

# Mathematical Methods subject report

2021 cohort

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# Contents

<b>Introduction</b> .....	<b>1</b>
Audience and use .....	1
Report preparation .....	1
<b>Subject data summary</b> .....	<b>2</b>
Subject completion .....	2
Units 1 and 2 results .....	2
Units 3 and 4 internal assessment (IA) results .....	2
Total marks for IA .....	2
IA1 marks .....	3
IA2 marks .....	4
IA3 marks .....	5
External assessment (EA) marks .....	6
Final subject results .....	7
Final marks for IA and EA .....	7
Grade boundaries .....	7
Distribution of standards .....	7
<b>Internal assessment</b> .....	<b>8</b>
Endorsement .....	8
Confirmation .....	8
<b>Internal assessment 1 (IA1)</b> .....	<b>10</b>
Problem-solving and modelling task (20%) .....	10
Assessment design .....	10
Assessment decisions .....	12
<b>Internal assessment 2 (IA2)</b> .....	<b>17</b>
Examination — short response (15%) .....	17
Assessment design .....	17
Assessment decisions .....	19
<b>Internal assessment 3 (IA3)</b> .....	<b>23</b>
Examination — short response (15%) .....	23
Assessment design .....	23
Assessment decisions .....	25
<b>External assessment</b> .....	<b>29</b>
Summative external assessment (EA) — Examination (50%) .....	29
Assessment design .....	29
Assessment decisions .....	29
<b>Senior External Examination</b> .....	<b>40</b>
Distribution of standards .....	40
Assessment decisions .....	40

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# Introduction

Despite the challenges brought about by the COVID-19 pandemic, Queensland's education community can look back on 2021 with satisfaction at having implemented the first full assessment cycle in the new Queensland Certificate of Education (QCE) system. That meant delivering three internal assessments and one external assessment in each General subject.

This report analyses that cycle — from endorsing summative internal assessment instruments to confirming internal assessment marks, and designing and marking external assessment. It also gives readers information about:

- applying syllabus objectives in the design and marking of internal and external assessments
- patterns of student achievement.

The report promotes continuous improvement by:

- identifying effective practices in the design and marking of valid, accessible and reliable assessments
- recommending where and how to enhance the design and marking of valid, accessible and reliable assessment instruments
- providing examples of best practice where relevant, possible and appropriate.

## Audience and use

This report should be read by school leaders, subject leaders and teachers to:

- inform teaching and learning and assessment preparation
- assist in assessment design practice
- assist in making assessment decisions
- help prepare students for external assessment.

The report is publicly available to promote transparency and accountability. Students, parents, community members and other education stakeholders can learn about the assessment practices and outcomes for General subjects (including alternative sequences (AS) and Senior External Examination (SEE) subjects, where relevant) and General (Extension) subjects.

## Report preparation

The report includes analyses of data and other information from endorsement, confirmation and external assessment processes. It also includes advice from the chief confirmer, chief endorser and chief marker, developed in consultation with and support from QCAA subject matter experts.



# Subject data summary

## Subject completion

The following data includes students who completed the General subject or AS.

For the purposes of this report, while the 2021 summative units for the AS are AS units 1 and 2, this information will be included with the General summative Units 3 and 4.

**Note:** All data is correct as at 17 December 2021. Where percentages are provided, these are rounded to two decimal places and, therefore, may not add up to 100%.

Number of schools that offered the subject: 416.

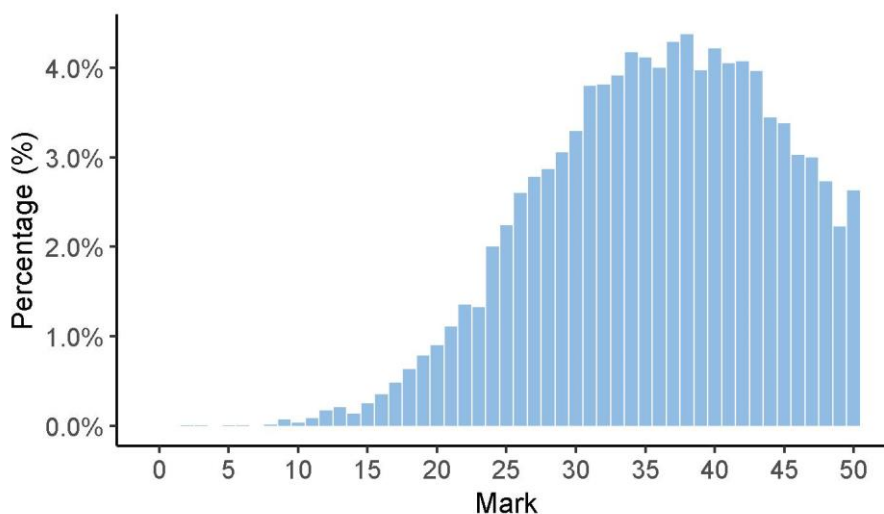
Completion of units	Unit 1	Unit 2	Units 3 and 4
Number of students completed	13778	12006	9838

## Units 1 and 2 results

Number of students	Satisfactory	Unsatisfactory
Unit 1	12664	1114
Unit 2	10164	1842

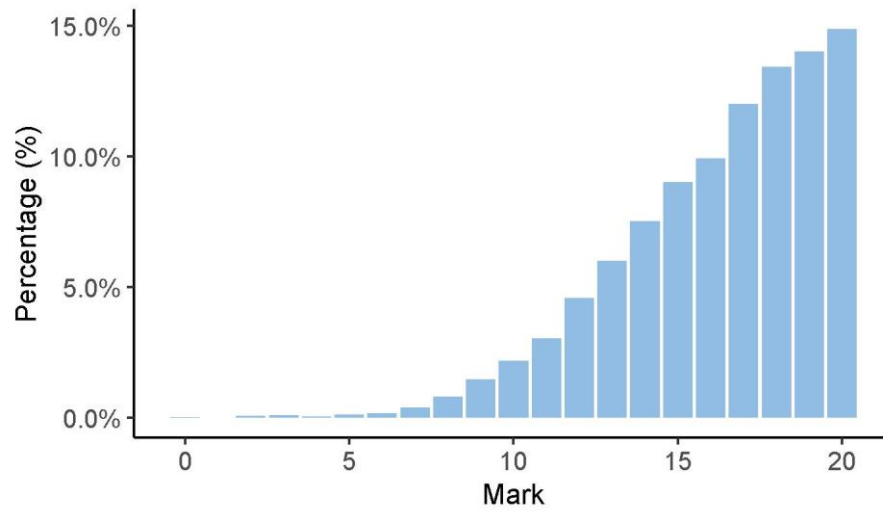
## Units 3 and 4 internal assessment (IA) results

