Mathematical Methods marking guide

External assessment 2022

Paper 1: Technology-free (55 marks)

Paper 2: Technology-active (55 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4

Queensland Government
Queensland Curriculum & Assessment Authority
For all Queensland schools

Purpose

This marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded.

Where no response to a question has been made, a mark of ‘N’ will be recorded.

Allow FT mark/s — refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Multiple choice
Paper 1: Technology-free (55 marks)

Question	Response
1	A
2	D
3	C
4	C
5	D
6	B
7	B
8	A
9	A
10	D

Short response

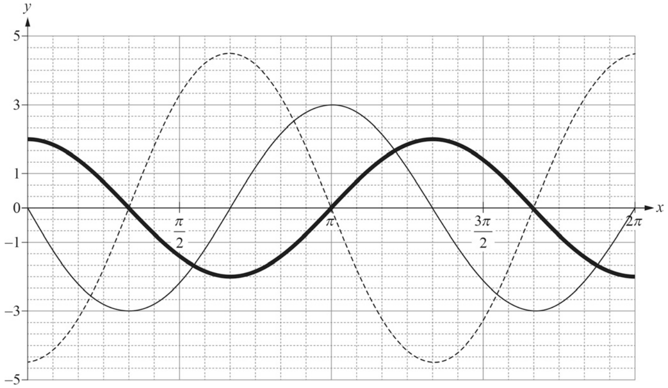
Q	Sample response	The response:
11a)	Change from log to index form and rearrange $2x = e^5$ $x = \frac{e^5}{2}$	• correctly rearranges equation to remove $\ln$ **[1 mark]** • correctly determines x **[1 mark]**
11b)	Using log laws $\log_4\left(\frac{4x+16}{x^2-2}\right) = 1$ $(4x+16) = 4(x^2-2)$ $x^2 - x - 6 = 0$ $(x-3)(x+2) = 0$ $x = 3, -2$	• correctly applies the log law **[1 mark]** • correctly determines quadratic equation to solve **[1 mark]** • determines possible values for x **[1 mark]**

Q	Sample response	The response:
12a)	<table><tr><td>x_i</td><td>0</td><td>1</td></tr><tr><td>p_i</td><td>$\frac{1}{5}$</td><td>$\frac{4}{5}$</td></tr></table> $E(X) = \sum p_i x_i$ $= \frac{1}{5} \times 0 + \frac{4}{5} \times 1$ $= \frac{4}{5}$	• correctly determines the mean **[1 mark]**
12b)	$Var(X) = \sum p_i (x_i - \mu)^2$ $= \frac{1}{5} \times \left(\frac{-4}{5}\right)^2 + \frac{4}{5} \times \left(\frac{1}{5}\right)^2$ $= \frac{4}{25}$	• correctly determines the variance **[1 mark]**
12c)	$Standard\ deviation = \sqrt{Variance}$ $= \frac{2}{5}$	• determines the standard deviation **[1 mark]**

Q	Sample response	The response:
13a)	$f'(x) = 6e^{2x+1}$	• correctly determines the derivative **[1 mark]**
13b)	$g(x) = \frac{\ln(x)}{x}$ Let $u = \ln(x)$ and $v = x$ $\therefore \frac{du}{dx} = \frac{1}{x}$ and $\frac{dv}{dx} = 1$ $g'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $= \frac{x \times \frac{1}{x} - \ln(x) \times 1}{x^2}$ $= \frac{1-\ln(x)}{x^2}$ $g'(e) = \frac{1-\ln(e)}{(e)^2} = 0$	• correctly identifies the use of the product or quotient rule **[1 mark]** • correctly determines the derivative **[1 mark]** • determines the derivative at the given value **[1 mark]**
13c)	$h(x) = x\sin(x)$ Let $u = x$ and $v = \sin(x)$ $\therefore \frac{du}{dx} = 1$ and $\frac{dv}{dx} = \cos(x)$ $h'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$ $= x \times \cos(x) + \sin(x) \times 1$ $= x\cos(x) + \sin(x)$ Let $u = x$ and $v = \cos(x)$ $\therefore \frac{du}{dx} = 1$ and $\frac{dv}{dx} = -\sin(x)$ $h''(x) = u\frac{dv}{dx} + v\frac{du}{dx} + \cos(x)$ $= x \times -\sin(x) + \cos(x) \times 1 + \cos(x)$ $= 2\cos(x) - x\sin(x)$	• correctly identifies the use of the product rule **[1 mark]** • correctly determines the derivative **[1 mark]** • correctly identifies the use of the product rule and that $\frac{d}{dx}(h(x) + g(x)) = \frac{d}{dx}h(x) + \frac{d}{dx}g(x)$ **[1 mark]** • determines the second derivative **[1 mark]** • simplifies the second derivative **[1 mark]**

