Mathematical Methods marking guide

Sample external assessment 2020

Paper 1: Technology-free (60 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government
Queensland Curriculum & Assessment Authority
For all Queensland schools

Introduction

The Queensland Curriculum and Assessment Authority (QCAA) has developed mock external assessments for each General senior syllabus subject to support the introduction of external assessment in Queensland.

An external assessment marking guide (EAMG) has been created specifically for each mock external assessment.

The mock external assessments and their marking guides were:

- developed in close consultation with subject matter experts drawn from schools, subject associations and universities
- aligned to the external assessment conditions and specifications in General senior syllabuses
- developed under secure conditions.

Purpose

This document consists of an EAMG and an annotated response.

The EAMG:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of '0' will be recorded.

Where no response to a question has been made, a mark of 'N' will be recorded.

External assessment marking guide

Multiple-choice

Question	Response
1	D
2	A
3	A
4	B
5	B
6	D
7	A
8	A
9	C
10	A

Short response

Question 11 (5 marks)

Sample response	The response
a) $\frac{3}{x}$	correctly states the derivative **[1 mark]**
b) Using the product rule $-x^3\sin(x^4+1).4x^3+\cos(x^4+1).3x^2$	provides a statement identifying the use of the product rule **[1 mark]** correctly determines the first term **[1 mark]** correctly determines the second term **[1 mark]**
c) $\frac{3e^{4x}}{4}+c$	correctly determines the integral **[1 mark]**

Question 12 (3 marks)

Sample response	The response
a) $x = \log_6 9 + \log_6 4$ $= \log_6 36$ $36 = 6^x$ $x = 2$	correctly simplifies the expression **[1 mark]**
Method 1: Changing to index form b) $x = \log_{64} 16$ $16 = 64^x$ $4^2 = 4^{3x}$	correctly establishes the index equation **[1 mark]**
$3x = 2$ $x = \frac{2}{3}$	correctly simplifies the expression **[1 mark]**
OR	**OR for Method 2**
Method 2: Using the change of base rule b) $x = \frac{\log_4 16}{\log_4 64}$	correctly establishes the logarithmic expression using base 4 **[1 mark]**
$x = \frac{2}{3}$	correctly simplifies the expression **[1 mark]**

Question 13 (4 marks)

Sample response	The response
a) $2x + 4 = 25$	correctly develops the linear equation **[1 mark]**
$x = \frac{21}{2}$	correctly determines x **[1 mark]**
b) $\ln\left(\frac{7-x}{10}\right) = \ln(x)$	
$\frac{7-x}{10} = x$	correctly develops the linear equation **[1 mark]**
$x = \frac{7}{11}$	correctly determines x **[1 mark]**

Question 14 (5 marks)

Sample response	The response
a) $a = \frac{d}{dt}\left(\frac{1}{\pi} + 3\sin(t)\right)$	provides a statement identifying the use of derivatives **[1 mark]**
$a = 3\cos(t)$	correctly determines the derivative **[1 mark]**
b) $s = \int \frac{1}{\pi} + 3\sin(t)\, dt$	
$s = \frac{t}{\pi} - 3\cos(t) + c$	correctly determines the integral of $\frac{1}{\pi}$ **[1 mark]** correctly determines the integral of $3\sin(t)$ **[1 mark]**
Given $s\left(\frac{\pi}{3}\right) = 4$	
$4 = \frac{\frac{\pi}{3}}{\pi} - 3\cos\left(\frac{\pi}{3}\right) + c$ $c = 4 - \frac{1}{3} + \frac{3}{2}$ $c = \frac{31}{6}$	determines constant for displacement function **[1 mark]**
$\therefore s = \frac{t}{\pi} - 3\cos(t) + \frac{31}{6}$	

Question 15 (9 marks)

Sample response	The response
a) $f'(x) = 0 = x(2-x)e^{-x+4}$ Use null factor theorem $x = 0, 2$ Use given graph of $f(x)$ to identify maximum at $x = 2$ Maximum point $(2, f(2)) = (2, 4e^2)$	provides a statement identifying the x-intercepts of $f'(x)$ as the critical points **[1 mark]** correctly determines the coordinates of the maximum point **[1 mark]**
b) $f''(2) = (4 - 8 + 2)e^2$ $f''(x) < 0 \therefore$ maximum	substitutes value into second derivative **[1 mark]** verifies nature of stationary point **[1 mark]**

c) $f''(x) = 0$ $= (x^2 - 4x + 2)e^{-x+4}$ Use null factor theorem $(x^2 - 4x + 2) = 0$	correctly establishes the quadratic equation **[1 mark]**
Solving the quadratic equation: $x = \frac{4 \pm \sqrt{8}}{2}$	correctly determines the x- coordinates of the two points of inflection **[1 mark]**
d) $f''(x) < 0$ curve is concave down or $f''(x) > 0$ curve is concave up	provides evidence of relationship between concavity and second derivative **[1 mark]**
for $\frac{4-\sqrt{8}}{2} < x < \frac{4+\sqrt{8}}{2}$ the curve is concave down	determines interval where curve is concave down **[1 mark]**
for $x < \frac{4-\sqrt{8}}{2}$ and $x > \frac{4+\sqrt{8}}{2}$ the curve is concave up	determines intervals where curve is concave up **[1 mark]**