### Total marks for IA

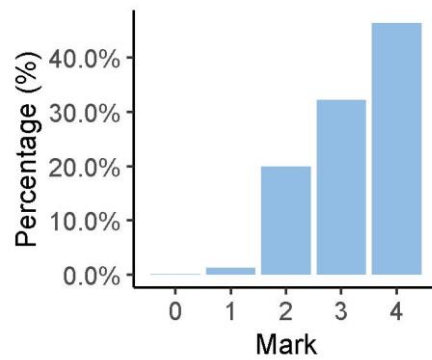


## IA1 marks

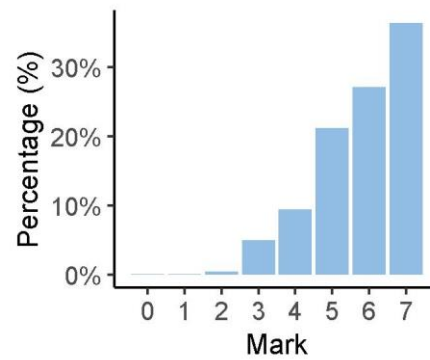
### IA1 total



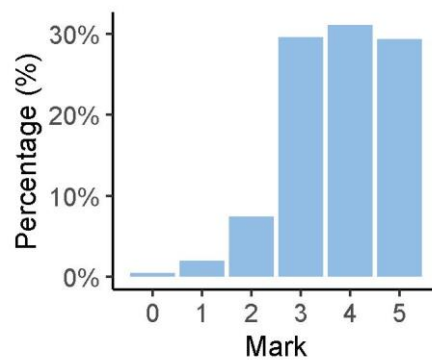
### IA1 Criterion: Formulate



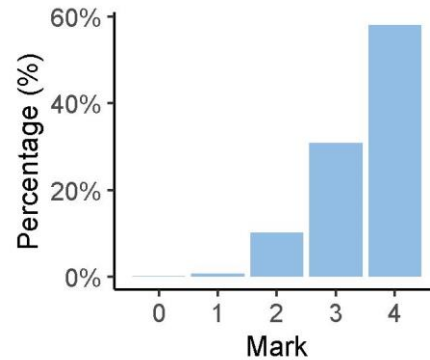
### IA1 Criterion: Solve



### IA1 Criterion: Evaluate and verify

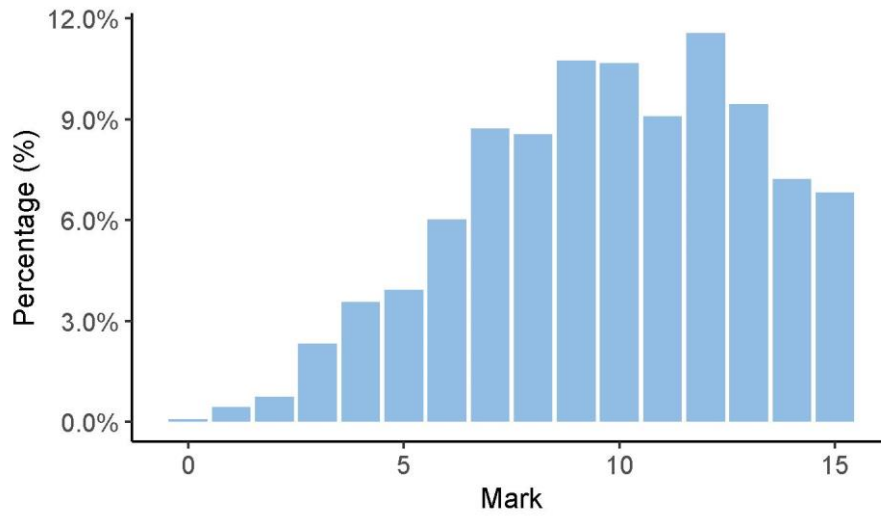


### IA1 Criterion: Communicate

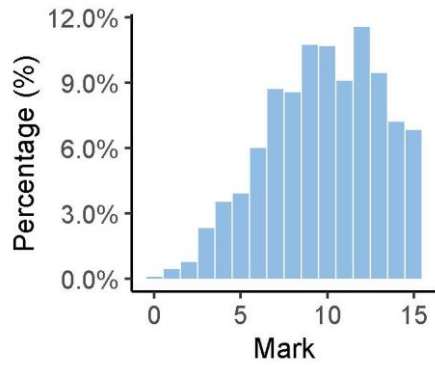


## IA2 marks

### IA2 total

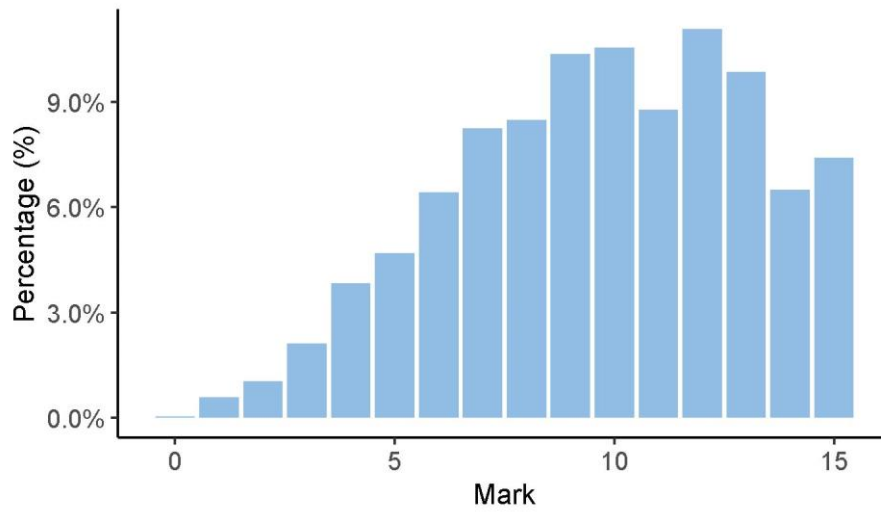


### IA2 Criterion: Foundational knowledge and problem-solving

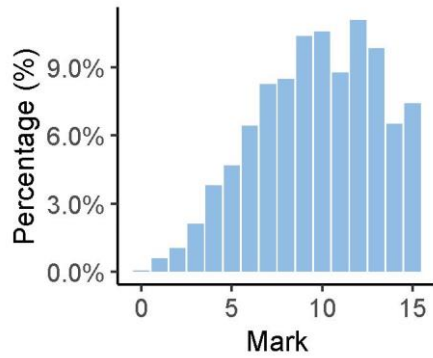


## IA3 marks

### IA3 total

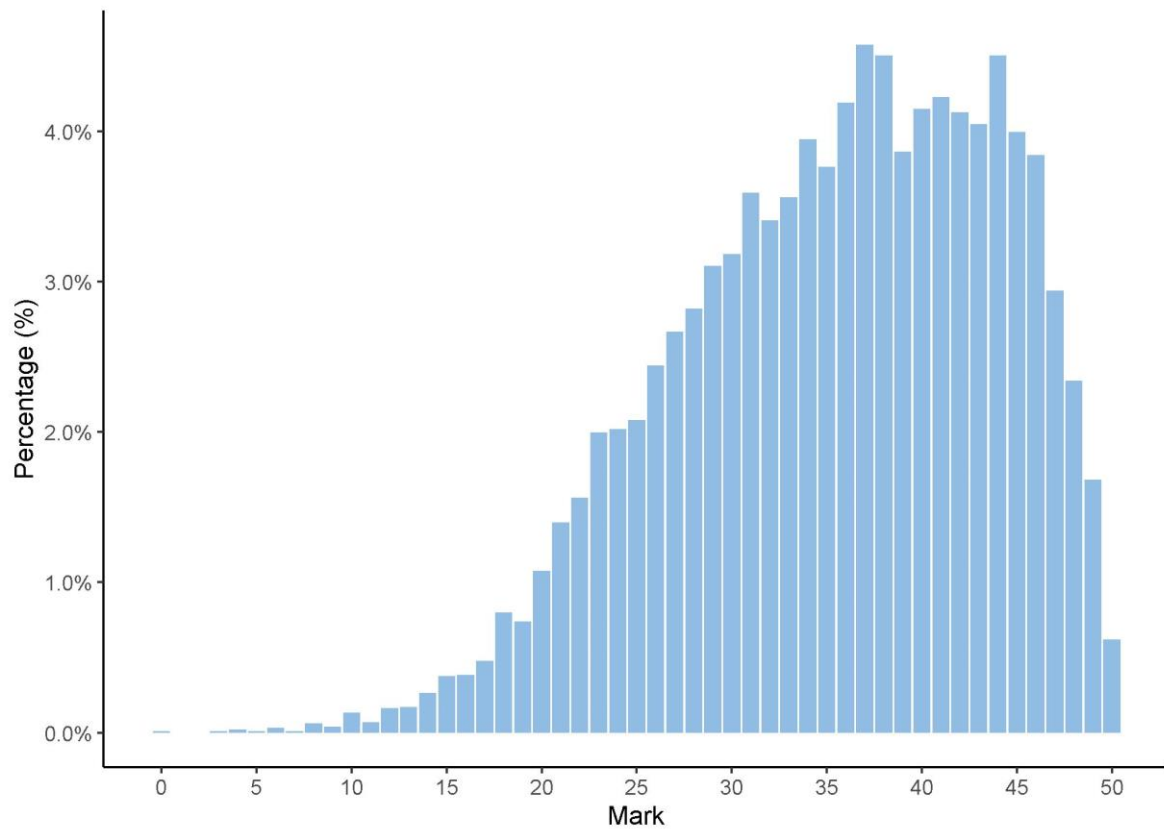


### IA3 Criterion: Foundational knowledge and problem-solving



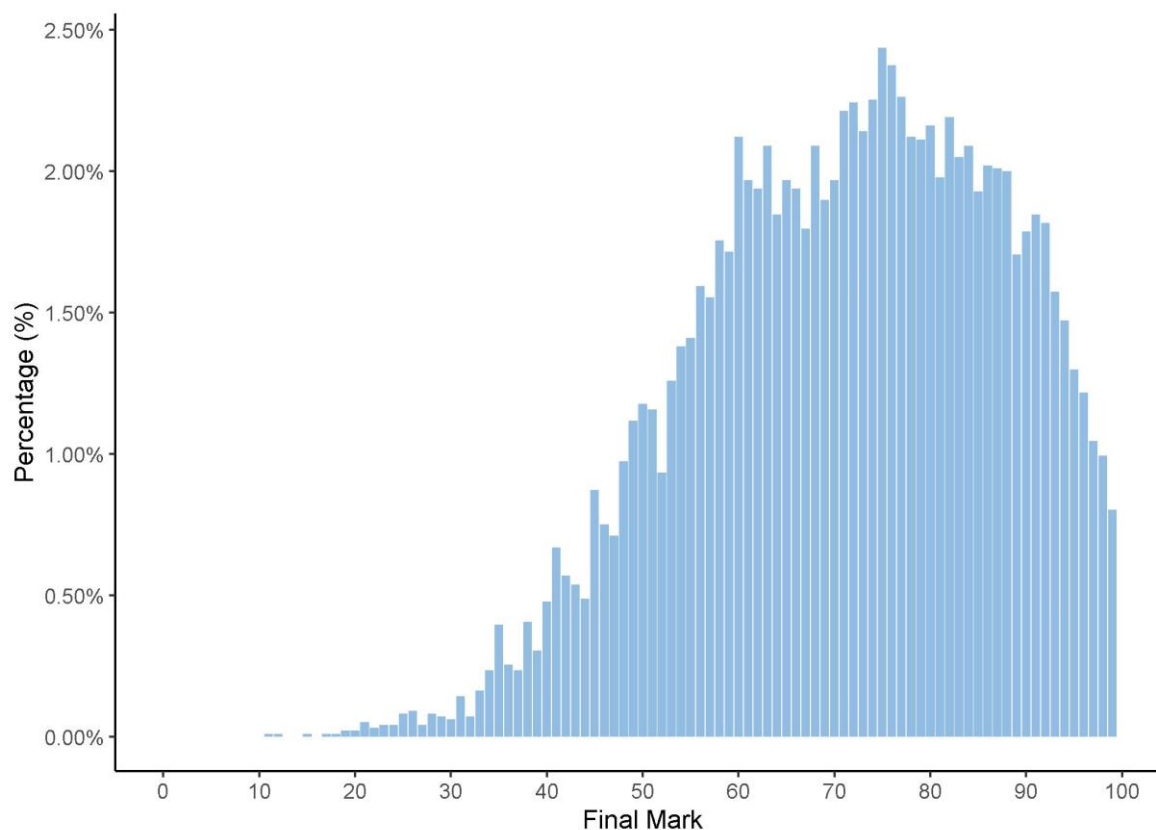


## External assessment (EA) marks



# Final subject results

## Final marks for IA and EA



## Grade boundaries

The grade boundaries are determined using a process to compare results on a numeric scale to the reporting standards.

