 marks

IA2 total

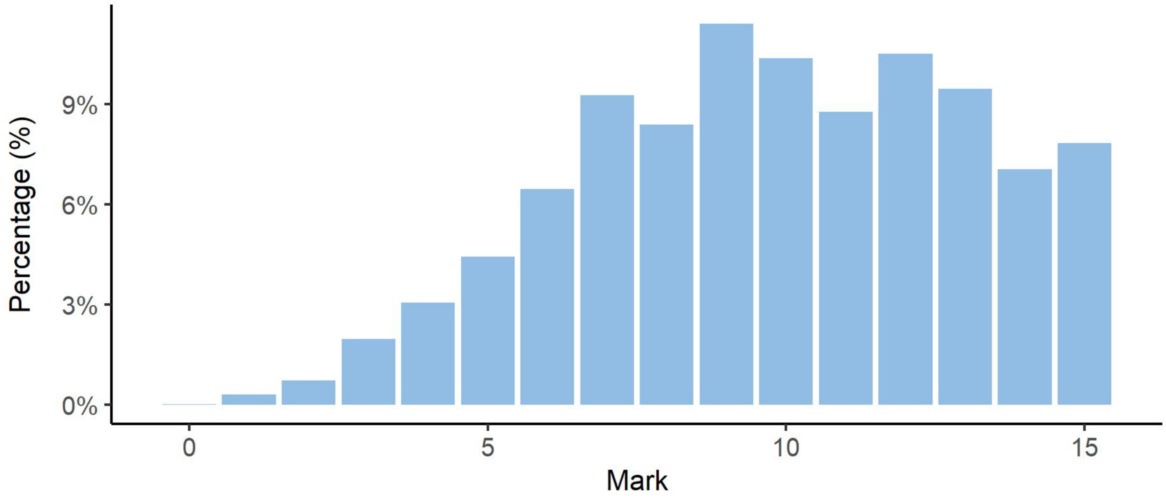


IA2 Criterion: Foundational knowledge and problem-solving

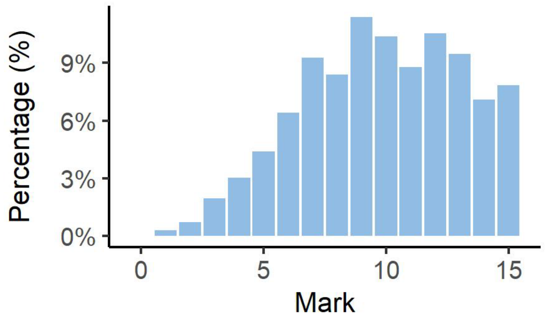


IA3 marks

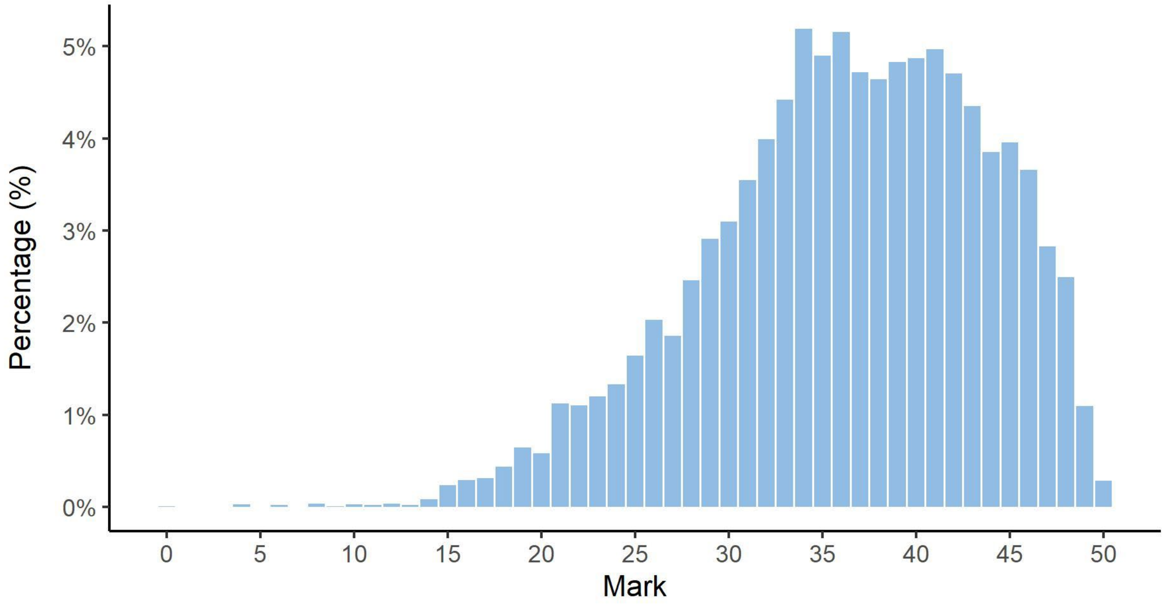
IA3 total



IA3 Criterion: Foundational knowledge and problem-solving

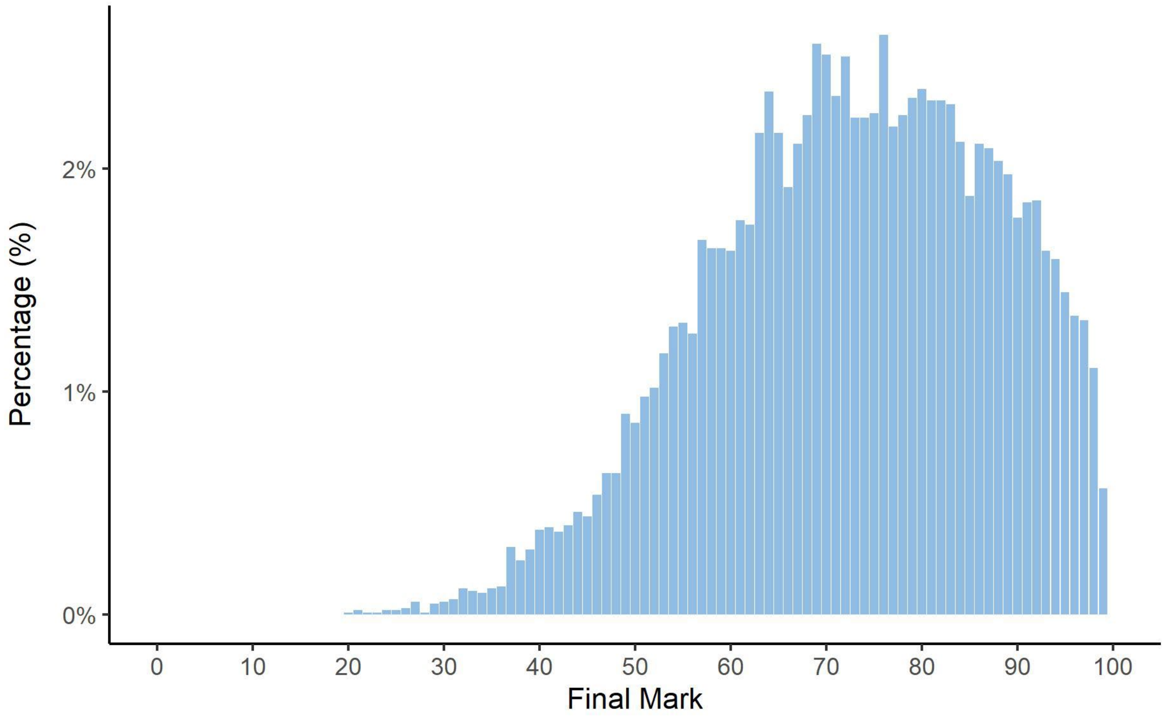


External assessment (EA) marks



Final subject results

Final marks for IA and EA



Grade boundaries

The grade boundaries are determined using a process to compare results on a numeric scale to the reporting standards.

Standard	A	B	C	D	E
Marks achieved	100–84	83–67	66–45	44–20	19–0

Distribution of standards

The number of students who achieved each standard across the state is as follows.

Standard	A	B	C	D	E
Number of students	2752	4040	3022	381	0

Internal assessment

The following information and advice relate to the assessment design and assessment decisions for each IA in Units 3 and 4. These instruments have undergone quality assurance processes informed by the attributes of quality assessment (validity, accessibility and reliability).

Endorsement

Endorsement is the quality assurance process based on the attributes of validity and accessibility. These attributes are categorised further as priorities for assessment, and each priority can be further broken down into assessment practices.

Data presented in the Assessment design section identifies the reasons why IA instruments were not endorsed at Application 1, by the priority for assessments. An IA may have been identified more than once for a priority for assessment, e.g. it may have demonstrated a misalignment to both the subject matter and the assessment objective/s.

Refer to *QCE and QCIA policy and procedures handbook v4.0*, Section 9.5.

Percentage of instruments endorsed in Application 1

Number of instruments submitted	IA1	IA2	IA3
Total number of instruments	424	424	420
Percentage endorsed in Application 1	64%	22%	37%

Confirmation

Confirmation is the quality assurance process based on the attribute of reliability. The QCAA uses provisional criterion marks determined by teachers to identify the samples of student responses that schools are required to submit for confirmation.

Confirmation samples are representative of the school's decisions about the quality of student work in relation to the instrument-specific marking guide (ISMG), and are used to make decisions about the cohort's results.

Refer to *QCE and QCIA policy and procedures handbook v4.0*, Section 9.6.

The following table includes the percentage agreement between the provisional marks and confirmed marks by assessment instrument. The Assessment decisions section of this report for each assessment instrument identifies the agreement trends between provisional and confirmed marks by criterion.

Number of samples reviewed and percentage agreement

IA	Number of schools	Number of samples requested	Number of additional samples requested	Percentage agreement with provisional marks
1	416	2743	116	87.98%
2	415	2303	0	97.83%
3	415	2272	0	99.04%

Internal assessment 1 (IA1)

Problem-solving and modelling task (20%)

This assessment focuses on the interpretation, analysis and evaluation of ideas and information. It is an independent task responding to a particular situation or stimuli. While students may undertake some research in the writing of the problem-solving and modelling task, it is not the focus of this technique. This assessment occurs over an extended and defined period of time. Students will use class time and their own time to develop a response.