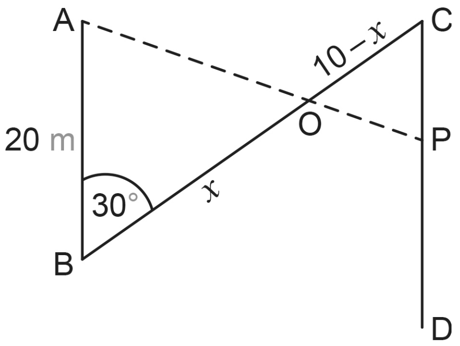
Q	Sample response	The response:
14a)	$V = \int 0.25e^{0.25t}\,dt$ $= e^{0.25t} + c$ when $t = 0, V = 0$ $\therefore 0 = e^{0.25\times 0} + c$ $\therefore c = -1$ $\therefore V = e^{0.25t} - 1$	• correctly determines the integral of the function $V(t)$ **[1 mark]** • determines the value of *c* **[1 mark]**
14b)	$V(8\ln(6)) = e^{0.25\times 8\ln(6)} - 1$ $= 36 - 1$ $= 35$ litres	• determines the simplified exponential term **[1 mark]** • determines number of litres **[1 mark]**
14c)	Using trapezoidal rule Volume after 3 hours = $\frac{1}{2}(0.25+0.53+2(0.32+0.41))$ Volume after 3 hours $= 1.12$ litres	• establishes expression for approximate number of litres of water in vessel after 3 hours **[1 mark]** • determines approximate number of litres **[1 mark]**

Q	Sample response	The response:
15	The function is decreasing when $f'(x) < 0$ and concave up when $f''(x) > 0$ $f'(x) = (x-4)e^x < 0$ when $x < 4$ $f''(x) = (x-4)e^x + e^x = e^x(x-3) > 0$ when $x > 3$ Therefore, the function is decreasing and concave up when $3 < x < 4$	• correctly describes conditions when the function is decreasing and concave up **[1 mark]** • correctly determines the interval where $f(x)$ is decreasing **[1 mark]** • correctly determines the interval where $f(x)$ is concave up **[1 mark]** • determines interval when function is decreasing and concave up **[1 mark]**

Q	Sample response	The response:
16	The provided functions are a sine and a cosine function, both with negative amplitudes and the same period. Therefore, the smooth line is $y' = -a\sin(bx)$, and the dotted line is $y'' = -A\cos(bx)$, with $a = 3$ and $A = 4.5$. Therefore, the primary function must be a cosine function with the same period i.e. $y = 2\cos\left(\frac{3}{2}x\right)$ 	• correctly identifies the smooth curve as the first derivative **[1 mark]** • provides mathematical reasoning for sketch **[1 mark]** • sketches the function **[1 mark]**

Q	Sample response	The response:
17	Using first integral $F(b) - F(a) = 117$ $b^3 - a^3 = 117 \ldots$ Equation I Using second integral $(b-1)^3 - a^3 = 56 \ldots$ Equation II Equation I – Equation II $b^3 - (b-1)^3 = 61$ $b^3 - (b^3 - 3b^2 + 3b - 1) = 61$ $3b^2 - 3b - 60 = 0$ $b^2 - b - 20 = 0$ $(b-5)(b+4) = 0$ $b = -4, 5$ Given $b > 1$ $\therefore b = 5$	• correctly establishes a formula for one of the integrals **[1 mark]** • determines equation in b **[1 mark]** • determines values of b **[1 mark]** • evaluates the reasonableness of solutions **[1 mark]**

