External assessment 2022

Question and response book

Mathematical Methods

Paper 1 — Technology-free

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are **not** permitted.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (45 marks)

- 9 short response questions

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

LUI

School code

School name

Given name/s

Family name

Book of books used

Attach your barcode ID label here

Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Do not write outside this box.

Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 45 marks.

Do not write on this page

This page will not be marked

Do not write outside this box.

Question 11 (5 marks)

Solve for x in the following.

a) $\ln(2x) = 5$ [2 marks]

b) $\log_4(4x+16) - \log_4(x^2-2) = 1$ [3 marks]

Do not write outside this box.

Question 12 (3 marks)

The probability that a debating team wins a debate can be modelled as a Bernoulli distribution. Given that the probability of winning a debate is $\frac{4}{5}$

a) Determine the mean of this distribution. [1 mark]

b) Determine the variance of this distribution. [1 mark]

c) Determine the standard deviation of this distribution. [1 mark]

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Question 13 (9 marks)

a) Determine the derivative of $f(x) = 3e^{2x+1}$ [1 mark]

b) Given that $g(x) = \dfrac{\ln(x)}{x}$, determine the simplest value of $g'(e)$. [3 marks]

c) Determine the second derivative of $h(x) = x\sin(x)$. (Give your answer in simplest form.) [5 marks]

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Question 14 (6 marks)

The rate that water fills an empty vessel is given by $\frac{dV}{dt} = 0.25e^{0.25t}$ (in litres per hour), $0 \leq t \leq 8\ln(6)$, where t is time (in hours).

a) Determine the function that represents the volume of water in the vessel (in litres). [2 marks]

The vessel is full when $t = 8\ln(6)$.

b) Determine the volume of water, to the nearest litre, the vessel can hold when full. [2 marks]

The table shows the approximate rate the water flows into the vessel at certain times.

t	$\frac{dV}{dt}$
0	0.25
1	0.32
2	0.41
3	0.53

Do not write outside this box.

c) Use information from the table and the trapezoidal rule to determine the approximate volume of water in the vessel after three hours. [2 marks]

Question 15 (4 marks)

The derivative of a function is given by $f'(x) = e^x(x-4)$.

Determine the interval on which the graph of $f(x)$ is both decreasing and concave up.