The problem-solving and modelling task must use subject matter from one or both of the following topics in Unit 3:

- Topic 2: Further differentiation and applications 2
- Topic 3: Integrals.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	81
Authentication	48
Authenticity	26
Item construction	15
Scope and scale	21

*Each priority might contain up to four assessment practices.

Total number of submissions: 424.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- featured appropriate scope and scale to allow students to address all stages of the problem-solving and modelling approach, e.g. designing a rollercoaster to ensure smoothness or designing a rollercoaster to determine the area enclosed underneath but not both
- required students to use functions studied in Unit 3 when developing a response to tasks based on Topic 2, e.g. ‘By using exponential, logarithmic or trigonometric functions …’ or ‘Use functions studied in Unit 3 as part of your response’

- provided opportunity for students to develop a unique response, e.g. providing an open-ended task, individual datasets or models so students made choices about how to use the data and what concepts and techniques were relevant to solve the problem
- featured realistic contexts that were accessible to students, e.g. designing a logo, designing a waterslide, finding the area of a lake or modelling the rate of population growth/decline.

Practices to strengthen

It is recommended that assessment instruments:

- indicate the types of functions required to solve the problem (for tasks referencing subject matter from Topic 2), so as to avoid students solving the problem using Unit 2 subject matter, e.g. 'Using functions found in Unit 3' or 'By considering exponential, logarithmic and/or trigonometric functions'
- feature objective, rather than subjective, task constraints or specifications that assist students to develop a response in alignment to syllabus subject matter, e.g. if designing a 'thrilling' rollercoaster, provide a mathematical definition of 'thrill' in the stimulus to remove any research component.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	6
Language	21
Layout	8
Transparency	14

*Each priority might contain up to four assessment practices.

Total number of submissions: 424.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- featured a specific task or issue that was written in a straightforward manner and that was explicit about the nature of the problem
- featured a clear layout, where text and other items (e.g. MathType, tables and graphs) appeared aligned and in their entirety on the page. See *Developing summative internal assessment instruments: Endorsement user guide*, available from the Endorsement application in the QCAA Portal
- used appropriate language, diagrams and images
- contained a scenario or context that was accessible to students, e.g. designing a rollercoaster, waterslide or racetrack or developing a logo.

Practices to strengthen

It is recommended that assessment instruments:

- are reviewed before submission to check for typographical, grammatical, punctuation and spelling errors
- are reviewed before submission to check for references to the previous year's problem-solving and modelling task, particularly in the scaffolding section.