Q	Sample response	The response:
18	$\int_1^a 2x - 2\,dx = 0.36$ $x^2 - 2x\Big\vert_1^a = 0.36$ $(a^2 - 2a) - (1 - 2) = 0.36$ $a^2 - 2a + 1 = 0.36$ $a^2 - 2a + 0.64 = 0$ $\therefore a = \frac{2 \pm \sqrt{4 - 4 \times 1 \times 0.64}}{2}$ $\therefore a = \frac{2 \pm \sqrt{1.44}}{2}$ $\therefore a = \frac{2 \pm 1.2}{2}$ $\therefore a = 1.6 \text{ or } 0.4$ $\text{Given } 1 \le x \le 2$ $\therefore a = 1.6$	• correctly determines the definite integral **[1 mark]** • determines the quadratic equation **[1 mark]** • determines values of a **[1 mark]** • evaluates the reasonableness of solutions **[1 mark]**

Q	Sample response	The response:
19		• correctly uses all of the given information to draw a labelled diagram **[1 mark]**
	Total area OBA and OCP (A_T)(using area rule) $= \frac{1}{2} \times 20 \times x \times \sin 30° + \frac{1}{2} \times (10-x) \times CP \times \sin 30°$	• correctly establishes a formula for the total area **[1 mark]**
	Given $\frac{CP}{AB} = \frac{OC}{OB}$ $\therefore \frac{CP}{20} = \frac{10-x}{x}$ $\therefore CP = \frac{20(10-x)}{x}$ $A_T = \frac{1}{2} \times 20 \times x \times \sin 30° + \frac{1}{2} \times (10-x) \times \frac{20(10-x)}{x} \times \sin 30°$ $= 5x + \frac{5}{x}(10-x)^2$	• correctly determines an expression for *CP* in terms of x **[1 mark]**
	$A_T = \frac{10x^2 - 100x + 500}{x}$	• determines simplified version of the formula for total area **[1 mark]**

Q	Sample response	The response:
	Differentiate A_T $A_T = 10x + 500x^{-1} - 100$ $A'_T = 10 - 500x^{-2}$ Let $A'_T = 0$ $\therefore 0 = 10 - 500x^{-2}$ $\therefore \frac{500}{x^2} = 10$ $\therefore \quad x = \pm\sqrt{50}$ x is a positive length $\therefore \quad x = \sqrt{50}$ Verifying using $f''(x)$ $f'(x) = 10 - 500x^{-2}$ $f''(x) = \frac{1000}{x^3}$ $f''\left(\sqrt{50}\right) > 0 \therefore \text{minimum}$ Therefore when $x = \sqrt{50}$ the total area is minimised.	• determines an equation to solve for stationary points **[1 mark]** • evaluates the reasonableness of solutions **[1 mark]** • verifies solution **[1 mark]**

Marking guide

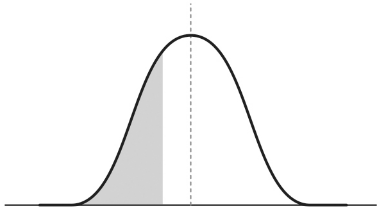
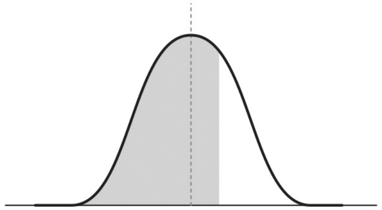
Multiple choice
Paper 2: Technology-active (55 marks)

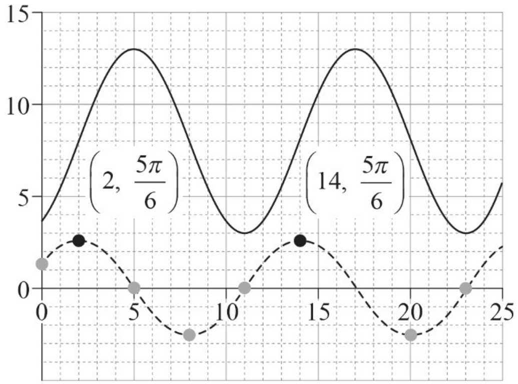
Question	Response
1	D
2	B
3	C
4	C
5	B
6	B
7	D
8	C
9	B
10	C

Note that question 3 has been updated to indicate option C as the correct answer.