Standard	A	B	C	D	E
Marks achieved	100–84	83–66	65–44	43–20	19–0

## Distribution of standards

The number of students who achieved each standard across the state is as follows.

Standard	A	B	C	D	E
Number of students	2547	3762	3024	497	7



# Internal assessment

The following information and advice pertain to the assessment design and assessment decisions for each IA in Units 3 and 4. These instruments have undergone quality assurance processes informed by the attributes of quality assessment (validity, accessibility and reliability).

## Endorsement

Endorsement is the quality assurance process based on the attributes of validity and accessibility. These attributes are categorised further as priorities for assessment, and each priority can be further broken down into assessment practices.

Data presented in the Assessment design section identifies the reasons why IA instruments were not endorsed at Application 1, by the priority for assessments. An IA may have been identified more than once for a priority for assessment, e.g. it may have demonstrated a misalignment to both the subject matter and the assessment objective/s.

Refer to the quality assurance tools for detailed information about the assessment practices for each assessment instrument.

### Percentage of instruments endorsed in Application 1

Number of instruments submitted	IA1	IA2	IA3
Total number of instruments	418	418	415
Percentage endorsed in Application 1	66%	29%	53%

## Confirmation

Confirmation is the quality assurance process based on the attribute of reliability. The QCAA uses provisional criterion marks determined by teachers to identify the samples of student responses that schools are required to submit for confirmation.

Confirmation samples are representative of the school's decisions about the quality of student work in relation to the ISMG and are used to make decisions about the cohort's results. If further information is required about the school's application of the ISMG to finalise a confirmation decision, the QCAA requests additional samples.

Schools may request a review where an individual student's confirmed result is different from the school's provisional mark in one or more criteria and the school considers this result to be an anomaly or exception.

The following table includes the percentage agreement between the provisional marks and confirmed marks by assessment instrument. The Assessment decisions section of this report for each assessment instrument identifies the agreement trends between provisional and confirmed marks by criterion.

**Number of samples reviewed and percentage agreement**

<b>IA</b>	<b>Number of schools</b>	<b>Number of samples requested</b>	<b>Number of additional samples requested</b>	<b>Percentage agreement with provisional marks</b>
<b>1</b>	411	2466	235	87.86%
<b>2</b>	411	2280	0	99.27%
<b>3</b>	410	2252	0	98.54%



## Internal assessment 1 (IA1)

### Problem-solving and modelling task (20%)

This problem-solving and modelling task must use subject matter from one or both of the following topics in Unit 3:

- Topic 2: Further differentiation and applications 2
- Topic 3: Integrals.

The problem-solving and modelling task is an assessment instrument developed in response to a mathematical investigative scenario or context. It requires students to respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Students must provide a response to a specific task or issue that is set in a context that highlights a real-life application of mathematics. The task requires students to use relevant stimulus material involving the selected subject matter and must have sufficient scope to allow students to address all the stages of the problem-solving and modelling approach. Technology must be used.

### Assessment design

#### Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

#### Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	81
Authentication	15
Authenticity	24
Item construction	9
Scope and scale	31

\*Each priority might contain up to four assessment practices.

Total number of submissions: 418.

## Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided realistic contexts that related to subject matter and were accessible to students, e.g. designing a logo or stained-glass window, finding the volume of a pool or pond, modelling the rate of population growth, investigating climate change, or designing a rollercoaster or go-cart track
- provided opportunity for students to develop a unique response, e.g. providing an open-ended task, individual datasets, or models such that students made choices about how to use the data, and what concepts and techniques were relevant to solve the problem
- followed conventions for item construction where stimulus items were relevant to the task, e.g. including relevant images or data.

## Practices to strengthen

It is recommended that assessment instruments:

- use the approach to problem-solving and mathematical modelling as specified in the syllabus (Section 1.2.4 Figure 4) without scaffolding or task instructions that indicate to students how to solve the problem, as this interferes with students' ability to demonstrate their knowledge and understanding of the relevant criteria and to provide a unique, authentic response
- include task constraints or specifications that assist students to develop a response of an appropriate scope and scale, e.g. if designing a 'smooth' rollercoaster ride, provide a mathematical definition of 'smooth' in the stimulus to remove any research component and encourage students to be more discerning in their application of mathematical concepts in order to address all aspects of the problem-solving and modelling process within specified conditions
- assess subject matter within the scope and scale of the syllabus, e.g. volumes of solids of revolution about an axis is part of the Specialist Mathematics syllabus
- include checkpoints that reflect the school's assessment policy and clearly indicate when and how teachers provide feedback on one draft.

## Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

### Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	6
Language	31
Layout	4
Transparency	20

\*Each priority might contain up to four assessment practices.

Total number of submissions: 418.

## Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- featured a clear layout, where text and other items (e.g. MathType, tables and graphs) appeared aligned and in their entirety on the page
- featured a specific task or issue that was written in a straightforward manner and explicit about the nature of the problem
- used appropriate language, diagrams and images.

## Practices to strengthen

It is recommended that assessment instruments:

- use the language of the assessment objectives, e.g. 'evaluate the reasonableness of solutions by considering assumptions'
- are reviewed before submission to check for typographical, grammatical, punctuation and spelling errors.

## Additional advice

- It is recommended that schools develop an exemplar response to the problem-solving and modelling task prior to the task being submitted for endorsement. This does not have to be a full report but rather what would be expected for full marks being awarded in all criteria. This would:
  - enhance marker accuracy and reliability across the teaching team
  - ensure that students have an opportunity to demonstrate all aspects of the ISMG within the conditions specified by the syllabus.

## Assessment decisions

### Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

### Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Formulate	91.5%	7.52%	0.73%	0.24%
2	Solve	96.36%	2.67%	0.73%	0.24%
3	Evaluate and verify	93.69%	5.58%	0.73%	0%
4	Communicate	98.3%	0.73%	0.97%	0%

## Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- making judgments about the 'accurate use of complex procedures to reach a valid solution' within the Solve criterion, the identification of complex procedures was clearly annotated, and the validity of the solution was mentioned in the student response and annotated by the marker
- making judgments about the use of technology within the Solve criterion, markers clearly identified when the use was 'accurate and appropriate' as opposed to just 'using technology' and the 'superficial use of technology'
- making judgments about the justifications of decisions using mathematical reasoning in the Evaluate and verify criterion, the reasoning was clearly identified in the justification
- making judgments about strengths and limitations of the solution and/or model in the Evaluate and verify criterion, judgments accurately identified where responses documented the relevant strengths and limitations as opposed to statements
- making judgments about the vocabulary and conventions and the organisation of the response in the Communicate criterion, judgments accurately identified where responses contained technical and procedural vocabulary and correct and concise organisation of the response.

## Samples of effective practices

The following are excerpts from responses that illustrate the characteristics for the criterion at the performance level indicated. The excerpts may provide evidence of more than one criterion. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

This student response excerpt has been included:

- to demonstrate the accurate use of complex procedures (integration techniques) to validate a solution obtained using technology.