Additional advice

- In the scaffolding section of the task, schools are allowed to state, 'The approach to problem-solving and mathematical modelling (Syllabus section 1.2.4, Figure 4) must be used.' Schools do not have to include a generic flowchart or task-specific flowchart. Where a school chooses to use a task-specific flowchart, any changes made from the generic flowchart should be aligned with the instrument's proposed context and task.
- Schools should include checkpoints that clearly indicate when and how teachers provide feedback on one draft.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Formulate	91.83%	7.21%	0.72%	0.24%
2	Solve	94.95%	3.85%	0.96%	0.24%
3	Evaluate and verify	95.91%	3.13%	0.24%	0.72%
4	Communicate	98.32%	0.72%	0.72%	0.24%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- in making judgments across all criteria, student responses were annotated to clearly identify the key characteristics and alignment to the ISMG, e.g.
  - in the Solve criterion, supporting judgments about the complexity of procedures by identifying the characteristics that make the response complex
  - in the Solve criterion, clearly identifying when the response has 'accurate and appropriate use of technology' rather than 'use of technology'

- in the Communicate criterion, for the top performance-level descriptors, clearly identifying where the student has used technical and procedural vocabulary, and how the response can be read independently of the task sheet.

Samples of effective practices

The following excerpts demonstrate documentation of assumptions and observations, accurate use of complex procedures and appropriate annotation of an ISMG.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Excerpt 1

2.1 Observations

- The vaccination reduces the number of cases by 11.3% per month.
- The countries have the following cases per month:

	Ebyam	Doog
January	5700	4900
February	7700	6900
March	13700	12400
April	14700	14900
May	16200	17400
June	15600	17100
July	14200	16600
August	11700	16100
September	9700	15600
October	9200	15900
November	8200	12900
December	7700	11900

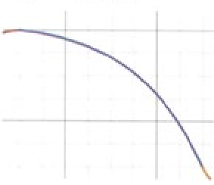
- The model should use a combination of higher order polynomials, trigonometric, logarithmic and exponential functions. The large range of functions will increase the accuracy of the model which is created.
- The maximum rate of infection must be determined for each country. This will provide a good indication of the point within the first year that the outbreak is spreading most rapidly.
- Vaccinations are implemented after 12 months. Therefore, the exponential decay function should be implemented at this point.

2.2 Assumptions

- The vaccine is taken by everyone immediately and are effective immediately. Therefore, the decreasing exponential function can be implemented immediately after 12 months.
- The modelled equations will be rounded to 2 decimal places within the report but not within the graphs. The unrounded equations can be found within appendix 1.
- It is assumed that the reported cases are a total of all cases from across the month. Therefore, the interval 0 to 1 will represent the month of January and so on. This assumption helps makes it easier to determine the time of points along the model.
- It is assumed that when new infections are reduced to less than one infection per month for a period of 3 months the disease is eradicated. This will allow a mathematical approach to finding the point at which the disease is no longer spreading.

Excerpt 2

Graph 2 - Exponential



General form of exponential:
$y = be^{k(x+c)} + d$
Substitute in two points to make two equations
(50,40) (60,38)
(1) $40 = be^{k(50+c)} + 42$
(2) $38 = be^{k(60+c)} + 42$
Solve equation **(1)** to find b
$$b = \frac{-2}{e^{k(50+c)}}$$
Substitute new value into equation **(2)** to solve for **k**
$$-4 = \frac{-2e^{k(60+c)}}{e^{k(50+c)}}$$
$$4e^{k(50+c)} = 2e^{k(60+c)}$$
Complexity here
$$ln4 + k(50 + c) = ln2 + k(60 + c)$$
$$ln4 - ln2 = 10k$$
$$\frac{ln4 - ln2}{10} = k$$
$$k = 0.0693147$$
Therefor the equation of the parabola is:
$y = -13.558e^{0.0693147(x-77.6107)} + 4$

Differentiate equation
$$y' = \frac{4698843513e^{\frac{693147(x(\frac{776107}{10000})}{10000000}}}{5000000000}$$
Substitute in steepest x value (90)
$$m = \frac{4698843513e^{\frac{693147(90(\frac{776107}{10000})}{10000000}}}{5000000000}$$
$m = 2.085$
$tan^{-1}(2.085) = 64.38°$

$tan^{-1}(2.085) = 1.123588242$
$1.123588242 \times 30 = 33.71$
Therefore, the drop factor for descent 1 is **33.71.**

An exponential function was used to gradually decline the track from its highest point to provide the first descent and first drop factor. In the equation b is the vertical stretch, k is the horizontal stretch by a factor of $\frac{1}{k}$, c is the horizontal translation and d is a vertical translation/asymptote. math language

The d value of the exponential was already known from the hand drawing being the asymptote, $d = 42$. Using two coordinates on the hand drawing (50,40) (60,38), two equations were made. The first equation was used to find an equation for b. This b value was then substituted into equation 2 to solve for the k value.

The gradient was then found to find the maximum angle that the exponential had to ensure it was not over the maximum allowed value of 70°. This was done by differentiating the equation and substituting the steepest x value into the differentiated equation.

The angle in radians was then found and multiplied by the total verticle distance of the drop to find the drop factor.

Complex calc.

Excerpt 3

The student work has the following characteristics:	Marks
• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model • justification of decisions made using mathematical reasoning.	4–5
• statements about the reasonableness of solutions by considering the context of the task • statements about relevant strengths and limitations of the solution and/or model • statements about decisions made relevant to the context of the task.	2–3
• statement about a decision and/or the reasonableness of a solution.	1
• does not satisfy any of the descriptors above.	0

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- the ISMG is clearly annotated to show which characteristics of the performance-level descriptors are evident in the student work, and the correct mark allocation for each criterion
- within the Formulate criterion, the difference between 'documentation' and 'statement' must be clear when making judgments, e.g.
  - responses that demonstrate 'documentation of appropriate assumptions' not only include assumptions related to the student's model/solution but also evidence to support the

assumption, e.g. the likely effect of an important assumption and how this is considered in the model/solution, or the impact of not making the assumption

  - responses that demonstrate ‘accurate documentation of relevant observations’ provide evidence to support observations (data or information) used in a student’s model/solution. This can be demonstrated by explaining how the observations were collected, the source of the observations, what made the observations valid and reliable, or identifying a specific feature of an observation that made it relevant to the model/solution, e.g. from the plotted data points it is clearly observed that there is one turning point at (x, y) and that the curve is symmetrical about that turning point
  - evidence of both assumptions and observations can be found not only in the introductory section but throughout the response, e.g. plotting a series of points somewhere in the response, observing that the points appear to be sinusoidal, and therefore assuming that a periodic function should be used to model the situation. It is important that teachers annotate this to identify where these characteristics are evident
- within the Evaluate and verify criterion, evaluation of the reasonableness of solutions requires consideration of ‘results, assumptions and observations’ (not just statements about possible reasonableness or unreasonableness of the solution), e.g.
  - responses need to demonstrate that the results, assumptions and observations have been considered to appraise and justify the solutions. Any assumptions and observations introduced throughout the report could be used while evaluating the reasonableness of solutions
  - the evaluation could include the use of technology to verify solutions or the use of both mathematical and everyday language to justify solutions.