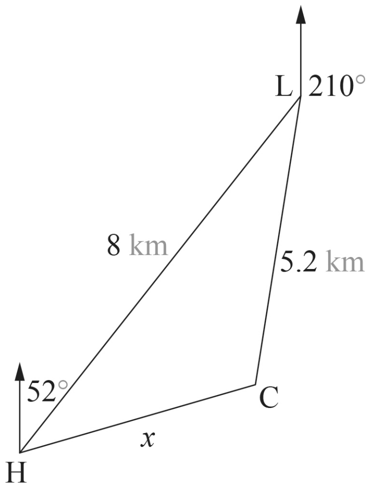
Short response

Q	Sample response	The response:
11a)	Mean number of sales $= np$ $= 25 \times 0.2$ $= 5$	• correctly substitutes into formula for mean **[1 mark]** • correctly determines the mean **[1 mark]**
11b)	Standard deviation of number of sales $= \sqrt{np(1-p)}$ $= \sqrt{25 \times 0.2 \times (1-0.2)}$ $= 2$	• correctly substitutes into formula for standard deviation **[1 mark]** • correctly determines the standard deviation **[1 mark]**
11c)	$1 - \binom{n}{0} 0.2^0 0.8^n \geq 0.88$ $\therefore 1 - 0.8^n \geq 0.88$ $\therefore 0.8^n \leq 0.12$ $\therefore n \geq \log_{0.8}(0.12)$ $\therefore n \geq 9.50179$ $\therefore$ minimum number of customers would be 10.	• correctly determines the required inequation **[1 mark]** • correctly determines the unknown in the inequation **[1 mark]** • determines the minimum number of customers **[1 mark]**

Q	Sample response	The response:
12a)	 $P(x < 12000) = 0.3085$	• correctly uses an appropriate mathematical representation **[1 mark]** • correctly determines the probability **[1 mark]**
12b)	 $x = 14\,561.38$ kilometres	• correctly uses an appropriate mathematical representation **[1 mark]** • correctly determines the value of x **[1 mark]**

Q	Sample response	The response:
13a)	$w(2) = 8$ $\therefore a + b\sin(0) = 8$ $\therefore a = 8$ $w(11) = 3$ $\therefore 8 + b\sin\left(\frac{3\pi}{2}\right) = 3$ $\therefore b \times -1 = -5$ $\therefore b = 5$	• correctly determines a **[1 mark]** • correctly determines b **[1 mark]**
13b)	 The rate of change at 8am is $\frac{5\pi}{6}$. Using sketch $t = 14$ At 8 pm the rate is the same (for the first time).	• determines rate when $t = 2$ **[1 mark]** • determines first time when rate is the same as $t = 2$ **[1 mark]**

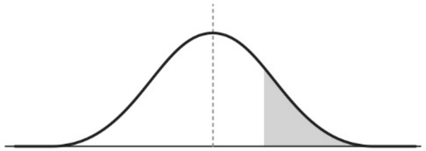
Q	Sample response	The response:
14a)	Using GDC to determine confidence interval associated with $n = 200, \hat{p} = 0.25, z = 1.96$ $(0.19, 0.31)$	• correctly identifies all of the information required to establish the confidence interval **[1 mark]** • correctly determines the confidence interval **[1 mark]**
14b)	Combining results $n = 450, \hat{p} = \frac{11}{45}$ Using GDC $(0.2047, 0.2842)$	• correctly determines n and $\hat{p}$ for the combined sample **[1 mark]** • determines confidence interval **[1 mark]**
14c)	By combining the results, the sample size is increased and the confidence interval width is reduced. The new sample statistic provides a better estimate for the population parameter.	• identifies changed width of confidence interval **[1 mark]** • evaluates the reasonableness of Khadija's suggestion **[1 mark]**
14d)	Using approximation to the normal distribution $\text{Mean} = 0.24$ $\text{Standard deviation} = \sqrt{\frac{0.24 \times 0.76}{200}} = 0.0302$ Using GDC $P(\hat{p} > 0.30) = 0.0235$	• correctly determines the mean and standard deviation of the normal distribution **[1 mark]** • determines the probability **[1 mark]**

