LUI

School code $\square$

$\square$


Family name $\square$


## Mathematical Methods

## Paper 1 - Technology-free

## Time allowed

- Perusal time - 5 minutes
- Working time - 90 minutes


## General instructions

- Answer all questions in this question and response book.
- Calculators are not permitted.
- QCAA formula book provided.
- Planning paper will not be marked.


## Section 1 (10 marks)

- 10 multiple choice questions


## Section 2 (45 marks)

- 10 short response questions


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## Section 1

## Instructions

- Choose the best answer for Questions 1-10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.



## Section 2

## Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
- On the additional pages, write the question number you are responding to.
- Cancel any incorrect response by ruling a single diagonal line through your work.
- Write the page number of your alternative/additional response, i.e. See page ...
- If you do not do this, your original response will be marked.
- This section has 10 questions and is worth 45 marks.


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## QUESTION 11 (5 marks)

Determine the derivative with respect to $x$ of the following functions.
a) $y=\left(e^{x}+1\right)^{3}$
b) $y=\frac{\sin (x)}{x^{2}}$ (Give your answer in simplest form.)
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## QUESTION 12 (5 marks)

Solve for $x$ in the following.
a) $\log _{2}(5 x+7)=5$
[2 marks]
b) $\quad \log _{10}(x+3)+\log _{10}(x-3)=\log _{10}(9 x-29)$

[^0]
## QUESTION 13 (5 marks)

Consider the functions $f(x)=x^{2}$ and $g(x)=4 x$.
a) Determine the $x$-coordinates of the points of intersection of the graphs of the two functions.
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b) Use the results from Question 13a) to calculate the area enclosed by the graphs of $f(x)$ and $g(x)$.
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[^1]
## QUESTION 14 (4 marks)

Consider the function $f(x)=\ln (3 x+4)$, for $x>\frac{-4}{3}$
a) Determine $f^{\prime}(x)$.
[1 mark]
b) Determine the $x$-intercept of the graph of $f(x)$.
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c) Determine the gradient of the tangent to the graph of $f(x)$ at the $x$-intercept.
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## QUESTION 15 (4 marks)

In the isosceles triangle $A B C$, angle $C$ is $120^{\circ}$ and side $a$ is 4 cm .
a) Draw the triangle, indicating all given information.
$\square$
Note: If you make a mistake in the diagram, cancel it by ruling a single diagonal line through your work and use the additional diagram on page 17 of this question and response book.
b) Calculate the area of the triangle in $\mathrm{cm}^{2}$. (Give your answer in simplest form.)
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## QUESTION 16 (4 marks)

A tangent is drawn at the point $(1, e)$ on the graph of the function $y=e^{2-x}$ as shown.


Determine the area of the shaded triangle.

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## QUESTION 17 (3 marks)

In any five-day working week Leonardo either catches a bus to work or uses another form of transportation. On average, he catches the bus to work on three of the five days. His decision on any given day is independent of his decision on any other day.

Determine the probability that Leonardo catches a bus to work on exactly one day in a given five-day working week.

[^2]
## QUESTION 18 (4 marks)

The graph of $y=f(x)$, where $f(x)$ is the quadratic function $f(x)=a x^{2}+b x+4$, is shown.
Two regions of the area between the graph of $y=f(x)$ and the $x$-axis are shaded.


Region $P$ has an area of $\frac{13}{6}$ units $^{2}$ and Region $Q$ has an area of $\frac{43}{6}$ units $^{2}$.
Determine the values of $a$ and $b$.

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## QUESTION 19 (4 marks)

A firm aims to have $95 \%$ confidence in estimating the proportion of office workers who respond to an email in less than an hour to within $\pm 0.05$.

A survey has never been undertaken before, so no past data is available.
The firm believes that if the proportion is 0.5 , then this will result in the largest variability in the sample proportion.

Based on this, determine the sample size needed using the approximate value of $z=2$ for the $95 \%$ confidence interval.

Justify the choice of 0.5 for the proportion.

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## QUESTION 20 (7 marks)

The population of rabbits $(P)$ on an island, in hundreds, is given by $P(t)=t^{2} \ln (3 t)+6, t>0$, where $t$ is time in years.

Determine the intervals of time when the population is increasing and the intervals when it is decreasing.
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## ADDITIONAL PAGE FOR STUDENT RESPONSES

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## ADDITIONAL PAGE FOR STUDENT RESPONSES

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## ADDITIONAL RESPONSE SPACE FOR QUESTION 15

If you want this diagram to be marked, rule a single diagonal line through the diagram on page 7 .


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[^1]:    Do not write outside this box.

[^2]:    Do not write outside this box.

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