**Solve  
(6–7 marks)**

- accurate use of complex procedures to reach a valid solution
- discerning application of mathematical concepts and techniques relevant to the task
- accurate and appropriate use of technology

**Excerpt 1****Complex example: Bottle 1; Section 2**

$$f(x) = 0.0872619(x - 11.4482)^2 + 2.59164$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

To integrate this function, it first needs to be expanded and then squared and simplified.

$$f(x) = 0.0872619(x - 11.4482)^2 + 2.59164$$

$$f(x) = 0.0872619(x^2 - 22.8964x + 131.0613) + 2.59164$$

$$f(x) = 0.0872619x^2 - 1.998x + 11.4367 + 2.59164$$

$$f(x) = 0.0872619x^2 - 1.998x + 14.0283$$

$$\{f(x)\}^2 = (0.0872619x^2 - 1.998x + 14.0283)^2$$

$$\{f(x)\}^2 = (0.0872619x^2 - 1.998x + 14.0283)(0.0872619x^2 - 1.998x + 14.0283)$$

$$\{f(x)\}^2 = 0.00761464x^4 - 0.176347x^3 + 1.224136x^2 + 0.176347x^3 + 3.992x^2 - 28.0285x + 1.224136x^2 - 28.0285x + 196.7932$$

$$\{f(x)\}^2 = 0.00761464x^4 - 0.348699x^3 + 6.44028x^2 - 56.0571x + 196.793$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

$$V = \pi \int_8^{15} (0.00761464x^4 - 0.348699x^3 + 6.44028x^2 - 56.0571x + 196.793) dx$$

$$V = \pi [0.0015229x^5 - 0.087175x^4 + 2.14676x^3 - 28.0285x^2 + 196.793x]_8^{15}$$

$$V = \pi \{ (0.0015229(15)^5 - 0.087175(15)^4 + 2.14676(15)^3 - 28.0285(15)^2 + 196.793(15)) - (0.0015229(8)^5 - 0.087175(8)^4 + 2.14676(8)^3 - 28.0285(8)^2 + 196.793(8)) \}$$

$$V = \pi(634.0041 - 572.4915)$$

$$V = \pi \times 61.5126$$

$$V = 193.25\text{cm}^3 = 193.25\text{ml}$$

The volume for this section calculated using Wolfram Alpha was  $193.38\text{cm}^3$ . This difference is accounted for by rounding errors.

This student response excerpt has been included:

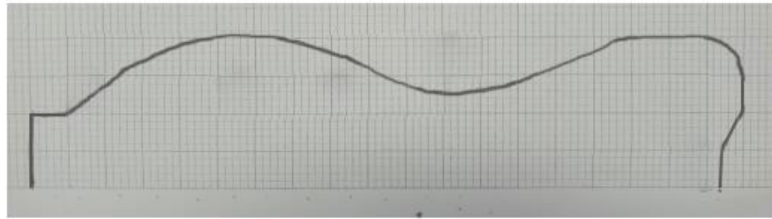
- to demonstrate accurate and appropriate use of technologies
- to demonstrate accurate use of complex procedures to obtain a valid solution.

**Solve  
(6–7 marks)**

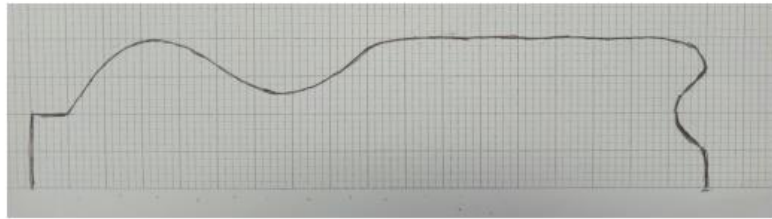
- accurate use of complex procedures to reach a valid solution
- discerning application of mathematical concepts and techniques relevant to the task
- accurate and appropriate use of technology

**Excerpt 1****Designs****Bottle Design 1**

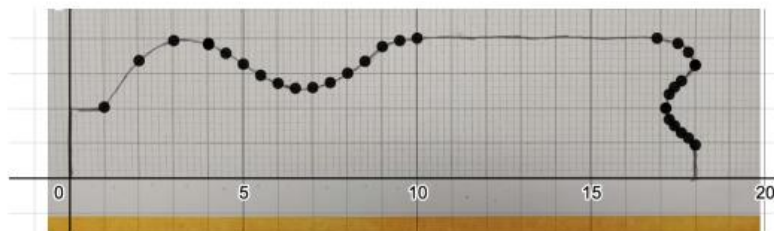
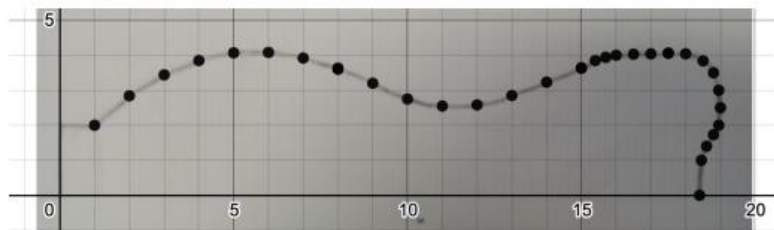
This design is loosely based upon a 600ml Powerade bottle with a wider top portion and larger cavity.

**Bottle Design 2**

This was designed without any influence from other bottles, but after reviewing it does resemble a Fanta 600ml bottle.



Both of these images were uploaded to Desmos, and points were manually fitted to the bottles to enable regression to be used to model the bottle with functions. See below:



**Solve  
(6–7 marks)**

- accurate use of complex procedures to reach a valid solution
- discerning application of mathematical concepts and techniques relevant to the task
- accurate and appropriate use of technology

**Excerpt 2**Solve

Before commencing design, it would be sensible to consider what relative height and radius would give the required volume.

Comparison of height vs radius for cylinders with 600 ml starting at a height of 12 cm

Height (cm)	Radius (cm)
12	3.99
14	3.69
16	3.45
18	3.26
20	3.09

Example calculation for height of 12 cm

$$V = \pi r^2 h$$

$$r = \sqrt{\frac{V}{\pi h}}$$

$$r = \sqrt{\frac{600}{\pi \times 12}}$$

$$r = \sqrt{15.9}$$

$$r = 3.99 \text{ cm}$$

With these calculations, a preferable bottle design could be engineered with any of the calculated heights. To attempt to replicate a common bottle, the designs will be in the range of 18 to 20 cm. The average radius will be between 3 and 4 centimetres.

**Practices to strengthen**

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- within the Formulate criterion, a clear distinction is made between ‘documentation’ and ‘statement’, and ‘assumptions’ and ‘observations’ when making judgments. In this criterion
  - clear demonstration of the ‘documentation of appropriate assumptions’ could include assumptions related to the student’s model/solution, as well as evidence to support the assumption. This could be in the form of a reference, historical data relating to the topic, and/or explaining the likely effect of an important assumption and how this is considered in the model/solution, or the impact of not making the assumption
  - clear demonstration of ‘accurate documentation of relevant observations’ could provide evidence to support observations (information/data) used in the model/solution such as explaining how the observations were collected, the source of the observations, what made the observations valid and reliable, or identifying a specific feature of an observation that made it relevant to the model/solution, e.g. from the plotted data points, it is clearly observed that there is one turning point at (x, y) and that the curve is symmetrical about that turning point. It could also contain a reference.

**Additional advice**

- Schools should check that the most current version of the ISMG from the syllabus is being used to mark students’ responses in cases where a separate ISMG has been printed.
- The ISMG should be clearly highlighted or underlined when identifying relevant characteristics and indicating the subsequent mark allocation.
- When students exceed the response length, teachers should annotate the written response where they have ceased marking and indicate this on the appropriate criteria on the ISMG.



## Internal assessment 2 (IA2)

### Examination — short response (15%)

The examination assesses the application of a range of cognitions to a number of items using a representative sample of subject matter from all Unit 3 topics. Where relevant, the focus of this assessment should be on subject matter not assessed in the problem-solving and modelling task. Subject matter from Units 1 and 2 is considered assumed knowledge.

Student responses must be completed individually, under supervised conditions, and in a set timeframe (120 minutes plus 5 minutes perusal). The percentage allocation of marks must match the degree of difficulty specifications: ~20% complex unfamiliar, ~20% complex familiar, ~60% simple familiar.

### Assessment design

#### Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

#### Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	367
Authentication	0
Authenticity	6
Item construction	20
Scope and scale	69

\*Each priority might contain up to four assessment practices.

Total number of submissions: 418.

#### Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- featured questions that assessed a selection of subject matter that accurately reflected the intended learning of all topics in Unit 3
- featured a balance of items requiring both technology-free and technology-active responses, which could be identified by specifying the questions (or sections of the paper) requiring a technology-free or technology-active approach
- featured an appropriate number of questions that matched the degree of difficulty specifications in the syllabus and allowed students to respond in the set timeframe

- featured a correct marking scheme that indicated clearly how marks have been allocated; this assists schools to check the scope and scale of the assessment and promotes consistency in the awarding of marks.

### Practices to strengthen

It is recommended that assessment instruments:

- provide complex unfamiliar opportunities such that
  - relationships and interactions have a number of elements but are not scaffolded (e.g. not providing a series of parts that step through a problem)
  - all the information to solve the problem is not immediately identifiable, i.e. the required procedure is not clear from the way the problem is posed (e.g. by avoiding cues such as ‘Determine the area of...’ in a question involving integrals)
- require students to demonstrate knowledge and understanding of Unit 3 subject matter and do not solely assess subject matter from Units 1 and 2, e.g. solving an indicial equation which does not use ‘e’
- assess subject matter within the scope and scale of the syllabus, e.g. using related rates as instances of the chain rule is part of the Specialist Mathematics syllabus
- provide opportunities for students to respond to Assessment objective 4: evaluate the reasonableness of solutions.

### Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

#### Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	13
Language	88
Layout	28
Transparency	43

\*Each priority might contain up to four assessment practices.

Total number of submissions: 418.

### Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- featured a clear layout, where text and other items (e.g. MathType, tables and graphs) appeared aligned and in their entirety on the page
- featured correct language conventions, and were free of punctuation, grammatical, spelling and typographical errors
- featured adequate response space for each question.

## Practices to strengthen

It is recommended that assessment instruments:

- use the language of the assessment objectives, e.g. 'evaluate the reasonableness of ...'
- feature questions of an appropriate length so students are not disadvantaged by having to read excessive, or unnecessary, information, as this takes away time from solving the problem
- use instructions that do not conflict with opportunities within the paper, e.g. if instructions state 'Technology can be used unless analytical procedures are specified', then questions within the paper should reflect this practice.

## Assessment decisions

### Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

### Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	99.27%	0.73%	0%	0%

### Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- accurate marking schemes were submitted that clearly identified where part-marks are awarded for each question, and that aligned with the school's awarded marks
- alternative responses for questions were allowed for and explained in the marking scheme
- marking schemes for comparable and amended assessment pieces were clearly denoted as comparable and/or amended
- annotations were used to show where marks had been awarded
- cut-offs were correctly applied to the percentages calculated to accurately determine the provisional marks, i.e. results were not rounded to the nearest percentage before applying the ISMG, e.g. a student who receives >80% is awarded 13/15, whereas a student who receives exactly 80% is awarded 12/15.

### Samples of effective practices

The following are excerpts from responses that illustrate the characteristics for the criteria at the performance level indicated. The excerpts may provide evidence of more than one criterion. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

This excerpt has been included:

- to demonstrate the correct application of the percentage cut-off to determine the correct mark allocation, i.e. clearly indicates the response was awarded 56.5 out of 60 marks. This equates to 94.17% and is therefore allocated 15 marks.

**Excerpt 1**

56.5/60 · 94%

**Instrument-specific marking guide (IA2): Examination (15%)**

**Criterion: Foundational knowledge and problem-solving**

Assessment objectives

- select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
- comprehend mathematical concepts and techniques drawn from all Unit 3 topics
- communicate using mathematical, statistical and everyday language and conventions
- evaluate the reasonableness of solutions
- justify procedures and decisions by explaining mathematical reasoning
- solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics

The student work has the following characteristics:	Cut-off	Marks
<ul style="list-style-type: none"> <li>consistently correct selection, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>authoritative</u> and <u>accurate</u> command of mathematical concepts and techniques; <u>astute</u> evaluation of the <u>reasonableness of solutions</u> and use of mathematical reasoning to correctly <u>justify</u> procedures and decisions; and <u>fluent</u> application of mathematical concepts and techniques to <u>solve</u> problems in a <u>comprehensive</u> range of <u>simple familiar</u>, <u>complex familiar</u> and <u>complex unfamiliar</u> situations.</li> </ul>	> 93%	15
	> 87%	14
<ul style="list-style-type: none"> <li>correct selection, <u>recall</u> and <u>use</u> of facts, rules, definitions and procedures; <u>clear</u> communication of mathematical concepts and techniques; <u>considered</u> evaluation of the <u>reasonableness of solutions</u> and use of mathematical reasoning to <u>justify</u> procedures and decisions; and <u>proficient</u> application of mathematical concepts and techniques to <u>solve</u> problems in <u>simple familiar</u>, <u>complex familiar</u> and <u>complex unfamiliar</u> situations.</li> </ul>	> 80%	13
	> 73%	12

This student response excerpt has been included:

- to demonstrate where teachers have annotated the responses to clearly indicate where marks have been awarded
- to demonstrate annotation highlighting application of the school's marking scheme (Excerpt 1)
- to demonstrate annotations highlighting the application of the marking scheme when follow-through error is involved.

**Foundational knowledge and problem-solving (5.5 of a possible 6 marks)**

**Excerpt 1**

(Simple Familiar)

a)

$$3e^x + 2 = 14$$

$$3e^x = 12 \quad \downarrow \downarrow$$

$$e^x = 4 \quad \downarrow \downarrow$$

$$x = \ln(4) \quad \downarrow \downarrow$$

3.

b)

$$\log_6(x^2) - \log_6(2x) = 2$$

$$\log_6\left(\frac{x^2}{2x}\right) = 2 \quad \checkmark$$

$$\log_6\left(\frac{x}{2}\right) = 2 \quad \checkmark \quad x \neq 0$$

$$6^2 = \frac{x}{2} \quad \checkmark$$

$$\frac{x}{2} = 36 \quad \checkmark \quad 2 \cdot 36$$

$$x = 72 \quad \checkmark$$

5 1/2.

Foundational knowledge and problem-solving (5 of a possible 6 marks)

### Excerpt 2

Answers must be in simplest form.

a.  $y = e^{x^2 - 4x}$

b.  $f(x) = 2x + x \cos(5x)$

a)  $\frac{d}{dx} (e^{x^2 - 4x})$  FT  $\downarrow$  2.  
 $= 2x e^{x^2 - 4x}$

(Simple Familiar)

b)  $\frac{d}{dx} (2x + x \cos(5x))$   $\frac{d}{dx} = uV' + vU'$   
 $= 2 + 5x \sin(5x) + \cos(5x)$  3  
 let  $u = x$   $u' = 1$   
 $v = \cos(5x)$   $v' = -5 \sin(5x)$   
 $\therefore \frac{d}{dx} = x(-5 \sin(5x)) + \cos(5x) \times 1$   
 $= -5x \sin(5x) + \cos(5x)$

5

This student response excerpt has been included:

- to demonstrate where teachers have annotated the responses to provide clear feedback regarding the expected response to the question.

Foundational knowledge and problem-solving

### Excerpt 1

This excerpt provided a question involving a given trigonometric model. Students were required to use calculus methods to evaluate the validity of the model's use in a given scenario. The annotations give clear directions towards meeting the expected response.



$D(t) = 25 + 5 \sin\left(\frac{6t}{6}\right)$   
 $D'(t) = 25 + 5(0)$   
 $D(t) = 25$

Need to Derive  
 + then let  
 equal zero  
 + solve for t

$\therefore$  it is not at its maximum depth, the max. depth would occur at 3:00am when the model shows  $\frac{3\pi}{6}$  or  $\frac{\pi}{2}$

need to show working

### Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- schools update their marking schemes for an endorsed instrument to appropriately allocate marks to expected student responses and, when necessary, to correct errors in the questions or sample responses, change mark allocations, and/or accept alternative solutions. Changes to the marking scheme should be communicated in a timely manner to the QCAA, either through the amendment process or at the time of confirmation submission
- the correct marking scheme is used and submitted for a comparable assessment.



## Internal assessment 3 (IA3)

### Examination — short response (15%)

The examination assesses the application of a range of cognitions to a number of items using a representative sample of subject matter from all Unit 4 topics. Subject matter from Units 1, 2 and 3 is considered assumed knowledge. Student responses must be completed individually, under supervised conditions, and in a set timeframe (120 minutes plus 5 minutes perusal). The percentage allocation of marks must match the degree of difficulty specifications: ~20% complex unfamiliar, ~20% complex familiar, ~60% simple familiar.

### Assessment design

#### Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

#### Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	227
Authentication	0
Authenticity	1
Item construction	6
Scope and scale	30

\*Each priority might contain up to four assessment practices.

Total number of submissions: 415.

#### Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- featured a balance of items requiring both technology-free and technology-active responses, which could be identified by specifying the questions (or sections of the paper) requiring a technology-free or technology-active approach
- featured an appropriate number of questions that matched the degree of difficulty specifications in the syllabus and allowed students to respond in the set timeframe
- featured a correct marking scheme that indicated clearly how marks would be allocated as this assists schools to check the scope and scale of the assessment and promotes consistency in the awarding of marks.

## Practices to strengthen

It is recommended that assessment instruments:

- provide a balance of questions from all topics in Unit 4, specifically Topic 5
- provide complex unfamiliar opportunities such that
  - relationships and interactions have a number of elements but are not scaffolded (e.g. not providing a series of parts that step through a problem)
  - all the information to solve the problem is not immediately identifiable, i.e. the required procedure is not clear from the way the problem is posed (e.g. by avoiding cues such as ‘Determine the maximum volume’ or ‘Determine the expected value of  $f(x)$ ’)
- provide opportunities for students to respond to Assessment objective 4: evaluate the reasonableness of solutions
- feature questions that accurately reflect the syllabus subject matter, e.g. using technology to solve normal distribution problems instead of Cumulative Normal Distribution (CND) tables.

## Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

### Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	8
Language	52
Layout	9
Transparency	10

\*Each priority might contain up to four assessment practices.

Total number of submissions: 415.

## Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- featured a clear layout, where text and other items (e.g. MathType, tables and graphs) appeared aligned and in their entirety on the page
- featured correct language conventions, and were free of punctuation, grammatical, spelling and typographical errors
- featured adequate response space for each question.

## Practices to strengthen

It is recommended that assessment instruments:

- use the language of the assessment objectives, e.g. ‘Evaluate the reasonableness of ...’ instead of ‘State any assumptions and their associated effects’ or ‘Evaluate the strengths and limitations’

- use appropriate mathematical language, e.g. 'trigonometric functions' instead of 'trig functions'.