Additional advice

- When student responses exceed the specified conditions for word length and/or page count, teachers must annotate the written response to indicate how the school’s assessment policy has been applied. See Section 8.2.6 of the *QCIA and QCE policy and procedures handbook v4.0* for advice on managing response length. Communication with the QCAA is recommended if further advice is required about managing responses that exceed the allowable length.
- As appendixes are not to be marked, students should provide evidence relevant to criteria descriptors within the report (e.g. the use of complex procedures and use of technology should be included within the report), but repeated calculations or large datasets are better placed in the appendixes.
- For larger cohorts with multiple classes and different teachers, schools should apply a range of moderation processes to ensure consistent application of the ISMG for all students.

Internal assessment 2 (IA2)

Examination (15%)

This examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 3 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	284
Authentication	0
Authenticity	9
Item construction	30
Scope and scale	74

*Each priority might contain up to four assessment practices.

Total number of submissions: 424.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- featured questions that assessed a selection of subject matter that accurately reflected the intended learning of all topics in Unit 3
- featured a balance of technology-free and technology-active responses by either
  - splitting the instrument into two sections (technology-free and technology-active)
  - indicating technology-free responses on a calculator-allowed examination by using a picture, statement or giving cues such as ‘Use analytical procedures’ or ‘Show algebraic working’
- provided an appropriate number of questions that matched the degree of difficulty specifications in the syllabus and allowed students to respond in the set timeframe
- provided a correct marking scheme that indicated clearly how marks had been allocated; this assisted schools to check the scope and scale of the assessment and promoted consistency in the awarding of marks.

Practices to strengthen

It is recommended that assessment instruments:

- provide complex unfamiliar opportunities so
  - relationships and interactions have a number of elements but are not scaffolded, e.g. not providing a series of parts that step through a problem
  - all the information to solve the problem is not immediately identifiable, i.e. the required procedure is not clear from the way the problem is posed, e.g. by avoiding cues such as ‘By using integrals …’ or ‘Use differentiation to …’
- require students to demonstrate knowledge and understanding of Unit 3 subject matter and do not solely assess subject matter from Units 1 and 2, e.g. solving indicial equations that have bases other than e is part of Unit 2 and determining derivatives involving only polynomial functions is part of Unit 2
- feature appropriate cues in technology-active questions that have marks allotted for an algebraic response, e.g. ‘Use an analytical procedure to …’ and ‘Evaluate the reasonableness of this result by showing algebraic working’
- assess subject matter within the scope and scale of the syllabus, e.g. using reciprocal trigonometric functions is part of the Specialist Mathematics syllabus and should not be assessed in Mathematical Methods.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	14
Language	97
Layout	26
Transparency	40

*Each priority might contain up to four assessment practices.

Total number of submissions: 424.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- featured a clear layout, where text and other items (e.g. MathType, tables and graphs) appeared aligned and in their entirety on the page (see *Developing summative internal assessment instruments: Endorsement user guide*, available from the Endorsement application in the QCAA Portal)
- used consistent formatting, including use of MathType, throughout the instrument
- used correct language conventions, and were free of punctuation, grammatical, spelling and typographical errors

- provided realistic contexts, where appropriate, which were accessible to students, e.g. population of animals or position of a particle.

Practices to strengthen

It is recommended that assessment instruments:

- are reviewed before submission to check for errors in mathematical notation and conventions, e.g. ensuring integrals of the form $\int f(x)\,dx$ include dx
- are reviewed before submission to check for typographical, grammatical, punctuation and spelling errors.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	97.83%	1.45%	0.48%	0.24%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- there was clear alignment between a school's submitted marking scheme and the awarded marks for each question, particularly in situations where part marks were awarded. This was most effective where schools provided detailed marking schemes indicating how marks were allocated
- the 'greater than x%' cut-offs were correctly applied to the percentage calculations to determine accurate provisional marks, e.g.
  - results were not rounded to the nearest percentage before applying the ISMG
  - a student who received >73% was awarded 12/15, while a student who received 73% (exactly) was awarded 11/15
- there was a clear indication of the total marks awarded to the student, the total possible marks, and the associated percentage before applying the ISMG.

Samples of effective practices

The following excerpts demonstrate clear indication of the application of the percentage cut-off on the ISMG, and clear allocation of marks, including how follow-through (FT) marks and implied marks have been awarded.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Excerpt 1

Instrument-specific marking guide (IA2): Examination (15%)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.

$\frac{36.5}{45} = 81.1\%$

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10

Excerpt 2

$a(t) = 6t + 4$

$v(t) = 3t^2 + 4t + c$ ✓

since $v(0) = -4$ m/s

$-4 = 3(0)^2 + 4(0) + c$ ✓

$= c$

$\therefore c = -4$

$\therefore v(t) = 3t^2 + 4t - 4$

$\therefore x(t) = t^3 + 2t^2 - 4t + c$ ← integral error ($-\frac{1}{2}$)

since $x(0) = -7$

$-7 = 0^3 + 0^2 - 4(0) + c$ ✓ ↓ F.T. errors

$= c$

$c = -7$

$\therefore x(t) = t^3 + t^2 - 4t - 7$ ($2\frac{1}{2}$)

Velocity and Position at $t = 2$

$v(2) = 3(2)^2 + 4(2) - 4$ ✓

$= 16$ m/s.

$x(2) = 2^3 + 2^2 - 4(2) - 7$

$= -1$

Excerpt 3

✓✓

$\int_0^5 (13 + 58e^{-0.6t})\,dt =$ distance travelled in 1st 5 seconds

✓✓ imp.

$= 204.36$ metres ✓

≈ 204.4 metres travelled over 1st 5 seconds

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- schools check that the correct and up-to-date marking scheme is uploaded at confirmation, i.e. it matches all questions in the endorsed instrument, including mark allocations for alternative responses, where applicable, and the total number of marks for each question matches the endorsed instrument
- any comparable assessments have matching marking schemes provided at confirmation
- when awarding follow-through marks, teachers annotate student responses to identify the error and where the awarding of follow-through marks has occurred
- teachers annotate student responses to identify that implied marks have been awarded, where applicable.

Additional advice

- Confirmation involves reviewing responses to an endorsed assessment instrument or a comparable assessment instrument. If a change to an endorsed assessment instrument is necessary then communication with the QCAA is essential. If the change is identified prior to implementation, request an amendment via the Endorsement application. If an error is identified during implementation or post-implementation, contact the QCAA. See ‘Amending an endorsed assessment’ (webpage on the QCAA website).

