Mathematical Methods marking guide

Sample external assessment 2020

Paper 2: Technology-active (60 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government | Queensland Curriculum & Assessment Authority

For all Queensland schools

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Introduction

The Queensland Curriculum and Assessment Authority (QCAA) has developed mock external assessments for each General senior syllabus subject to support the introduction of external assessment in Queensland.

An external assessment marking guide (EAMG) has been created specifically for each mock external assessment.

The mock external assessments and their marking guides were:

- developed in close consultation with subject matter experts drawn from schools, subject associations and universities
- aligned to the external assessment conditions and specifications in General senior syllabuses
- developed under secure conditions.

Purpose

This document consists of an EAMG and an annotated response.

The EAMG:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of '0' will be recorded.

Where no response to a question has been made, a mark of 'N' will be recorded.

External assessment marking guide

Multiple-choice

Question	Response
1	A
2	D
3	B
4	A
5	A
6	C
7	C
8	A
9	D
10	C

Short response

Question 11 (4 marks)

Sample response	The response
a) $P = 0.06$	correctly determines P **[1 mark]**
b) $2.06 = \log_{10}\left(\frac{p_i}{8.71\times10^{-6}}\right)$	establishes logarithmic equation **[1 mark]**
Using application on GDC p_i=0.001	determines p_i **[1 mark]**
Increasing the P value by 2 increases the p_i value (chance of possible impact) by a factor of 100 (not 1000 times as stated).	evaluates reasonableness of the statement **[1 mark]**

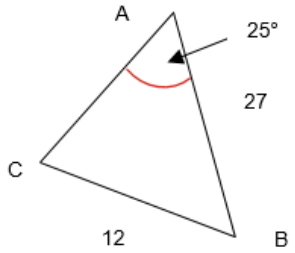
Question 12 (5 marks)

Sample response	The response
a) The situation consists of 45 repeated independent trials and each trial results in two possible outcomes only. I.e. donor is a universal donor or donor is not a universal donor.	identifies the relevant concept used **[1 mark]**
b) mean $= 45 \times 0.09 = 4.05$	correctly determines the mean **[1 mark]**
standard deviation $= \sqrt{45 \times 0.09 \times 0.91}$ $= 1.92$	correctly determines the standard deviation **[1 mark]**
c) Using Binomial probability application on GDC $n = 45, p = 0.09$, lower = 0, upper =3	uses an appropriate mathematical representation **[1 mark]**
$P(x \leq 3) = 0.41$	correctly determines the probability **[1 mark]**

Question 13 (6 marks)

Sample response	The response
a) $M(t) = (16 + 3t^2)^{\frac{-1}{2}}$ Using chain rule	provides a statement identifying the use of the chain rule **[1 mark]**
$M'(t) = -3t(16 + 3t^2)^{\frac{-3}{2}}$	correctly determines the derivative **[1 mark]**
b) $N(t) = \frac{-600}{\sqrt{16+3t^2}} + c$	correctly determines the indefinite integral **[1 mark]**
c) $N'(4) = 1800 \times 4 \times (16 + 3 \times 4^2)^{\frac{-3}{2}}$ $N'(4) = 14.063$	correctly determines $N'(4)$ **[1 mark]**
d) Given $N(0) = 0$ $N(0) = 0 = \frac{-600}{\sqrt{16}} + c$ $c = 150$	determines c **[1 mark]**
$N(4) = \frac{-600}{\sqrt{16 + 48}} + 150$ $N(4) = 75$	determines $N(4)$ **[1 mark]**

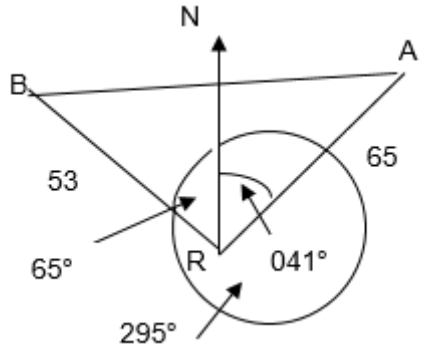
Question 14 (5 marks)

Sample response	The response
	uses convention for labelling triangles to construct a diagram **[1 mark]**
Using Sine rule: $\frac{12}{\sin 25°} = \frac{27}{\sin C°}$	correctly establishes an equation in $C°$ **[1 mark]**
$C° = 71.97°$ or $108.03°$	determines solutions for angle C **[1 mark]**
$\therefore B° = 83.03°$ or $46.97°$	determines solutions for angle B **[1 mark]**
Using Sine rule: $\frac{b}{\sin 83.03°} = \frac{12}{\sin 25°}$ $b = 28.18$ $\frac{b}{\sin 46.97°} = \frac{12}{\sin 25°}$ $b = 20.76$	determines associated lengths b **[1 mark]**

Question 15 (8 marks)