## Assessment decisions

### Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

#### Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	98.54%	0.49%	0.24%	0.73%

### Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- accurate marking schemes were submitted which aligned with the school's awarded marks, and the marking scheme detailed what, where and how marks were awarded
- alternative responses for questions were allowed for and explained in the marking scheme
- marking schemes for comparable and amended assessment pieces were submitted in a timely manner and were clearly denoted as comparable and/or amended
- annotations were used when marking student work to show where marks had been awarded
- cut-offs were correctly applied to the percentages calculated to accurately determine the provisional marks, i.e. results were not rounded to the nearest percentage before applying the ISMG (e.g. a student who receives >80% is awarded 13/15, whereas a student who receives exactly 80% is awarded 12/15).

#### Samples of effective practices

The following are excerpts from responses that illustrate the characteristics for the criteria at the performance level indicated. The excerpts may provide evidence of more than one criterion. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

This excerpt has been included:

- to demonstrate how the mark and conversion has been clearly identified on the ISMG, i.e. clearly indicates the response was awarded 54 marks out of a possible 64 marks. This equates to 84.4% and the response is therefore allocated 13 marks. The calculated percentage is not rounded to the nearest whole number when using the ISMG.

**Excerpt 1**

**Instrument-specific marking guide (IA3): Examination (15%)**

**Criterion: Foundational knowledge and problem-solving**

Assessment objectives

- select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
- comprehend mathematical concepts and techniques drawn from all Unit 4 topics
- communicate using mathematical, statistical and everyday language and conventions
- evaluate the reasonableness of solutions
- justify procedures and decisions by explaining mathematical reasoning
- solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

$\frac{54}{64} \approx 84.4\%$

The student work has the following characteristics:	Cut-off	Marks
<ul style="list-style-type: none"> <li>consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.</li> </ul>	> 93%	15
	> 87%	14
<ul style="list-style-type: none"> <li>correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.</li> </ul>	> 80%	13
	> 73%	12
<ul style="list-style-type: none"> <li>thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.</li> </ul>	> 67%	11
	> 60%	10
<ul style="list-style-type: none"> <li>selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.</li> </ul>	> 53%	9
	> 47%	8
<ul style="list-style-type: none"> <li>some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.</li> </ul>	> 40%	7
	> 33%	6
<ul style="list-style-type: none"> <li>infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.</li> </ul>	> 27%	5
	> 20%	4
<ul style="list-style-type: none"> <li>isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.</li> </ul>	> 13%	3
	> 7%	2
<ul style="list-style-type: none"> <li>isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.</li> </ul>	> 0%	1
<ul style="list-style-type: none"> <li>does not satisfy any of the descriptors above.</li> </ul>		0

This student response excerpt has been included:

- to demonstrate annotations of where marks have been awarded.

<p><b>Foundational knowledge and problem-solving</b></p>	<p><b>Excerpt 1</b></p> <p>Determine the values of <math>a</math> and <math>b</math>.</p> <p>Polynomial <math>y''(x)</math></p> $y' = 4ax^3 + 3bx^2 \quad ✓$ $y'' = 12ax^2 + 6bx \quad ✓$ <p>POI at <del>the</del> <math>x=1</math> when <math>y''=0</math></p> $0 = 12a(1)^2 + 6b(1)$ $0 = 12a + 6b \quad // \rightarrow \text{rearrange for } a$ $-6b = 12a$ $-\frac{b}{2} = a \quad \textcircled{5}$ <p>When <math>x=1</math>, gradient is 5 or <math>y'=5</math></p> $5 = 4a(1)^3 + 3b(1)^2$ $5 = 4a + 3b \quad ✓✓$ <table border="0" style="width: 100%;"> <tr> <td style="width: 50%; vertical-align: top;"> <math display="block">\text{let } a = -\frac{b}{2}</math> <math display="block">5 = 4\left(-\frac{b}{2}\right) + 3b</math> <math display="block">5 = -2b + 3b</math> <math display="block">5 = b \quad ✓✓</math> <math display="block">b = 5 \quad ✓✓</math> </td> <td style="width: 50%; vertical-align: top;"> <math display="block">\therefore \text{let } b = 5</math> <math display="block">a = -\frac{5}{2} \quad //</math> <math display="block">= -2.5</math> <math display="block">\therefore a = -2.5 \text{ and } b = 5.</math> </td> </tr> </table>	$\text{let } a = -\frac{b}{2}$ $5 = 4\left(-\frac{b}{2}\right) + 3b$ $5 = -2b + 3b$ $5 = b \quad ✓✓$ $b = 5 \quad ✓✓$	$\therefore \text{let } b = 5$ $a = -\frac{5}{2} \quad //$ $= -2.5$ $\therefore a = -2.5 \text{ and } b = 5.$
$\text{let } a = -\frac{b}{2}$ $5 = 4\left(-\frac{b}{2}\right) + 3b$ $5 = -2b + 3b$ $5 = b \quad ✓✓$ $b = 5 \quad ✓✓$	$\therefore \text{let } b = 5$ $a = -\frac{5}{2} \quad //$ $= -2.5$ $\therefore a = -2.5 \text{ and } b = 5.$		

This student response excerpt has been included:

- to demonstrate annotations of where marks have been awarded.

<b>Foundational knowledge and problem-solving</b>	<p><b>Excerpt 1</b></p> <p>find corresponding z-scores:</p> <p><math>z_1 \approx -1.2265</math> ✓</p> <p><math>z_2 \approx 1.7507</math> ✓</p> <p>using equation: <math>z = \frac{x - \mu}{\sigma}</math></p> <p>① <math>-1.2265 = \frac{0.2 - \mu}{\sigma}</math> ✓</p> <p><math>\sigma = \frac{0.2 - \mu}{-1.2265}</math> ✓</p> <p>② <math>1.7507 = \frac{0.9 - \mu}{\sigma}</math> ✓</p> <p>let <math>\sigma = \frac{0.2 - \mu}{-1.2265}</math> ✓</p> <p><math>1.7507 = \frac{0.9 - \mu}{\frac{0.2 - \mu}{-1.2265}}</math> ✓</p> <p>Using GC. ✓</p> <p><math>\mu \approx 0.4884</math></p> <p>∴ the approx. average score was 48.84%</p> <p style="text-align: center; color: red; font-size: 2em;">4</p>
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### Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- schools update their marking schemes for an endorsed instrument to appropriately allocate marks to expected student responses and, when necessary, to correct errors in the questions or sample responses, change mark allocations, and/or accept alternative solutions. Changes to the marking scheme should be communicated in a timely manner to the QCAA, either through the amendment process or at the time of confirmation submission
- the correct marking scheme is used and submitted for a comparable assessment.



# External assessment

External assessment (EA) is developed and marked by the QCAA. The external assessment for a subject is common to all schools and administered under the same conditions, at the same time, on the same day.

## Summative external assessment (EA) — Examination (50%)

### Assessment design

The assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the syllabus. The examination assessed subject matter from Units 3 and 4.

The examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (45 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (45 marks).

### Assessment decisions

Assessment decisions are made by markers by matching student responses to the external assessment marking guide (EAMG). The external assessment papers and the EAMG are published in the year after they are administered.

### General multiple choice item responses

There were 10 multiple choice items in Paper 1 and 10 multiple choice items in Paper 2.

#### Percentage of student responses to each option

##### Note:

- The correct answer is **bold** and in a **blue** shaded table cell.
- Some students may not have responded to every question.

#### Paper 1

Question	A	B	C	D
1	<b>80.3</b>	3.93	15.04	0.55
2	3.11	<b>67.85</b>	27.22	1.52
3	8.58	7.98	<b>77.01</b>	6.21
4	21.58	10.82	<b>52.61</b>	14.47



Question	A	B	C	D
5	5.2	8.98	20.9	<b>64.58</b>
6	<b>43.58</b>	24.3	13.88	16.7
7	2.81	7.03	<b>82.09</b>	7.73
8	18.95	<b>55.77</b>	14.94	9.42
9	27.47	<b>50.66</b>	9.45	11.5
10	1.85	3.53	<b>83.1</b>	11.04

## Paper 2

Question	A	B	C	D
1	<b>86</b>	7.16	5.44	1
2	<b>44.36</b>	15.67	34.88	4.64
3	4.82	10.14	11.41	<b>72.69</b>
4	11.33	36.9	<b>46.01</b>	4.88
5	1.77	5.47	2.94	<b>89.52</b>
6	<b>61.36</b>	13.02	24.19	0.88
7	11.11	<b>73.68</b>	8.62	5.87
8	8.26	5.51	15.76	<b>69.89</b>
9	1.73	2.89	3.5	<b>91.29</b>
10	<b>82.68</b>	4.27	4.64	7.87

## Effective practices

Overall, students responded well to:

- applications of trigonometric functions in physical contexts (e.g. area, side-length or angle calculations) and, when used to model displacement, velocity or acceleration of simple motion
- opportunities to demonstrate selection, recall and use of rules and procedures related to differential and integral calculus across a mix of function types and applications in simple familiar situations
- opportunities to demonstrate understanding of logarithmic laws (using any base) and the interpretation and use of a situation involving a logarithmic scale.