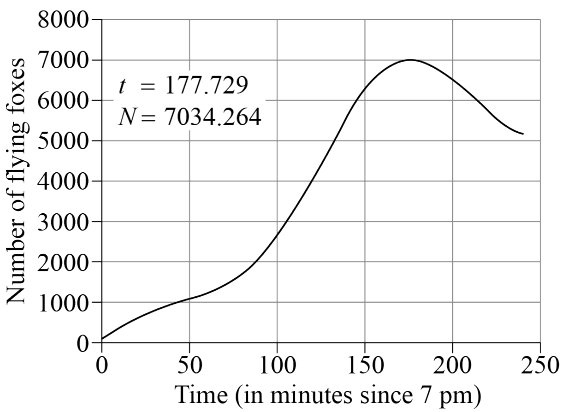
Q	Sample response	The response:
15	 $H =$ $h = 5.2$ $L = 22°$ $l = x$ $C =$ $c = 8$ Using cosine rule $x^2 = 8^2 + 5.2^2 - 2 \times 8 \times 5.2 \times \cos 22°$ $x = 3.7280$ km Using sine rule $\frac{3.7280}{\sin 22°} = \frac{8}{\sin C}$ $C = 53.5019°$ or $C = 126.4981°$ Bearing = 360 – 126.4981° + 30 ∴ walk back at a bearing of $263°30'$ T	• correctly establishes the diagram using bearing and distance information **[1 mark]** • correctly determines angle L **[1 mark]** • determines unknown distance **[1 mark]** • substitutes into appropriate rule **[1 mark]** • determine obtuse angle at C **[1 mark]** • determines direction needed to return to youth hostel using the obtuse angle for C **[1 mark]**

Q	Sample response	The response:
		• shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
16	$\int_{-2}^{2} \frac{a(4-x^2)}{32} dx = 1$ Using GDC (solving for equation) $a = 3$ $P(-1 \leq X \leq 1)$ $= \int_{-1}^{1} \frac{a(4-x^2)}{32} dx$ $= 0.6875$	• correctly identifies required integral equation to solve **[1 mark]** • correctly determines the value of a **[1 mark]** • correctly identifies interval **[1 mark]** • determines probability **[1 mark]**

Q	Sample response	The response:
17	Total displacement of the snail $\int_0^{15} 1.4\ln(1+t^2)\,dt = \mathbf{76.0431}$ cm Velocity of the ant $= \int 2\,dt$ $= 2t + c$ $\text{Displacement}_{\text{ant from 12 to 15min}} = \text{Displacement}_{\text{snail from 0 to 15min}}$ $\therefore \int_{12}^{15} 2t + c = 76.0431$ Solving numerically on GDC $c = -1.6523$ Therefore, velocity of ant at $t = 12$ $= 2 \times 12 - 1.6523$ $= 22.3477$ cm min^{-1} along the ant's path.	• correctly determines the total displacement of the snail **[1 mark]** • establishes an equation linking the ant and the snail **[1 mark]** • determines constant **[1 mark]** • determines velocity of ant **[1 mark]**

Q	Sample response	The response:
18	 $P(IQ \geq 120) = 0.1057$ Using binomial distribution $n = 9, p = 0.1057$ Using GDC $P(x \leq 2) = 0.9391$	• correctly determines the probability of IQ ≥ 120 **[1 mark]** • correctly recognises context is suitable for modelling as a binomial **[1 mark]** • determines the required probability **[1 mark]**

Q	Sample response	The response:
19	Number of flying foxes entering the region $= \int 42\sin\left(0.03t - \frac{\pi}{3}\right) + 71$ $= -1400\cos\left(0.03t - \frac{\pi}{3}\right) + 71t + c_1$ Number of flying foxes leaving the region $L(t)$ $= \int 42\sin\left(0.04t - \frac{\pi}{3}\right) + 42$ $= -\frac{42}{0.04}\cos\left(0.04t - \frac{\pi}{3}\right) + 42t + c_2$ Number of flying foxes in region $N(t)$ $= -1400\cos\left(0.03t - \frac{\pi}{3}\right) + 71t + c_1$ $-\left(-1050\cos\left(0.04t - \frac{\pi}{3}\right) + 42t + c_2\right)$ Given number of flying foxes in the region at time $t =$ 0 is 100 $N(t) = -1400\cos\left(0.03t - \frac{\pi}{3}\right) + 1050\cos\left(0.04t - \frac{\pi}{3}\right) + 29t + 275$	• correctly determines the function to model the total number of flying foxes in the region **[1 mark]**
	Using GDC to sketch $N(t)$  	• uses an appropriate mathematical method to identify the maximum values **[1 mark]**
	Using a GDC, the maximum point is identified (177.729, 7034.264). Maximum of 7034 flying foxes in the region at 9:58 pm.	• determines the maximum number of flying foxes and time, i.e. the coordinates of the turning point **[1 mark]** • states the number of flying foxes as a whole number and the time after 7 pm it occurs **[1 mark]**