Internal assessment 3 (IA3)

Examination (15%)

This examination assesses the application of a range of cognitions to a number of items, drawn from Unit 4 Topics 1–5. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	217
Authentication	0
Authenticity	2
Item construction	17
Scope and scale	59

*Each priority might contain up to four assessment practices.

Total number of submissions: 420.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided questions that assessed a selection of subject matter that accurately reflected the intended learning of all topics in Unit 4, including a representative sample of Topic 5
- included a balance of technology-free and technology-active questions. In instruments that allowed access to technology for the entire paper, appropriate cues were provided to direct students to use an analytical procedure, e.g. 'Use algebraic techniques to …'
- provided a correct marking scheme that indicated clearly how marks had been allocated; this assisted schools to check the scope and scale of the assessment and promoted consistency in the awarding of marks
- included questions that explicitly provided opportunities to address all assessment objectives, including Assessment objective 4: Evaluate the reasonableness of solutions.

Practices to strengthen

It is recommended that assessment instruments:

- provide a balance of questions from all topics in Unit 4, including Topic 5
- provide complex unfamiliar opportunities so
  - relationships and interactions have a number of elements but are not scaffolded, e.g. not providing a series of parts that step through a problem
  - all the information to solve the problem is not immediately identifiable, i.e. the required procedure is not clear from the way the problem is posed, e.g. by avoiding cues such as 'Use the second derivative test to …'
- provide opportunities for students to respond to Assessment objective 4: Evaluate the reasonableness of solutions.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	6
Language	72
Layout	12
Transparency	36

*Each priority might contain up to four assessment practices.

Total number of submissions: 420.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- featured a clear layout, where text and other items (e.g. MathType, tables and graphs) appeared aligned and in their entirety on the page
- used correct and consistent mathematical notation throughout the instrument
- featured correct language conventions, and were free of punctuation, grammatical, spelling and typographical errors.

Practices to strengthen

It is recommended that assessment instruments:

- use the language of the assessment objectives, e.g. 'evaluate the reasonableness of solutions' instead of 'discuss' or 'check'
- are reviewed before submission to check for typographical, grammatical, punctuation and spelling errors.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	99.04%	0.48%	0.48%	0%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

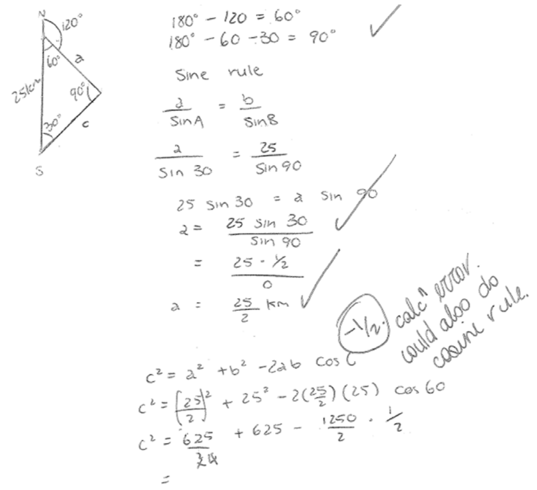
- there was clear alignment between a school's submitted marking scheme and the awarded marks for each question, particularly in situations where part marks were awarded. This was most effective where schools provided detailed marking schemes indicating how marks were allocated
- the 'greater than x%' cut-offs were correctly applied to the percentage calculations to determine accurate provisional marks, e.g.
  - results were not rounded to the nearest percentage before the ISMG was applied
  - a student who received > 73% was awarded 12/15, while a student who received 73% (exactly) was awarded 11/15
- there was a clear indication of the total marks awarded to the student, the total possible marks, and the associated percentage before applying the ISMG.

Samples of effective practices

The following excerpts demonstrate the use of annotations of student responses to show where marks were awarded and how calculation errors were accounted for in the awarding of marks. The marked parts of the response are clear because the student has ruled a line through the parts of their response that are to be cancelled.

Note: The characteristic/s identified may not be the only time the characteristic/s has occurred throughout a response.

Excerpt 1



$180° - 120 = 60°$
$180° - 60 - 30 = 90°$

Sine rule

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 30} = \frac{25}{\sin 90}$

$25 \sin 30 = a \sin 90$

$a = \frac{25 \sin 30}{\sin 90}$

$= \frac{25 \cdot ½}{0}$

$a = \frac{25}{2}$ km

(−½) calc error. could also do cosine rule.

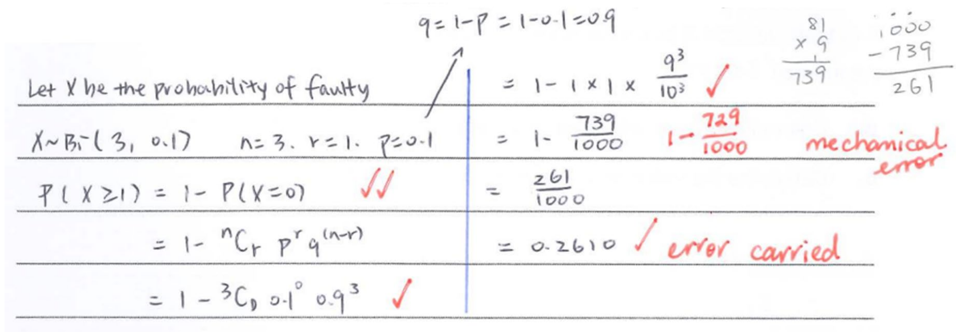
$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = \left[\frac{25}{2}\right]^2 + 25^2 - 2(\frac{25}{2})(25) \cos 60$

