External assessment 2022

Question and response book

Mathematical Methods

Paper 2 — Technology-active

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator **permitted**.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (45 marks)

- 9 short response questions

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Queensland Government | QCAA Queensland Curriculum & Assessment Authority

LUI

School code

School name

Given name/s

Family name

Book of books used

Attach your barcode ID label here

Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 45 marks.

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This page will not be marked

Do not write outside this box.

Question 11 (7 marks)

A salesperson has a 20% probability of making a sale to each customer who enters the store. Each sale is independent of all other sales.

a) Determine the mean number of sales on a day where 25 customers enter the store. [2 marks]

b) Determine the standard deviation of the number of sales on a day where 25 customers enter the store. [2 marks]

c) Determine the minimum number of customers who would have to enter the store to have an 88% chance or more of making at least one sale. [3 marks]

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Question 12 (4 marks)

Suppose that the distance travelled by vehicles in a year can be modelled by a normal distribution. In 2021, vehicles travelled a mean of 13 700 km with a standard deviation of 3400 km.

a) Determine the probability that a vehicle chosen at random travelled less than 12 000 km in 2021. [2 marks]

b) Determine the value of x where 60% of vehicles travelled less than x km in 2021. [2 marks]

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Question 13 (4 marks)

A sandy beach has a fence on one side and ocean on the other. The width of the beach is the distance (in metres) from the fence to the water's edge. The width, $w(t)$, at a certain point is given by

$$w(t) = a + b\sin\left(\frac{\pi}{6}t - \frac{\pi}{3}\right),\ 0 \le t \le 24$$

where t is time (in hours) since 6 am. The width of the beach is 8 metres at 8 am and 3 metres at 5 pm.

a) Determine a and b. [2 marks]

b) Determine the rate of change of the width of the beach at 8 am and the first time after this when this rate of change is repeated. [2 marks]

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Question 14 (8 marks)

Ravi randomly sampled 200 different pet owners in Brisbane and found that 50 celebrate their pet's birthday.

a) Determine an approximate 95% confidence interval for the proportion of Brisbane pet owners who celebrate their pet's birthday. [2 marks]

Two of Ravi's friends also randomly sampled Brisbane pet owners. The results are shown in the table.

Friend's name	Number sampled	Number who celebrate their pet's birthday
Khadija	100	26
Tim	150	34

Khadija suggested a more precise estimate for the proportion of Brisbane pet owners who celebrate their pet's birthday could be obtained by combining their results.

b) Using all available data, determine an approximate 95% confidence interval for the proportion of Brisbane pet owners who celebrate their pet's birthday. [2 marks]

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c) Use the results from Questions 14a) and 14b) to evaluate the reasonableness of Khadija's suggestion. [2 marks]

The proportion of all Brisbane pet owners who celebrate their pet's birthday is 0.24.

d) Using the normal approximation, determine the probability that in a randomly selected sample of size 200, more than 30% of pet owners celebrate their pet's birthday. [2 marks]

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Question 15 (7 marks)

A hiker begins her journey at a youth hostel (*H*) and walks for 8 km on a bearing of 052°T to her lunch stop (*L*). She then walks on a bearing of 210°T for 5.2 km until she reaches a campsite (*C*).

Determine the direction she would need to walk in a straight line to return directly to the youth hostel.

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Question 16 (4 marks)

The time spent waiting in a queue at a certain supermarket is given by $(X+11)$ minutes, where X is a random variable with the probability density function

$$f(x)=\begin{cases}\dfrac{a\left(4-x^2\right)}{32}, & -2\le x\le 2\\ 0, & \text{otherwise}\end{cases}$$

Determine the probability of waiting between 10 and 12 minutes in a queue at this supermarket.

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Question 17 (4 marks)

A snail is travelling along a straight path from point *A*. The snail's velocity (cm min^{-1}) is modelled by $v(t) = 1.4\ln\left(1+t^2\right)$, where t is time (in minutes) for $0 \le t \le 15$.

An ant passes point *A* 12 minutes after the snail and follows the snail's path. The ant moves with a constant acceleration of 2 cm min^{-2} and passes the snail at $t = 15$ minutes.

Determine the ant's velocity at point *A*.

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Question 18 (3 marks)

The intelligence quotient (IQ) of individuals in a population is normally distributed, with a mean of 100 and a standard deviation of 16.

Nine individuals are chosen at random from the population.

Determine the probability that no more than two of the individuals have an IQ of at least 120.

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Question 19 (4 marks)

Flying foxes enter and leave a fruit-growing region every evening. The rate at which the flying foxes enter the region is modelled by the function

$$A(t) = 42\sin\left(0.03t - \frac{\pi}{3}\right) + 71,\ 0 \le t \le 240$$

The rate at which the flying foxes leave the region is modelled by the function

$$L(t) = 42\sin\left(0.04t - \frac{\pi}{3}\right) + 42,\ 0 \le t \le 240$$

Both $A(t)$ and $L(t)$ are measured in animals per minute and t is measured in minutes after 7 pm.

There are 100 flying foxes in the region at 7 pm.

Determine the maximum number of flying foxes in the region and the time that this occurs.

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End of paper

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