Sample response	The response
a) $n = 100,\ p = 0.64$ $\sigma = \sqrt{\frac{0.64 \times 0.36}{100}} = 0.048$ Using statistical application on GDC	correctly determines the mean and standard deviation **[1 mark]**
$P(\hat{p} > 0.7) = 0.11$	determines probability **[1 mark]**
b) $0.04 = 1.96\sqrt{\frac{0.64 \times 0.36}{n}}$ Using application on GDC	establishes equation using given information and confidence interval definition **[1 mark]**
$n \sim 553$	determines reasonable value for size of sample **[1 mark]**
c) $0.02 = 1.96\sqrt{\frac{0.64 \times 0.36}{n}}$ Using application on GDC $n \sim 2213$	establishes equation using given information and confidence interval definition **[1 mark]**
∴halving the margin of error has resulted in a sample size that is four times as large	justifies decision using mathematical reasoning **[1 mark]**
d) Sample proportion is $\hat{p} = 0.64$ and the sample size is 25, ∴ the number of people aged 16 and over in Australia who would support changing single use plastic bags is $0.64 \times 25 = 16$	correctly determines the number of people that support the change **[1 mark]**
$P(\hat{p} = 0.64) = P(X = 16)$ $= \binom{25}{16} 0.64^{16} \times 0.36^{9}$ $= 0.16$	correctly determines the probability **[1 mark]**

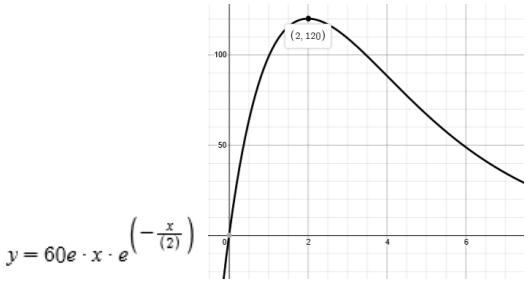
Question 16 (4 marks)

Sample response	The response
	communicates the information using an appropriate mathematical representation **[1 mark]**
$(\text{AB})^2 = 53^2 + 65^2 - 2 \times 53 \times 65 \cos 106°$	correctly determines the angle at R (65°) **[1 mark]**
$AB = 94.5153$ km	determines distance AB **[1 mark]**
Time for Ship B to travel this distance $t = \frac{94.5153}{30} = 3.15\ hours$	determines time **[1 mark]**

Question 17 (3 marks)

Sample response	The response
Using application on GDC to determine quadratic model $p(t) = 0.014t^2 + .083t + 7$	correctly determines the model for $p(t)$ **[1 mark]**
Amount of pollution = $\int_0^{30} p(t)dt$	provides statement identifying use of definite integral **[1 mark]**
Using application on GDC: Amount of pollution = 373 units	determines total amount of pollution **[1 mark]**

Question 18 (5 marks)

Sample response	The response
a) $M'(t) = -Abte^{-bt} + Ae^{-bt}$	correctly determines the derivative **[1 mark]**
$M'(2) = 0 = -2Abe^{-2b} + Ae^{-2b}$ $0 = Ae^{-2b}(-2b + 1)$ $\therefore b = \frac{1}{2}$	determines b **[1 mark]**
$M(t) = Ate^{\frac{-t}{2}}$ Using given information ((2,120) lies on the curve) $120 = 2Ae^{-1}$ $A = 60e$	determines exact value of A **[1 mark]**
b) Use graphing application on GDC  (2,120) is the maximum point of the model as given in the question	uses an appropriate mathematical representation to communicate approach **[1 mark]** evaluates reasonableness of solution **[1 mark]**

Question 19 (5 marks)

Sample response	The response
$B - A \sim N(78 - 80, 12^2 + 10^2)$	recognises use of normally distributed variable for difference of times of contestants **[1 mark]**
Mean = -2 Standard deviation = $\sqrt{244}$	correctly determines the values for the mean and standard deviation **[1 mark]**
Contestant B is faster when $\text{Time}_{\text{Contestant B}} - \text{Time}_{\text{Contestant A}} < 0$	justifies procedure used to determine the solution **[1 mark]**
Use normal probability application on GDC lower limit = -100, upper limit = 0, mean = -2, σ = 15.6205	uses an appropriate mathematical representation to communicate approach **[1 mark]**
$\therefore$ approximately 55% chance that contestant B will run the maze faster than contestant A	solves for probability **[1 mark]**

Question 20 (5 marks)

Sample response	The response
Determine horizontal distance to Fence B $4 = \ln(5x + e) - 1$ Using solving application on GDC $x = 29.139$ Area of paddock $=\int_0^{29.139} \ln(5x + e) - 1\ dx$ $= 89.5917\ km^2$ Half the area = $44.7958\ km^2$	correctly determines horizontal distance to Fence B **[1 mark]** determines the area under the curve **[1 mark]**
$\int_0^k \ln(5x + e) - 1\ dx = 44.7958\ km^2$ Using solving application on GDC $k = 17.2433$	communicates appropriate mathematical representation (definite integral with unknown upper limit) **[1 mark]**
The farmer should locate the fence approximately 17.24 kilometres from the point O.	solves for placement of fence **[1 mark]** shows logical organisation communicating key steps **[1 mark]**