$c^2 = \frac{625}{24} + 625 - \frac{1250}{2} \cdot \frac{1}{2}$

$=$

Excerpt 2



Let X be the probability of faulty

$X \sim Bi(3, 0.1)$ $n = 3.\ r = 1.\ p = 0.1$

$P(X \geq 1) = 1 - P(X = 0)$ ✓✓

$= 1 - {}^nC_r\, p^r q^{(n-r)}$

$= 1 - {}^3C_0\, 0.1^0\, 0.9^3$ ✓

$q = 1 - p = 1 - 0.1 = 0.9$

$= 1 - 1 \times 1 \times \frac{9^3}{10^3}$ ✓

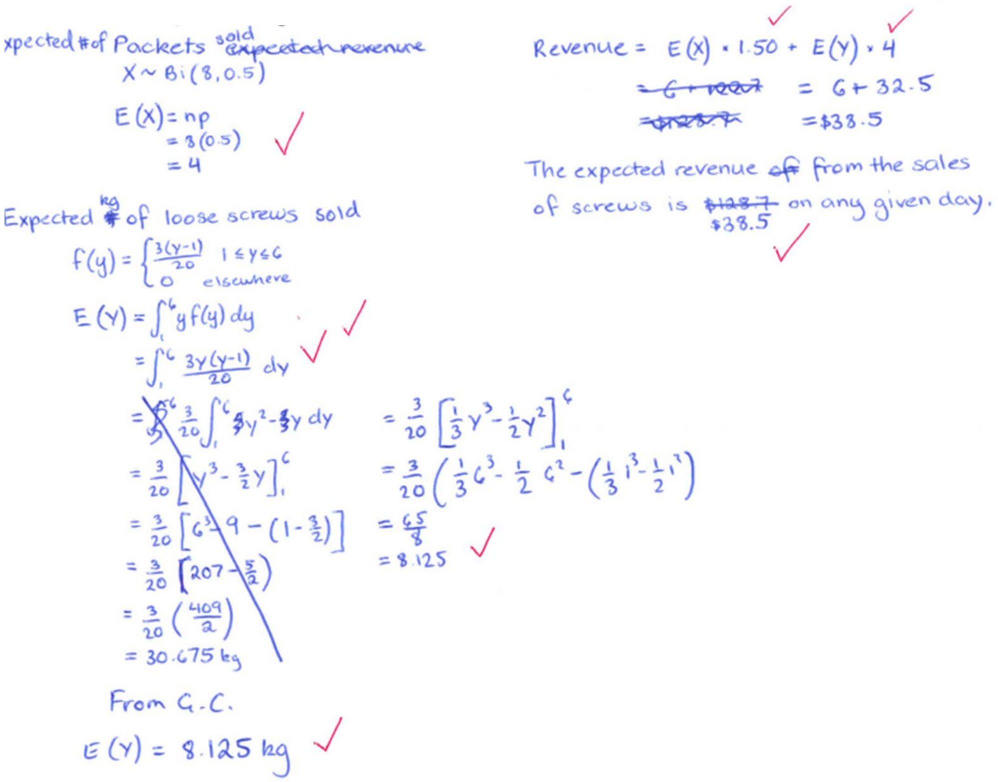
$= 1 - \frac{739}{1000}$ $1 - \frac{729}{1000}$ mechanical error

$= \frac{261}{1000}$

$= 0.2610$ ✓ error carried

81 × 9 = 739; 1000 − 739 = 261

Excerpt 3



Expected # of Packets sold ~~expected revenue~~

$X \sim Bi(8, 0.5)$

$E(X) = np$
$= 8(0.5)$
$= 4$ ✓

Expected kg ~~#~~ of loose screws sold

$f(y) = \begin{cases} \frac{3(y-1)}{20} & 1 \leq y \leq 6 \\ 0 & \text{elsewhere} \end{cases}$

$E(Y) = \int_1^6 y f(y)\, dy$ ✓

$= \int_1^6 \frac{3y(y-1)}{20}\, dy$ ✓

~~$= \frac{3}{20}\int_1^6 3y^2 - 3y\, dy$~~
~~$= \frac{3}{20}\left[y^3 - \frac{3}{2}y\right]_1^6$~~
~~$= \frac{3}{20}\left[6^3 - 9 - (1 - \frac{3}{2})\right]$~~
~~$= \frac{3}{20}\left[207 - \frac{5}{2}\right)$~~
~~$= \frac{3}{20}\left(\frac{409}{2}\right)$~~
~~$= 30.675$ kg~~

$= \frac{3}{20}\left[\frac{1}{3}y^3 - \frac{1}{2}y^2\right]_1^6$

$= \frac{3}{20}\left(\frac{1}{3}6^3 - \frac{1}{2}6^2 - (\frac{1}{3}1^3 - \frac{1}{2}1^2)\right)$

$= \frac{65}{8}$

$= 8.125$ ✓

From G.C.

$E(Y) = 8.125$ kg ✓

Revenue $= E(X) \cdot 1.50 + E(Y) \cdot 4$ ✓ ✓

~~$= 6 + 122.7$~~ $= 6 + 32.5$

~~$= \$128.7$~~ $= \$38.5$

The expected revenue ~~of~~ from the sales of screws is ~~\$128.7~~ \$38.5 on any given day. ✓

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- schools check that the correct and up-to-date marking scheme is uploaded at confirmation, i.e. it matches all questions in the endorsed instrument, including mark allocations for alternative responses, where applicable, and the total number of marks for each question matches the endorsed instrument
- any comparable assessments have matching marking schemes provided at confirmation
- teachers annotate student responses to clearly identify where all marks have been awarded, including
  - responses that are fully correct
  - how calculation errors and follow-through marks are accounted for
- teachers apply a consistent strategy when a student provides more than one response without indicating the response to be marked, e.g. teachers mark the first response that appears top to bottom, left to right.

Additional advice

- Confirmation involves reviewing responses to an endorsed assessment instrument or a comparable assessment instrument. If a change to an endorsed assessment instrument is necessary then communication with the QCAA is essential. If the change is identified prior to implementation, request an amendment via the Endorsement application. If an error is identified during or post implementation, contact the QCAA. See ‘Amending an endorsed assessment’ (webpage on the QCAA website).

External assessment

External assessment (EA) is developed and marked by the QCAA. The external assessment for a subject is common to all schools and administered under the same conditions, at the same time, on the same day.

Examination (50%)

Assessment design

The assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the syllabus. The examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (45 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (45 marks).

The examination assessed subject matter from Units 3 and 4.

The assessment required students to respond to multiple choice and short response questions.

Assessment decisions

Assessment decisions are made by markers by matching student responses to the external assessment marking guide (EAMG). The external assessment papers and the EAMG are published in the year after they are administered.

Multiple choice question responses

There were 10 multiple choice questions in Paper 1.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Paper 1

Question	A	B	C	D
1	**34.36**	8.49	51.41	5.5
2	1.66	11.29	10.64	**75.99**
3	5.28	34.19	**48.26**	11.58
4	11.67	17.71	**57.94**	11.58
5	0.64	1.96	18.07	**78.93**
6	20.99	**59.89**	9.74	9.07
7	56.26	**29.89**	9.76	3.07
8	**67.06**	6.16	24.19	2.24
9	**29.43**	29.4	29.83	10.35
10	7	5.03	6.52	**81.05**

There were 10 multiple choice questions in Paper 2.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Paper 2

Question	A	B	C	D
1	5.87	12.28	7.11	**74.13**
2	10.6	**57.06**	19.36	12.49
3	23.1	27.82	**21.14***	27.16
4	7.42	8.9	**79.39**	3.53
5	37.87	**37.66**	13.12	9.2
6	6.43	**57.21**	4.67	31
7	14.74	9.17	23.82	**51.13**
8	11.89	10.44	**68.07**	8.94
9	5.78	**69.37**	2.52	21.69
10	1.52	2.4	**93.21**	2.28

* Question 3 has been updated to indicate both C and D were awarded a mark due to an identified misconception. Option C is the correct answer.

