LUI | | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your
barcode ID label here

Book of books used

External assessment

Question and response book

Mathematical Methods

Paper 1 — Technology-free

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are not permitted.
- QCAA formula sheet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 10 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

Section 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Do not write outside this box.

Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has 10 questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

Do not write outside this box.

QUESTION 11 (3 marks)

Determine the derivative of each of the following with respect to x.

a) $y = \dfrac{1}{\sin(x)}$ *[1 mark]*

b) $y = x^2 \times e^{-x}$

Express your answer in factorised form. *[2 marks]*

Do not write outside this box.

QUESTION 12 (5 marks)

An object is moving in a straight line from a fixed point. The object is at the origin initially.

The acceleration a (in m s^{-2}) of the object is given by

$$a(t) = \pi \cos(\pi t) \qquad t \geq 0, \text{ where } t \text{ is time in seconds.}$$

The velocity at $t = 1$ is 0.5 m s^{-1}

a) Determine the initial acceleration. *[1 mark]*

b) Determine the initial velocity. *[2 marks]*

Do not write outside this box.

c) Determine the displacement after one second. *[2 marks]*

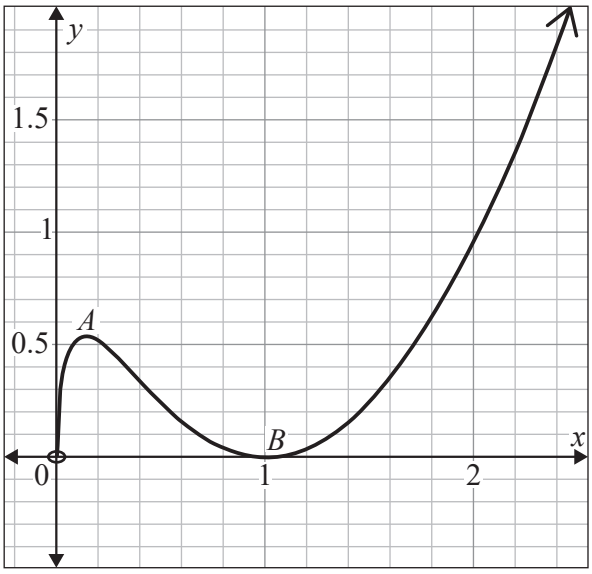
Do not write outside this box.

QUESTION 13 (7 marks)

A function is defined as $f(x) = x(\ln(x))^2$, $x > 0$.

The graph of the function is shown and has a local maximum at point A and a global minimum at point B.

The derivative of the function is given by $f'(x) = 2\ln(x) + (\ln(x))^2$, $x > 0$.



a) Verify that there is a stationary point at $x = 1$. *[2 marks]*

Do not write outside this box.

b) Determine the coordinates of A. *[3 marks]*

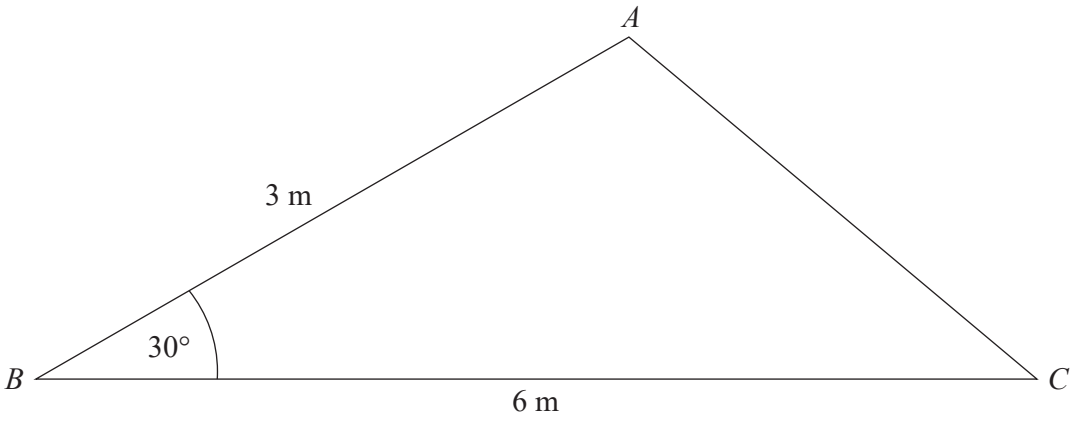
The graph of the function has a point of inflection at $x = e^p$

c) Determine p. *[2 marks]*

Do not write outside this box.

QUESTION 14 (3 marks)

Determine the area of the triangle shown.



Not drawn to scale

Do not write outside this box.

QUESTION 15 (4 marks)

Solve the following equations.