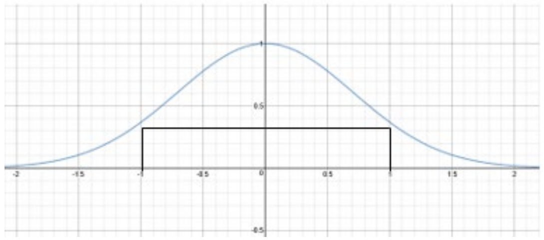
Question 16 (5 marks)

Sample response	The response
$y'' = 6x + 2a$ Given point of inflection $(1,6)$ Therefore $0 = 6 + 2a$ $a = -3$	correctly determines a **[1 mark]**
Point of inflection is a point on the function Therefore $b = 7$	determines b **[1 mark]**
$\therefore y = x^3 - 3x^2 + 7x + 1$ Gradient of tangent at any point $y'(x) = 3x^2 - 6x + 7$ Gradient of tangent at $(1,6)$ $y'(1) = 4$	determines gradient of tangent at point of inflection **[1 mark]**
Equation of tangent at $(1,6)$ $y = 4x + 2$	determines equation of tangent **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**

Question 17 (7 marks)

Sample response	The response
$f(x) = \begin{cases} a + bx^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$ pdf $\therefore \int_0^1 a + bx^2 dx = 1$	correctly determines the definite integral equation using pdf definition **[1 mark]**
$ax + \frac{bx^3}{3} \Big\|_0^1 = 1$	
$3a + b = 3$ (i)	correctly generates an equation in a and b **[1 mark]**
$E(X) = \int_0^1 x(a + bx^2)dx = \frac{3}{5}$	correctly determines the definite integral equation using $E(X)$ definition **[1 mark]**
$\frac{ax^2}{2} + \frac{bx^4}{4} \Big\|_0^1 = \frac{3}{5}$	
$10a + 5b = 12$ (ii) From (i) $b = -3a + 3$	correctly generates the second equation in a and b **[1 mark]**
Substitute into (ii) $10a - 15a + 15 = 12$	indicates use of simultaneous solving procedure **[1 mark]**
$a = \frac{3}{5}$ Substitute a into (i) $b = \frac{6}{5}$	determines a and b **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**

Question 18 (6 marks)

Sample response	The response
	provides evidence to support decision about dimensions of rectangle **[1 mark]**
Area (A) = $2x \times e^{-x^2}$ $A' = 2e^{-x^2}(-2x^2+1)$	correctly determines the derivative of the Area function **[1 mark]**
Stationary point/s $A' = 0$ $0 = 2e^{-x^2}(-2x^2+1)$ $x = \pm\dfrac{1}{\sqrt{2}}$	determines x −values of critical points **[1 mark]**
$A'(0) > 0$ $A'(2) < 0$ ∴ maximum point at $\frac{1}{\sqrt{2}}$	identifies x −value for maximum point with justification **[1 mark]**
Area = $2 \times \frac{1}{\sqrt{2}} \times e^{-\left(\frac{1}{\sqrt{2}}\right)^2}$ Area = $\frac{\sqrt{2}}{\sqrt{e}}$ square units	determines area of largest rectangle in simplest form **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**

Question 19 (6 marks)

Sample response	The response
Slope of the tangent $y' = 1 + \ln(x)$	correctly determines the expression for the gradient of the tangent line **[1 mark]**
Point of tangency $(x_0, x_0 \ln(x_0))$ The equation of the line passing through $(0, -e)$ $y + e = x(1 + \ln(x_0))$ *(i)*	correctly determines the tangent equation of the line passing through the given point **[1 mark]**
Substituting the coordinates of the point of tangency on the curve into (i) $x_0 \ln(x_0) + e = x_0 + x_0 \ln(x_0)$ Solving $\rightarrow x_0 = e$	determines x-coordinate of point on curve and tangent line **[1 mark]**
Point on the curve is (e, e) Slope of the tangent: $1 + \ln(e) = 2$	determines coordinates of point on curve and tangent line **[1 mark]**
Equation of tangent: $y = 2x - e$	determines equation of tangent **[1 mark]**
	shows logical organisation communicating key steps **[1 mark]**