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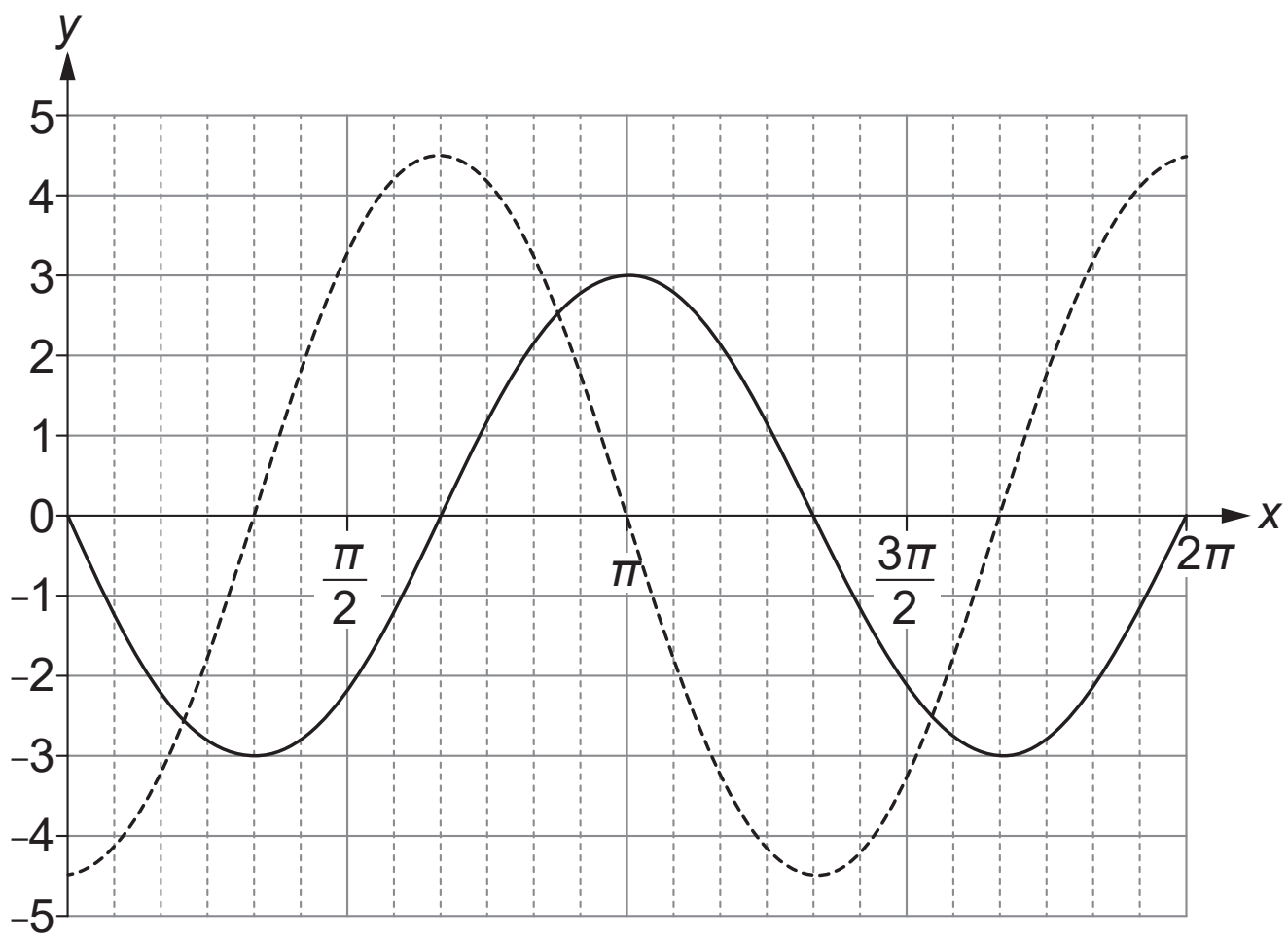
Question 16 (3 marks)

A section of the graphs of the first and second derivatives of a function are shown.

Sketch a possible graph of the function on the same axes over the domain $0 \leq x \leq 2\pi$.
Explain all reasoning used to produce the sketch.



Note: If you make a mistake in the graph, cancel it by ruling a single diagonal line through your work and use the additional response space on page 17 of this question and response book.

Do not write outside this box.

Question 17 (4 marks)

Determine the value of b given $\int_a^b 3x^2 dx = 117$ and $\int_a^{b-1} 3x^2 dx = 56$ for $b > 1$.

Question 18 (4 marks)

A percentile is a measure in statistics showing the value below which a given percentage of observations occur.

The continuous random variable X has the probability density function

$$f(x) = \begin{cases} 2x - 2, & 1 \leq x \leq 2 \\ 0 \quad , & \text{otherwise} \end{cases}$$

Determine the 36th percentile of X.

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Question 19 (7 marks)

Two triangles are said to be similar if their corresponding angles are congruent and the corresponding sides are in proportion, e.g. if ΔUVW is similar to ΔXYZ then

$\angle U = \angle X$, $\angle V = \angle Y$ and $\angle W = \angle Z$ and $\dfrac{UV}{XY} = \dfrac{VW}{YZ} = \dfrac{UW}{XZ}$

Two parallel walls AB and CD, where the northern ends are A and C respectively, are joined by a fence from B to C. The wall AB is 20 metres long, the angle $ABC = 30°$ and the fence BC is 10 metres long.

A new fence is being built from A to a point P somewhere along CD. The new fence AP will cross the original fence BC at O.

Let $OB = x$ metres, where $0 < x \leq 10$.

Determine the value of x that minimises the total area enclosed by ΔOBA and ΔOCP. Verify that this total area is a minimum.

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End of paper

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Additional page for student responses

Write the question number you are responding to.

Do not write outside this box.

