Mathematical Methods marking guide

External assessment

Paper 1: Technology-free (60 marks)

Paper 2: Technology-active (60 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government
Queensland Curriculum & Assessment Authority
For all Queensland schools

Purpose

This document is an External assessment marking guide (EAMG).

The EAMG:

- Provides a tool for calibrating external assessment markers to ensure reliability of results
- Indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- Informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded. Where no response to a question has been made, a mark of ‘N’ will be recorded.

Allow FT mark(s) – refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working – the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not evident in the student response, but by virtue of subsequent working there is sufficient evidence to award mark(s).

External assessment marking guide

Paper 1

Multiple choice

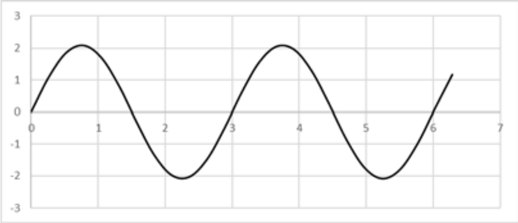
Question	Response
1	D
2	A
3	D
4	B
5	B
6	D
7	C
8	B
9	C
10	A

Short response

Q	Sample response	The response:
11	a) $f'(x) = \frac{-\cos(x)}{(\sin(x))^2}$	correctly determines the derivative **[1 mark]**
	b) $f'(x) = -x^2e^{-x} + 2xe^{-x}$ $= xe^{-x}(-x+2)$	correctly determines the derivative in expanded form **[1 mark]** determines factorised form of derivative **[1 mark]**
12	a) $a(0) = \pi\cos(\pi \times 0)$ $a(0) = \pi$ m s^{-2}	correctly determines the initial acceleration **[1 mark]**
	b) $\int a(t)dt = v(t)$ $v(t) = \sin(\pi t) + c$ Given $v = 0.5$ when $t = 1$ $0.5 = \sin(\pi) + c$ $c = 0.5$ $v(0) = \sin(0) + 0.5$ Initial velocity is 0.5 m s^{-1}	correctly determines the general function $v(t)$ **[1 mark]** determines initial velocity **[1 mark]**
	c) $\int v(t)dt = s(t)$ $s(t) = \frac{-1}{\pi}\cos(\pi t) + 0.5t + c$ Given $s = 0$ when $t = 0$ $0 = \frac{-1}{\pi}\cos(0) + 0.5 \times 0 + c$ $c = \frac{1}{\pi}$ $s(1) = \frac{-1}{\pi}\cos(\pi) + 0.5 + \frac{1}{\pi}$ $s(1) = 0.5 + \frac{2}{\pi}$ m	determines general function $s(t)$ **[1 mark]** determines displacement after one second **[1 mark]**

Q	Sample response	The response:
13	a) Given $f'(x) = 2\ln(x) + (\ln(x))^2$ Stationary point $f'(x) = 0$ $0 = 2\ln(x) + (\ln(x))^2$ $0 = \ln(x)\,(2 + \ln(x))$ $\ln(x) = 0 \therefore x = 1$	correctly identifies that the derivative equals 0 **[1 mark]** correctly shows there is a stationary point at $x = 1$ **[1 mark]**
	b) Stationary point $f'(x) = 0$ From a) $2 + \ln(x) = 0$ $\ln(x) = -2$ $x = e^{-2}$ $\therefore y = e^{-2} \times (\ln e^{-2})^2 = 4e^{-2}$ so $A(e^{-2}, 4e^{-2})$	correctly establishes the equation in x **[1 mark]** correctly determines the x-ordinate of A **[1 mark]** determines y-ordinate of A **[1 mark]**
	c) Using chain rule $f''(x) = \frac{2}{x} + \frac{2}{x}\ln(x)$ Point of inflection $f''(x) = 0$ $0 = \frac{2}{x} + \frac{2}{x}\ln(x)$ $0 = \frac{2}{x}(1 + \ln(x))$ $0 = 1 + \ln(x) \rightarrow x = e^{-1}$ $\therefore p = -1$	correctly establishes the equation in x equals 0 **[1 mark]** determines p **[1 mark]**