## Samples of effective practices

The following excerpts have been selected to illustrate effective student responses in one or more of the syllabus assessment objectives. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

## Short response

Item: Question 18 — Paper 1 (Technology-free)

Assessment objectives: 1, 2 and 5 — complex familiar

This question required students to identify the use of definite integrals to determine the area under two sections of a graph, resulting in two equations in two unknowns to be solved.

Effective student responses:

- correctly identified two definite integrals
- obtained two equations in two unknowns
- correctly solved simultaneous equations that involved fractional and negative coefficients.

This student response excerpt has been included:

- as it demonstrates a high-level response in the use of algebraic techniques involving fractional and negative coefficients within the context of integration as a measurement of area.

## Foundational knowledge and problem-solving

## Excerpt 1

Determine the values of  $a$  and  $b$ .

$$P = \int_0^1 f(x) dx \qquad Q = \int_0^1 f(x) dx$$

$$\therefore \frac{13}{6} = \int_{-1}^0 ax^2 + bx + 4 dx \qquad \therefore \frac{43}{6} = \int_0^1 ax^2 + bx + 4 dx$$

$$\frac{13}{6} = \left[ \frac{ax^3}{3} + \frac{bx^2}{2} + 4x \right]_{-1}^0 \qquad \frac{43}{6} = \left[ \frac{ax^3}{3} + \frac{bx^2}{2} + 4x \right]_0^1$$

$$\frac{13}{6} = - \left( \frac{a(-1)^3}{3} + \frac{b(-1)^2}{2} + 4(-1) \right) \qquad \frac{43}{6} = \frac{a(1)^3}{3} + \frac{b(1)^2}{2} + 4(1)$$

~~$$F(0) = 0$$~~

$$\therefore \frac{13}{6} = - \left( \frac{-a}{3} + \frac{b}{2} - 4 \right) \qquad \frac{43}{6} = \frac{a}{3} + \frac{b}{2} + 4$$

$$\frac{13}{6} = \frac{a}{3} - \frac{b}{2} + 4 \quad (1) \qquad \frac{43}{6} = \frac{a}{3} + \frac{b}{2} + 4 \quad (2)$$

~~$$\text{Equation (2) - (1):}$$~~

$$\frac{43}{6} - \frac{13}{6} = \frac{a}{3} + \frac{b}{2} + 4 - \left( \frac{a}{3} - \frac{b}{2} + 4 \right)$$

$$\frac{30}{6} = b$$

$$\therefore b = 5$$

Sub.  $b=5$  into (1)

$$\therefore \left( \frac{13}{6} = \frac{a}{3} - \frac{5}{2} + 4 \right) \times 6$$

$$13 = 2a - 15 + 24$$

$$4 = 2a$$

$$\therefore a = 2, \quad b = 5$$

Item: Question 19 — Paper 1 (Technology-free)

Assessment objectives: 1, 2, 3, 4, 5 and 6 — complex unfamiliar

This question required students to justify the choice of 0.5 for the proportion.

Effective student responses:

- justified the decision (of using 0.5) by clearly explaining their answer using mathematical reasoning.

These student response excerpts have been included:

- to demonstrate the use of alternative methods (both using clear mathematical reasoning) to justify the decision made for the choice of proportion.
  - Excerpt 1 shows the use of an analytical method involving the first derivative to identify the proportion value 0.5 and the second derivative test to justify that this provided a maximum value for the proportion.
  - Excerpt 2 shows the use of an alternative method that recognised a quadratic expression (with one stationary point) was needed to maximise the proportion and investigated numerical values near the choice value of 0.5.

<b>Foundational knowledge and problem-solving</b>	<p><b>Excerpt 1</b></p> <p><u>Justify the choice of 0.5 for the proportion.</u></p> <p>Justification for <math>\hat{p}=0.5</math></p> $ME = z \sqrt{\frac{p(1-p)}{n}}$ <p>so, the larger the value of <math>\hat{p}</math>, the larger the margin of error and the larger the variability in the sample proportion.</p> <p>to maximise <math>p</math>: let <math>y = p(1-p) = p - p^2</math></p> $y' = 1 - 2p$ $0 = 1 - 2p$ $p = 0.5$ <p><math>y'' = -2 \therefore p = 0.5</math> is maximum</p> <p>so <math>p(1-p)</math> is largest when <math>\hat{p} = 0.5</math></p>
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Foundational knowledge and problem-solving

### Excerpt 2

0.5 would result in the largest variability in the sample proportion as it produces the largest <sup>numerator</sup> ~~numerator~~  $\hat{p}(1-\hat{p})$ .

$$\text{Eg: } \frac{1}{2}(1-\frac{1}{2}) = \frac{1}{4} = \frac{4}{16} \Rightarrow \hat{p} = 0.5$$

$$\frac{1}{4}(1-\frac{1}{4}) = \frac{3}{16} \Rightarrow \hat{p} < 0.5$$

$$\frac{3}{4}(1-\frac{3}{4}) = \frac{3}{16} \Rightarrow \hat{p} > 0.5 \Rightarrow \frac{3}{16} < \frac{4}{16}$$

$\therefore \hat{p} = 0.5$  is a max.  
and hence 0.5 will result in the largest variability in the sample proportion.

Item: Question 20 — Paper 1 (Technology-free)

Assessment objectives: 1, 2, 3, 5 and 6 — complex unfamiliar

This question required students to determine intervals of time when a population is increasing and when it is decreasing.

Effective student responses:

- correctly determined one critical point
- used the second derivative to determine the nature of the critical point
- communicated when the population was increasing and decreasing.

This student response excerpt has been included:

- to demonstrate high-level understanding of the use of the second derivative test for finding local maxima and minima.

Foundational knowledge and problem-solving

### Excerpt 1

#### QUESTION 20 (7 marks)

The population of rabbits ( $P$ ) on an island, in hundreds, is given by  $P(t) = t^2 \ln(3t) + 6$ ,  $t > 0$ , where  $t$  is time in years.

Determine the intervals of time when the population is increasing and the intervals when it is decreasing.

$$P(t) = t^2 \ln(3t) + 6$$

$$P'(t) = 2t \ln(3t) + \frac{t^2}{t} \quad (\text{using product rule})$$

$$P'(t) = 2t \ln(3t) + t$$

$$0 = t(2 \ln(3t) + 1)$$

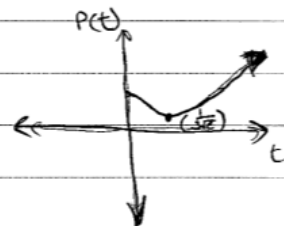
$$\therefore t=0 \quad \text{or} \quad 2 \ln(3t) + 1 = 0$$

not valid ( $t > 0$ )

$$\ln(3t) = -\frac{1}{2}$$

$$3t = e^{-1/2}$$

$$t = \frac{1}{3\sqrt{e}}$$



$$P''(t) = 2 \ln(3t) + \frac{2t}{t} + 1 \quad (\text{using product rule})$$

$$P''\left(\frac{1}{3\sqrt{e}}\right) = 2 \ln(3t) + 3$$

$$= 2 \ln\left(\frac{1}{\sqrt{e}}\right) + 3$$

$$= \ln\left(\frac{1}{e}\right) + 3$$

$$= -\ln(e) + 3$$

$$= 3 - 1$$

$$= 2$$

$\therefore t = \frac{1}{3\sqrt{e}}$  years is a minimum ( $P''(t) > 0$ )

( $\therefore$  Less than  $\frac{1}{3\sqrt{e}}$ , population is decreasing)  
and after  $\frac{1}{3\sqrt{e}}$ , population increasing)

$\therefore$  Population is decreasing between  $(0, \frac{1}{3\sqrt{e}})$  years.  
And population is increasing between  $(\frac{1}{3\sqrt{e}}, \infty)$  years

Item: Question 17 — Paper 2 (Technology-active)

Assessment objectives: 1, 2, 3, 4 and 6 — complex familiar

This question required students to construct mathematical models and evaluate the reasonableness of a claim.

Effective student responses:

- constructed two correct trigonometric models of populations using the information provided
- used addition of the constructed models to determine a model for the total animal population
- used the constructed models to evaluate the reasonableness of the stated claim.

This student response excerpt has been included:

- to demonstrate two trigonometric models constructed from the graphical information given
- to demonstrate the sum of the two models represented as both an equation and a graphical sketch
- to demonstrate clear evaluation of the reasonableness using the constructed models.