Additional page for student responses

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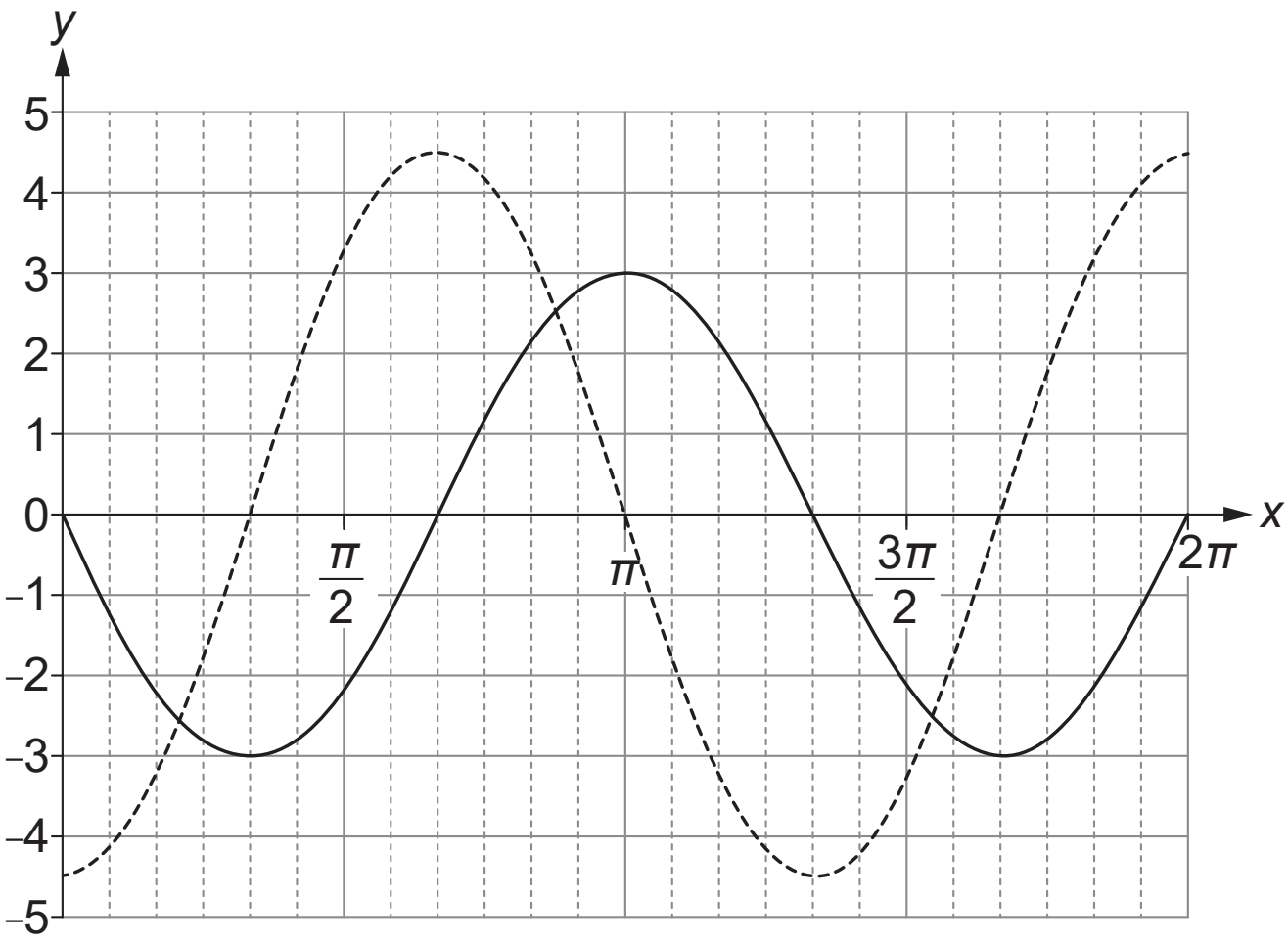
Additional response space for Question 16

If you want this graph to be marked, rule a single diagonal line through the graph on page 8.



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Clear zone — margin trimmed off after completion of assessment