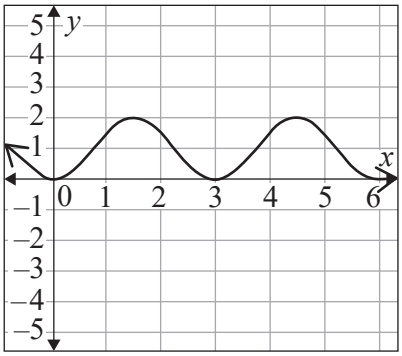
a) $4e^{x} = 100$ *[1 mark]*

b) $2 \log_4 x - \log_4(x - 1) = 1$ *[3 marks]*

Do not write outside this box.

QUESTION 16 (4 marks)

Consider the following graph of $f(x)$.



Identify the graph of the second derivative $f''(x)$ from the graphs in Diagram 1, Diagram 2 and Diagram 3.

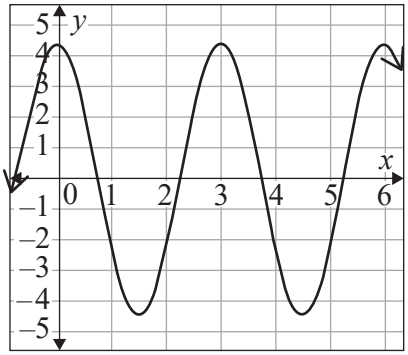


Diagram 1

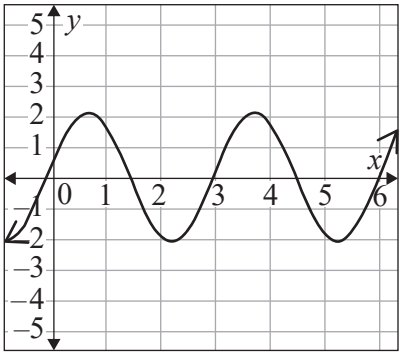


Diagram 2

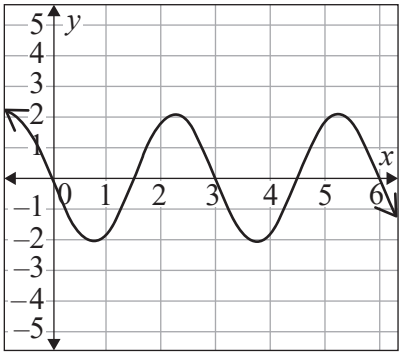


Diagram 3

Justify your decisions using mathematical reasoning.

Do not write outside this box.

QUESTION 17 (6 marks)

The volume of water in a tank is represented by a function of the form

$$V(t) = Ae^{kt}, \text{ where } V \text{ is in litres and } t \text{ is in minutes.}$$

Initially, the volume is 100 litres and it is decreasing by 50 litres per minute.

Determine the time at which the volume is decreasing at the rate of $\frac{50}{7}$ litres per minute.

Express your answer in the form $\ln(a)$.

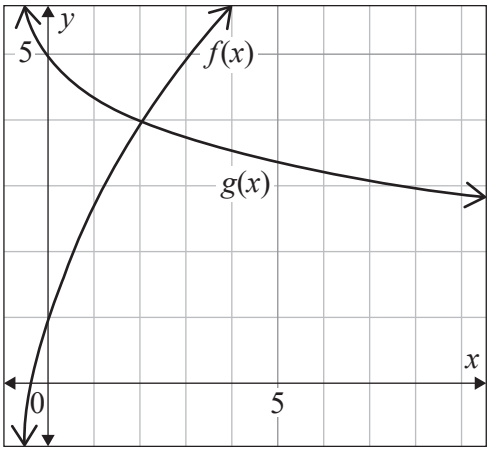
Do not write outside this box.

QUESTION 18 (6 marks)

The function $f(x)$ has the form given by $f(x) = 3 \log_2 (x + a) + b$

The function $g(x)$ has the form given by $g(x) = -\log_3 (x + c) + 5$

A section of the graphs of the two functions is shown.



Determine the values of a, b and c.

Do not write outside this box.

Do not write outside this box.

QUESTION 19 (6 marks)

A horizontal point of inflection is a point of inflection that is also a stationary point.

Determine the value/s of k for which the graph of $f(x) = \frac{\ln(x)}{k} - \frac{kx}{x+1}$ has only one horizontal point of inflection.

Do not write outside this box.

Do not write outside this box.

QUESTION 20 (6 marks)

At the end of the first stage of its growth cycle, a species of tree has a height of 5 metres and a trunk radius of 15 cm.

In the second stage of its growth cycle, the tree stays at this height for the next 10 years. However, the growth rate of the trunk radius (in cm per year) varies over the 10 years and is given by the function

$$r(t) = 1.5 + \sin\left(\frac{\pi t}{5}\right)$$

Assume the density (mass per unit volume) of the tree trunk is approximately 1 g/cm^3 and the tree trunk is in the shape of a cylinder.

Determine the ratio of the trunk's mass at the end of the second stage to its mass at the end of the first stage.

Do not write outside this box.

END OF PAPER

Do not write outside this box.

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Do not write outside this box.

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Do not write outside this box.

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Do not write outside this box.

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Do not write outside this box.