— Public use —

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Sample assessment 2020

Question and response book

Mathematical Methods

Paper 2 — Technology-active

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator permitted.
- QCAA formula sheet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 10 short response questions

Clear zone — margin trimmed off after completion of assessment

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Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 10 questions and is worth 50 marks.

QUESTION 11 (4 marks)

The Palermo Technical Impact Hazard Scale (P) is a logarithmic scale used by astronomers to rate the potential hazard of a Near-Earth Object (NEO).

Potential impacts with positive Palermo Scale values will generally indicate situations that merit some level of concern.

For the NEO asteroid Apophis, P is given by:

$$P = \log_{10}\left(\frac{p_i}{8.71 \times 10^{-6}}\right)$$

where p_i represents the impact probability.

a) Determine P for a p_i value of 1 in 100 000. *[1 mark]*

John believes that increasing the P value in 11a) by 2 will make the chance of impact 1000 times more likely.

b) Evaluate the reasonableness of this statement. *[3 marks]*

QUESTION 12 (5 marks)

People with type O negative blood are said to be 'universal donors'. In Australia, 9% of the population has this blood type.

On a given day, a random group of 45 people volunteer to donate blood.

a) Identify why this context is suitable for modelling as a binomial distribution. *[1 mark]*

b) Determine the mean and standard deviation of the number of people who are universal donors. *[2 marks]*

c) Determine the probability that no more than 3 of the donors are universal donors. *[2 marks]*

QUESTION 13 (6 marks)

a) Determine $\frac{dM}{dt}$ if $M(t) = \frac{1}{\sqrt{16+3t^2}}$ *[2 marks]*

Let $N(t) = \int 1800t(16 + 3t^2)^{\frac{-3}{2}}\, dt$

b) Using the result from 13a), determine $N(t)$ *[1 mark]*

c) Determine $N'(4)$ *[1 mark]*

The function $N(t)$ models the total number of customers, N, served by staff after t hours during an 8-hour workday ($0 \leq t \leq 8$).

At time $t = 0$, no customers had been served.

d) Determine $N(4)$ *[2 marks]*

QUESTION 14 (5 marks)

Solve for the two possible triangles using the information below:

A = 25°, $a = 12$ and $c = 27$

QUESTION 15 (8 marks)

Suppose the proportion of Australians who supported the removal of single-use plastic bags from supermarkets is 64%.

a) Using the normal approximation, determine the probability that, in a randomly selected sample of size 100, more than 70% of those surveyed supported the removal of the single-use plastic bags. *[2 marks]*

b) Determine the size of the sample required for the survey to achieve a margin of error of 4% in an approximate 95% confidence interval for this proportion. *[2 marks]*

c) Identify the effect that halving the margin of error has on the sample size obtained in 15b). *[2 marks]*

d) Determine the probability that in a randomly selected sample of size 25, the sample proportion is equal to the population proportion. *[2 marks]*

QUESTION 16 (4 marks)

Radar station R picks up signals from two ships.

Ship A is stationary and on a bearing $041°T$ from R and is 65 kilometres away.

Ship B is on a bearing $295°T$ from R and is 53 kilometres away.

Determine the time (in hours) for ship B to travel to ship A if ship B can travel at 30 kilometres per hour.

QUESTION 17 (3 marks)

During one 30-day period, the rate at which pollution passes into a nearby lake is measured every six days and results are given in the table below.

Day (t)	0	6	12	18	24	30
Rate of pollution in units per day $p(t)$	7	8	10	13	17	22

This information can be modelled as a quadratic function.

Determine the total amount of pollution entering the lake during this 30-day period.

QUESTION 18 (5 marks)

The amount of a certain drug in the bloodstream, M (mg), at any time t (hours) is modelled closely by $M(t) = Ate^{-bt}$ where A and b are parameters.

a) Determine the exact values of A and b if the maximum amount of the drug in the bloodstream was 120 mg at $t = 2$ hours. *[3 marks]*

b) Evaluate the reasonableness of your solution. *[2 marks]*

QUESTION 19 (5 marks)

Consider the following information when completing this question.

If X is normally distributed with mean μ and standard deviation σ then

$$X \sim N(\mu, \sigma^2)$$

If X and Y are two independent random normal variables such that

$$X \sim N(\mu_1, {\sigma_1}^2) \text{ and } Y \sim N(\mu_2, {\sigma_2}^2)$$

then

$$X - Y \sim N(\mu_1 - \mu_2, {\sigma_1}^2 + {\sigma_2}^2)$$

and

$$X + Y \sim N(\mu_1 + \mu_2, {\sigma_1}^2 + {\sigma_2}^2)$$

Contestant A and contestant B have trained to navigate an obstacle course.

The times for each contestant to run the obstacle course are independent of each other.

On any given day, the time to run the obstacle course for each contestant is normally distributed.

Let A be the run time (in minutes) for contestant A and B be the run time (in minutes) for contestant B with

$$A \sim N(80, 10^2)$$

$$B \sim \mathrm{N}(78, 12^2)$$

Determine the probability that contestant B runs the obstacle course faster than contestant A.

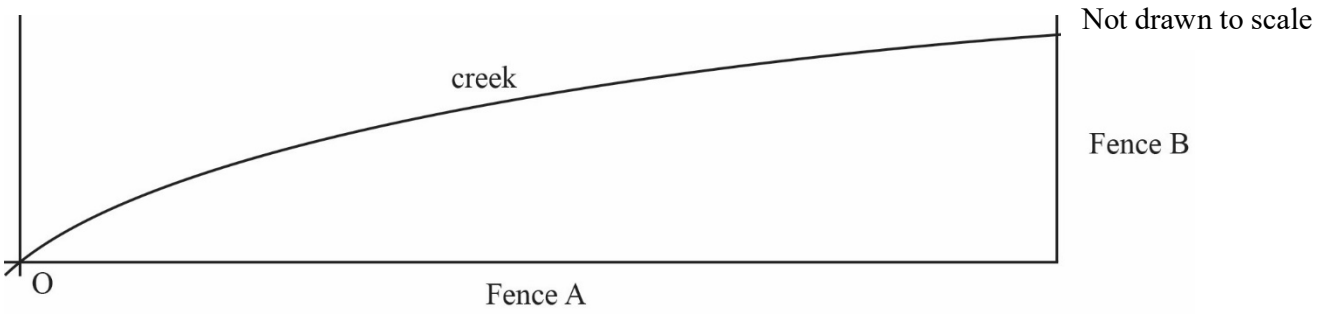
QUESTION 20 (5 marks)

A farmer has a paddock with straight fences on two sides (fence A and fence B) perpendicular to each other and bounded by a creek on the other side. Fence B is 4 kilometres long. The creek boundary can be modelled using

$$d = \ln(5x + e) - 1$$

where d is the perpendicular distance in kilometres from fence A to the creek and x is the distance in kilometres along fence A from the point O.

The farmer wants to divide his paddock area in half with a straight fence, parallel to fence B.



Determine where the farmer should locate the fence.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Write the question number you are responding to.

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