# Mathematical Methods 

## Paper 1 - Technology-free

## General instruction

- Work in this book will not be marked.


## Section 1

## QUESTION 1

$2 \log _{10}(x)-\log _{10}(3 x)$ is equal to
(A) $\log _{10}\left(\frac{x}{3}\right)$
(B) $\log _{10}\left(x^{2}-3 x\right)$
(C) $\frac{2 \log _{10}(x)}{\log _{10}(3 x)}$
(D) $-\log _{10}(x)$

## QUESTION 2

The table shows the time a technician has spent servicing photocopiers.

| Time (in minutes) | Frequency |
| :---: | :---: |
| $0 \leq t<5$ | 10 |
| $5 \leq t<10$ | 20 |
| $10 \leq t<15$ | 30 |
| $15 \leq t<20$ | 40 |
| $20 \leq t<25$ | 100 |

What is the probability that a given service required at least 10 minutes but less than 20 minutes?
(A) 0.15
(B) 0.35
(C) 0.70
(D) 0.85

## QUESTION 3

Determine $\int 10 e^{4 x} d x$
(A) $\frac{10 e^{4 x+1}}{4 x+1}+c$
(B) $40 e^{4 x}+c$
(C) $\frac{5}{2} e^{4 x}+c$
(D) $2 e^{5 x}+c$

## QUESTION 4

The second derivative of the function $f(x)$ is given by $f^{\prime \prime}(x)=\frac{2 x}{1+x^{2}}$
The interval on which the graph of $f(x)$ is concave up is
(A) $x<0$
(B) $x \leq 0$
(C) $x>0$
(D) $x \geq 0$

## QUESTION 5

The graph of $f^{\prime \prime}(x)$ is shown.


Which of the following could be the graph of $f^{\prime}(x)$ ?
(A)

(B)

(C)

(D)


## QUESTION 6

A random variable $X$ is the number of successes in a Bernoulli experiment with $n$ trials, each with a probability of success $p$ and a probability of failure $q$. The probability distribution table of $X$ is shown.

| $\boldsymbol{k}$ | $\boldsymbol{P}(\boldsymbol{X}=\boldsymbol{k})$ |
| :---: | :---: |
| 0 | $\frac{1}{81}$ |
| 1 | $\frac{8}{81}$ |
| 2 | $\frac{24}{81}$ |
| 3 | $\frac{32}{81}$ |
| 4 | $\frac{16}{81}$ |

Which values of $n, p$ and $q$ will generate this probability distribution?
(A) $n=4, p=\frac{2}{3}, q=\frac{1}{3}$
(B) $n=4, p=\frac{1}{3}, q=\frac{2}{3}$
(C) $n=5, p=\frac{2}{3}, q=\frac{1}{3}$
(D) $n=5, p=\frac{1}{3}, q=\frac{2}{3}$

## QUESTION 7

Determine $\int_{1}^{3}(2 x+3) d x$
(A) 2
(B) 4
(C) 14
(D) 16

## QUESTION 8

The continuous random variable $X$ has the probability density function

$$
f(x)=\left\{\begin{array}{rr}
\frac{3}{x^{2}}, & 1 \leq x \leq \frac{3}{2} \\
0, & \text { otherwise }
\end{array}\right.
$$

The mean of $X$ is
(A) $\ln \left(\frac{3}{2}\right)$
(B) $\ln \left(\frac{27}{8}\right)$
(C) $\ln \left(\frac{9}{2}\right)$
(D) 1

## QUESTION 9

A basket contains 10 green apples and 30 red apples. Three apples are drawn at random from the basket with replacement. Determine the probability that exactly two of the three apples are green.
(A) $\frac{3}{64}$
(B) $\frac{9}{64}$
(C) $\frac{10}{64}$
(D) $\frac{27}{64}$

## QUESTION 10

Handspans of teenagers are approximately normally distributed, with a mean of 15 cm and a standard deviation of 2 cm .

Which of the following groups is expected to be the largest?
(A) teenagers with handspans that are between 7 cm and 11 cm
(B) teenagers with handspans that are between 11 cm and 15 cm
(C) teenagers with handspans that are between 13 cm and 17 cm
(D) teenagers with handspans that are between 17 cm and 21 cm

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