Effective practices

Overall, students responded well to:

- opportunities to determine the mean, variance and standard deviation in situations involving Bernoulli and binomial distributions, with and without technology
- requirements to use the formulas of differentiation when applied to exponential, logarithmic and trigonometric functions in simple familiar situations
- opportunities to use technology to calculate probabilities associated with the normal distribution and to determine the approximate confidence interval for a sample proportion
- opportunities to use logarithmic laws to solve equations involving indices and logarithmic functions.

Samples of effective practices

Short response

The following excerpt is from Question 18 from Paper 1 (technology-free). It required students to understand the concept of a probability density function and use integration techniques to determine a percentile. The definition of a percentile was provided in the question.

Effective student responses:

- introduced an unknown variable for the required percentile and incorporated this variable into a definite integral that was equated to 0.36
- algebraically rearranged the result of the integration to form a quadratic equation equated to zero
- correctly solved the quadratic equation, obtaining two solutions as decimals, and identified the one solution that was in the required domain as the 36th percentile.

This excerpt has been included:

- as an example of a high-level response that demonstrates the use of integral and algebraic techniques involving decimals within the context of a given probability density function with a continuous random variable.

Let a be the value of X that is the 36^{th} percentile

$$0.36 = \int_1^a 2x - 2\,dx$$

$$0.36 = \left[x^2 - 2x\right]_1^a$$

$$0.36 = a^2 - 2a - 1 + 2$$

$$0.36 = a^2 - 2a + 1$$

$$0 = a^2 - 2a + 0.64$$

$$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 2.56}}{2}$$

$$= \frac{2 \pm \sqrt{1.44}}{2}$$

$$= \frac{2 \pm 1.2}{2}$$

reject $\frac{2-1.2}{2}$ as it is less than 1 $(x \geq 1)$

$\therefore a = \frac{3.2}{2} = 1.6$ $\quad\therefore$ the 36^{th} percentile of X is 1.6

The following excerpt is from Question 19 from Paper 1 (technology-free). It required students to minimise two enclosed areas between parallel walls using provided wall dimensions and information about angle and side properties found in similar triangles.

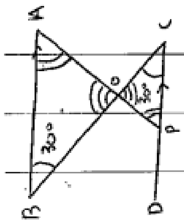
Effective student responses:

- used the given dimensions to create an appropriately labelled diagram where the lengths '*x*' and '*10-x*' were clearly identified
- formed an equation for the total area required, in terms of the variable '*x*', which had incorporated the similar triangle information that was provided in the question
- used calculus methods to determine the value of '*x*' that optimised the required area and to verify that this area was a minimum.

This excerpt has been included:

- as it demonstrates how the supplied information was used to create a correct equation for the total enclosed area of the two triangles

- as an example of a high-level algebraic manipulation to enable the first derivative calculation and the second derivative test for the minimum area to be correctly performed.



$BC = 10$ $\triangle OBA$ is similar to $\triangle OCP$ as all corresponding angles are congruent

$OB = x$

$OC = 10 - x$ $\therefore \frac{OC}{OB} = \frac{CP}{AB}$

$\angle ABO = \angle OCP$ (alternate)
$\angle OAB = \angle OPC$ (")
$\angle AOB = \angle COP$ (vertically opposite ∡)

$\frac{10-x}{x} = \frac{CP}{20}$

Area $\triangle OAB = \frac{1}{2}x(20)\sin(30)$ $CP = \frac{20(10-x)}{x}$

Area $\triangle OCP = \frac{1}{2}(10-x)\left(\frac{20(10-x)}{x}\right)\sin 30$

$\text{Area}_{total} = \left(\frac{1}{2}x(20)(\sin 30)\right) + \left(\frac{1}{2}(10-x)\left(\frac{20(10-x)}{x}\right)\sin 30\right)$

$= \left(10x\left(\frac{1}{2}\right)\right) + \left(\frac{10(10-x)^2}{x}\left(\frac{1}{2}\right)\right)$

$\text{Area}_{total} = 5x + \frac{10(10-x)^2}{2x}$

$= 5x + \frac{10(100-20x+x^2)}{2x}$

$= 5x + \frac{1000-200x+10x^2}{2x}$

$= 5x + \frac{500-100x+5x^2}{x}$

$= 5x + \frac{500}{x} - 100 + 5x$

$= 10x - 100 + \frac{500}{x}$

$\frac{dA}{dx} = 10 - \frac{500}{x^2}$

$0 = 10 - \frac{500}{x^2}$

$10 = \frac{500}{x^2}$

$10x^2 = 500$

$x^2 = 50$

$x = \pm\sqrt{50}$

$0 < x \leq 10 \quad \therefore x = \sqrt{50}$ m

$\frac{dA}{dx} = -500x^{-2} + 10$

$\frac{d^2A}{dx^2} = (-2)(-500)x^{-3}$

$= \frac{1000}{x^3}$

$= \frac{1000}{(\sqrt{50})^3}$

$= +ve$

$\therefore$ conc. up

$\therefore$ minimum

$\therefore$ the value of x that minimises the area enclosed by $\triangle OBA$ and $\triangle OCP$ is $\sqrt{50}$ metres

The following excerpt is from Question 17 from Paper 2 (technology-active). It required students to use integration methods to determine displacement given velocity, and velocity given acceleration, in a linear motion problem involving two moving creatures following the same path.

Effective student responses:

- correctly calculated the displacement of the 'snail' after 15 minutes
- correctly determined a velocity formula for the 'ant'
- used the calculated displacement of the 'snail' to determine the velocity of the 'ant' at a certain point on the path.

This excerpt has been included:

- as it demonstrates a thorough understanding of the calculus relationships between acceleration, velocity and displacement
- as it demonstrates high-level problem-solving based on the time reference of the 'ant', i.e. from 0 to 3 minutes, instead of the time for the 'snail' as stated in the question.