## Foundational knowledge and problem-solving

## Excerpt 1

Evaluate the reasonableness of Jane's claim.

$$\text{let Foxes} = f(x)$$

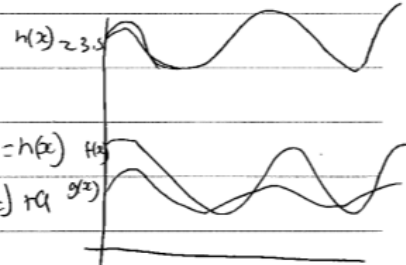
$$\text{let Rabbit} = g(x)$$

$$f(x) = 3.5 \cos\left(\frac{\pi}{6}t\right) + 11$$

$$g(x) = 2 \sin\left(\frac{\pi}{6}t\right) + 9$$

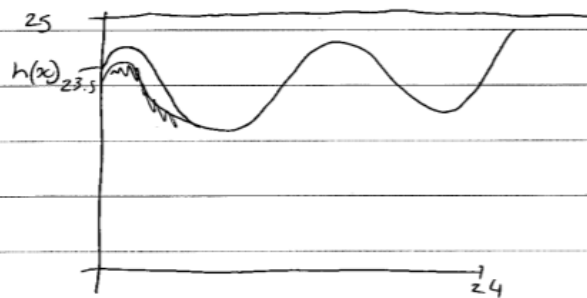
$$\text{Total population (in thousands)} = f(x) + g(x) = h(x)$$

$$h(x) = 3.5 \cos\left(\frac{\pi}{6}t\right) + 11 + 2 \sin\left(\frac{\pi}{6}t\right) + 9$$



to find if it exceeds 25000,

$$25 = 3.5 \cos\left(\frac{\pi}{6}t\right) + 11 + 2 \sin\left(\frac{\pi}{6}t\right) + 9$$



as line  $y=25$  doesn't intersect

$h(x)$ , the total number of rabbits never exceeds 25000.

also, the above equation is found to have no solutions hence also, never exceeds or even reaches 25000

Item: Question 19 — Paper 2 (Technology-active)

Assessment objectives: 1, 2, 3, 5 and 6 — complex unfamiliar

This question required students to determine a probability involving a uniformly distributed random variable where the definition of a probability density function was provided.

Effective student responses:

- determined the equation of the probability density function for the uniformly distributed random variable within the stated domain
- determined the mean and standard deviation values
- determined the area representing the required probability either using integral or graphical techniques.

This student response excerpt has been included:

- to demonstrate high-level understanding of the definition of a uniform distribution provided in the question and a graphical display to determine a probability.

<b>Foundational knowledge and problem-solving</b>	<p><b>Excerpt 1</b></p> <p>Determine the probability that the time between placing an order and delivery of the order will be within one standard deviation of the expected time.</p> <p><math>\therefore f(x) = \frac{1}{b-a} = \frac{1}{80}</math></p> <p><math>E(X) = \frac{a+b}{2} = \frac{100+180}{2} = 140 \text{ mins}</math></p> <p><math>SD(X) = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(180-100)^2}{12}} = 23.094 \approx 23 \text{ mins}</math></p> <p><math>P(\mu - \sigma &lt; X &lt; \mu + \sigma)</math>  <math>\mu - \sigma = 140 - 23 = 117</math>  <math>\mu + \sigma = 140 + 23 = 163</math></p> <p><math>P(117 &lt; X &lt; 163) = \frac{d-c}{b-a} = \frac{163-117}{180-100} = \frac{46}{80} = 0.575</math></p>
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Item: Question 20 — Paper 2 (Technology-active)

Assessment objectives: 1, 2, 3, 5 and 6 — complex unfamiliar

This question required students to solve an unfamiliar problem with subject matter across different domains of mathematics.

Effective student responses:

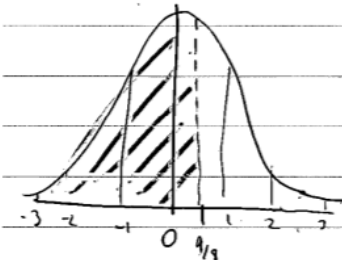
- identified the need to use the discriminant
- determined the range of values of the random variable



- determined the probability of the random variable being within the range.

This student response excerpt has been included:

- to demonstrate understanding of connections between subject matter across the Mathematical Methods course
- to demonstrate understanding of the role of the discriminant to determine the number of solutions to a quadratic equation
- to show the calculation of probabilities and quantiles associated with a given normal distribution using technology and use these to solve practical problems
- to demonstrate the correct use of algebraic techniques involving inequality.

<b>Foundational knowledge and problem-solving</b>	<p><b>Excerpt 1</b></p>  <p>Real roots if discriminant <math>&gt; 0</math></p> $\text{discriminant} = b^2 - 4ac$ $= 3^2 - 4 \times 1 \times 2B$ $\theta = 9 - 8B$ <p>let discriminant <math>&gt; 0</math></p> $0 < 9 - 8B$ $8B < 9$ $B < \frac{9}{8}$ <p><math>\therefore</math> Need to find <math>P(B &lt; \frac{9}{8})</math>, given</p> $\mu = 0, \sigma = 1$ <p><math>\therefore</math> using normalcdf(-1E99, <math>\frac{9}{8}</math>, 0, 1) = 0.8697</p> <p style="text-align: center;"><small>-1000</small></p> <p style="text-align: right;">= 0.87</p> <p>The probability of <span style="border: 1px solid black; padding: 2px;">END OF PAPER</span></p> <p><math>B</math> has <math>x^2 + 3x + 2B</math> having real roots is 0.87</p>
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### Practices to strengthen

It is recommended that when preparing students for external assessment, teachers consider:

- providing more opportunities for students to engage with algebraic manipulations of expressions and equations, especially those that involve fractional and negative coefficients. In Paper 1, Question 18 many students were unable to correctly manipulate the equations in this technology-free question. For the students that were successful at solving the simultaneous equations, the 'Elimination method' was the preferred technique. Similarly, in Paper 1, Question 11(b) the instruction 'Give your answer in simplest form' was given. Many

students either did not attempt the simplification or incorrectly simplified their response. This also suggests students find manipulating fractions manually challenging. This trend was also evident in Paper 1, Question 19 where many students were unable to manipulate the equation (expressed as a fraction) to solve for the sample size

- providing more opportunities for students to engage with complex situations, both familiar and unfamiliar, that require mathematical justification of procedures and decisions (Assessment objective 5), e.g. in Paper 1, Question 19, students were required to justify the choice of 0.5 for the proportion stated in the question. Many students were unable to justify this choice with appropriate mathematical reasoning, instead choosing to offer verbal justification or none at all
- providing opportunities for students to consolidate understanding of the use of the second derivative to explore the concepts of concavity, points of inflection, and finding local maxima and minima. It was evident from the external assessment, when determining intervals where a population was increasing or decreasing (Paper 1, Question 20), that many students misunderstood the use of the second derivative as an indicator of increasing and decreasing intervals
- providing more opportunities for students to construct mathematical models and use the models to solve practical problems. Students should be expected to verify and evaluate the usefulness of models using quantitative analysis. The opportunities for students should present a range of contexts that would allow students to engage with modelling, using logarithmic, exponential and trigonometric functions. In Paper 2, Question 17, students that could not determine mathematical models for rabbits and foxes (and therefore total population) struggled to find an acceptable alternative approach to investigate the problem with the appropriate comparative complexity
- providing more opportunities for students to engage with situations that require an evaluation of the reasonableness of solutions (Assessment objective 4), e.g. Paper 2, Question 17. Many students did not successfully demonstrate the evaluation. Most students tried to evaluate reasonableness with a written paragraph that contained little mathematical content. Stories of rabbits and foxes were often provided instead of numerical analysis
- providing more opportunities for students to investigate interval estimates from various types of distributions. Paper 2, Question 19 presented students with a probability density function for a uniformly distributed random variable. Being in the technology-active paper, many students attempted to calculate the required probability using technology (graphing calculator) and the normal distribution instead of the given uniform distribution. This suggests that, in general, students were very familiar with probability calculation techniques using normal distributions and performed these in favour of using the uniform distribution information provided in this question
- providing more opportunities for students to apply their problem-solving skills in situations where relationships have a number of elements such that connections are made with subject matter across the domains of mathematics. A complex unfamiliar question such as Paper 2, Question 20 highlighted this. A random variable with a normal distribution was presented within a quadratic equation, with the requirement for the equation to have real roots. This question resulted in a small number of correct solutions. Many students who got as far as the discriminant inequality often struggled to solve that inequality correctly. This question proved to be interesting to mark as students argued their alternative methods. One of the more successful 'alternative' approaches to solving the discriminant inequality was a graphical approach involving translation of the quadratic function.



# Senior External Examination

The Mathematical Methods Senior External Examination (SEE) is a standalone examination offered to eligible Year 12 students and adult learners. It contributes 100% to a student's final subject result.

The assessment was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the Mathematical Methods Senior External Examination syllabus.

The SEE consisted of two assessments:

- SEE 1 contributed 50% of the marks
- SEE 2 contributed 50% of the marks.

**Note:** The SEE information should be read in conjunction with the rest of the subject report.

Number of students who completed the Mathematical Methods Senior External Examination: 18.

## Distribution of standards

There were insufficient student enrolments in this subject to provide useful analytics.

## Assessment decisions

### Effective practices

Overall, students responded well to:

- using data provided in the stimulus material
- performing quantitative (residual) analysis
- observing shapes and key features of graphs to identify appropriate mathematical function types for modelling data
- using technology (graphics calculators) to generate equations of mathematical models.

### Practices to strengthen

It is recommended that when preparing students for the Senior External Examination, teachers consider:

- providing more opportunities for students to plot raw and residual data to reinforce the appropriate axes to use when graphing (identifying independent and dependant variables)
- providing more opportunities for students to understand concavity and points of inflection and their relationship with the second derivative
- providing a range of opportunities for students to develop their algebraic techniques when using the rules of calculus in complex situations, including the simplification of resulting expressions.