After 15 minutes, the snail is x cm from A, where

$$x = \int_0^{15} 1.4\ln(1+t^2)\,dt \quad \left(\text{displacement} = \int \text{velocity } dt\right)$$

$= 76.0431$ m from A

Let the ant have an acceleration function $a(t)$ where

$a(t) = 2$ (cm min^{-2})

$$v_{ant}(t) = \int a(t)\,dt = \int 2\,dt$$

$v_{ant}(t) = 2t + c$ ← c is the initial velocity of the ant at A

Over the three minutes between $t = 12$ and $t = 15$, the ant travels x ~~met~~ cm ie 76.0431 cm

Thus, $76.0431 = \int_0^3 2t + c\,dt$ $\left(\text{displacement} = \int \text{velocity } dt\right)$

From GDC nSolve, $c = 22.3477$

Thus, the ant moves at a velocity of 22.35 cm min^{-1} at point A

The following excerpt is from Question 19 from Paper 2 (technology-active). It required students to determine the maximum number of flying foxes in a region when provided with two trigonometric models for the rates of the animals entering and leaving the region. Integration of the rates was required to produce a model for the number of animals. Technology was required to optimise this model.

Effective student responses:

- correctly found the indefinite integral of each rate
- subtracted the indefinite integrals to obtain a model for the number of flying foxes and equated this to the initial population to determine the combined constant of integration
- graphically represented the 'number of flying foxes' model and used technology to determine the maximum number of animals and the time when this occurred.

This excerpt has been included:

- as an example of a high-level response that demonstrates the use of integration techniques and provides evidence of how technology was used to develop the two answers to the question.

R(t), rate of change = rate in − rate out

$= A(t) - L(t)$

$= 42\sin\left(0.07t - \frac{\pi}{7}\right) - 4 \times 2\sin\left(0.04t - \frac{\pi}{7}\right) + 72$

$= 42\left[\sin\left(0.07t - \frac{\pi}{7}\right) - \sin\left(0.04t - \frac{\pi}{7}\right)\right] + 29$

N, number of flying foxes

$N = \int R(t)$

$N = -1400\cos\left(0.07t - \frac{\pi}{7}\right) + 1050\cos\left(0.04t - \frac{\pi}{7}\right) + 29t + c$

@ $t = 0$, $N = 100$

∴

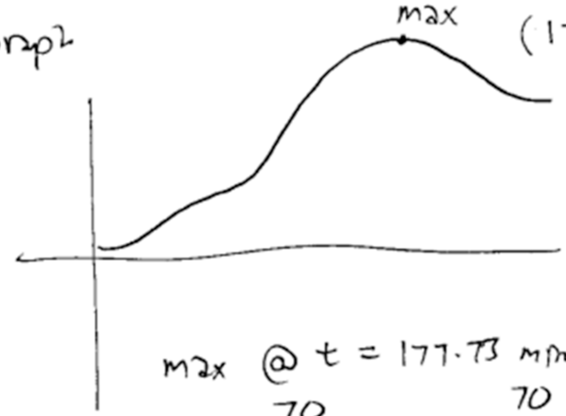
$100 = -175 + c$

$c = 275$

$N = -1400\cos\left(0.07t - \frac{\pi}{7}\right) + 1050\cos\left(0.04t - \frac{\pi}{7}\right) + 29t + 275$

Graph

max (177.73, 7034.26)



max @ $t = 177.73$ mins

$N = 7034.26 \approx 7034$ flying foxes

$\frac{177.73}{60} = 2.96$ hours

= 2 hours and 58 mins after 7pm

(9:58 pm)

∴

The max number of flying foxes is 7034 at 9:58pm

Practices to strengthen

It is recommended that when preparing students for external assessment, teachers consider:

- increasing students' opportunities to explore the concepts of probability density functions and probabilities that can be given by integrals. In Paper 1, Question 18, the students that made some progress could identify the required definite integral but were unable to develop a simplified quadratic equation (involving decimals) that could be solved in a technology-free situation. Paper 2, Question 16 also involved calculating a probability from a probability density function. Many students found this technology-active question difficult to solve
- enabling students to apply their problem-solving skills in a wide range of unfamiliar situations. A complex unfamiliar question such as Paper 1, Question 19 provided information about angle and side properties of similar triangles. This information was provided to assist students to develop an equation for the total area of an enclosure, which could then be optimised. Many students could make a start by drawing an appropriately labelled diagram using the provided information; however, they then had difficulty progressing beyond this stage to produce an equation in one variable that could be optimised using calculus and then verified using the second derivative test
- providing more opportunities for students to investigate scenarios where integration techniques are required to solve problems involving acceleration, velocity and displacement. In Paper 2, Question 17, many students could not use the velocity and acceleration information of two moving objects to determine a value for the velocity of one object at a given point
- providing more problem-solving opportunities where quantities are to be determined by integrating rates of change. Paper 2, Question 19 proved to be the most difficult question in the external assessment, with the fewest number of correct solutions obtained. It presented students with two rates of change, which needed to be integrated and combined to determine an overall total amount of flying foxes. The use of technology was then required to interpret the developed model to determine the maximum number of flying foxes and the time when this occurred.

Additional advice

- Students should be given sufficient opportunities to consolidate their understanding of concavity and the relationship with the second derivative. Students need to be able to examine second derivatives to determine intervals over the domain where they are either positive or negative.
- Students should be allowed to explore and compare the shapes of graphs of functions with the graphs of their derivatives and make connections between their shapes.
- Teachers should provide frequent opportunities for students to solve linear and non-linear simultaneous equations, with and without technology.
- Teachers should provide students with a range of contexts where the application of the sine and cosine rules are needed to model and solve problems.

Senior External Examination

The Mathematical Methods Senior External Examination (SEE) is a standalone examination offered to eligible Year 12 students and adult learners. It contributes 100% to a student's final subject result.

Assessment design

The assessment was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the Mathematical Methods Senior External Examination syllabus.

The SEE consisted of two assessments:

- SEE 1 contributed 50% of the marks
- SEE 2 contributed 50% of the marks.

Note: The SEE information should be read in conjunction with the rest of the subject report.

Number of students who completed the Mathematical Methods Senior External Examination: 17.

There were insufficient student enrolments in this subject to provide useful analytics.

Assessment decisions

Effective practices

Overall, students responded well to:

- the requirement to use the trapezoidal rule to approximate area
- the requirement to state a strength and limitation of a model based on graphical data
- the opportunity to rearrange equations.

Practices to strengthen

It is recommended that when preparing students for the Senior External Examination, teachers consider:

- increasing students' opportunities to plot data, use residuals and linear regression to determine the strength or reasonableness of models for data
- providing regular opportunities for students to develop their algebraic techniques when solving multi-step index or logarithmic equations
- enabling students to apply the rules of integral calculus over a wide range of function types.