Q	Sample response	The response:
14	$\angle ABC = 30°$ $\text{Area} = \frac{1}{2} \times a \times c \times \sin B$ $\text{Area} = \frac{1}{2} \times 6 \times 3 \times \sin 30°$ $\text{Area} = \frac{9}{2}\ \text{m}^2$	correctly substitutes into the area equation **[1 mark]** correctly determines the area **[1 mark]** correctly communicates the units **[1 mark]**
15	$e^x = 25$ $x = \ln(25)$ Using log laws $\log_4 \frac{x^2}{x-1} = 1$ Change from log to index form $\frac{x^2}{x-1} = 4$ $x^2 - 4x + 4 = 0$ Factorising $x = 2$	correctly determines x **[1 mark]** correctly establishes equation using log laws **[1 mark]** correctly establishes the quadratic equation **[1 mark]** determines x **[1 mark]**

Q	Sample response	The response:
16	$f(x)$ changes concavity at certain points and is increasing and decreasing between $x = 0$ and $x = 3$ and between $x = 3$ and $x = 6$	correctly identifies an appropriate method to determine $f''(x)$ graph **[1 mark]**
	Therefore, the sketch of $f'(x)$ must be positive and negative (above and below the x-axis) for these intervals.	correctly determines a relevant feature of the graph of $f(x)$ **[1 mark]**
		
	$f'(x)$ has maximum and minimum points at approximately $x = 0.8$ and $x = 2.4$. Therefore, the sketch of $f''(x)$ will cross the x-axis at these points.	correctly determines a relevant feature for $f''(x)$ **[1 mark]**
	Diagram 1 is $f''(x)$	correctly identifies Diagram 1 **[1 mark]**

Q	Sample response	The response:
17	At $t = 0, V = 100$ $V(0) = 100 = Ae^{k \times 0}$ $A = 100$ $\therefore V(t) = 100e^{kt}$ $V'(t) = 100ke^{kt}$ At $t = 0, V'(t) = -50$ $-50 = 100k$ $k = \frac{-1}{2}$ So $V(t) = 100e^{-0.5t}$ and $V'(t) = -50e^{-0.5t}$ Determine t when $V'(t) = \frac{-50}{7}$ $\frac{1}{7} = e^{-0.5t}$ $-0.5t = \ln\left(\frac{1}{7}\right)$ $t = -2\ln\left(\frac{1}{7}\right)$ $t = \ln(49)$	correctly determines A **[1 mark]** determines k **[1 mark]** establishes equation in t **[1 mark]** determines t **[1 mark]** determines t in required form **[1 mark]** shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
18	Using y-intercepts $1 = 3\log_2(a) + b$ (i) $5 = -\log_3(c) + 5$ (ii) From (ii) $c = 1$	correctly determines c **[1 mark]**
	Using point of intersection $(2,\ 4)$ $4 = 3\log_2(2 + a) + b$ (iii) Solving simultaneously (i) and (iii) $b = 1 - 3\log_2(a)$ (i) $b = 4 - 3\log_2(2 + a)$ (ii)	correctly establishes two equations in a and b **[1 mark]**
	Equating $1 - 3\log_2(a) = 4 - 3\log_2(2 + a)$ $3\log_2(2 + a) - 3\log_2(a) = 3$ $3(\log_2\left(\frac{2+a}{a}\right)) = 3$ $\frac{2+a}{a} = 2$	correctly selects procedure to solve for unknowns **[1 mark]**
	$a = 2$ $\therefore b = -2$	correctly determines a **[1 mark]** determines b **[1 mark]**
		shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
19	**Method 1** $f'(x) = \frac{1}{kx} - \left(\frac{k(x+1) - kx}{(x+1)^2}\right)$ Stationary points $f'(x) = 0$ $0 = \frac{1}{kx} - \left(\frac{k(x+1) - kx}{(x+1)^2}\right)$ $0 = \frac{1}{kx} - \frac{k}{(x+1)^2}$ $0 = x^2 + (2 - k^2)x + 1$ (i) The quadratic has real roots when discriminant ≥ 0 $(2 - k^2)^2 - 4 \geq 0$ $2 - k^2 \geq \pm 2$ There is only ONE phi $\therefore$ $2 - k^2 = \pm 2$ $k = 0$ (not valid) and $k^2 = 4$ so $k = 2, -2$ Sub into (i) to determine the x-ordinate of the stationary point. $\rightarrow x = 1$ For $k = 2$ $\therefore f''(x) = \frac{-1}{2x^2} + \frac{4}{(x+1)^3}$ $f''(1) = \frac{-1}{2} + \frac{4}{8}$ $f''(1) = 0$ For $k = -2$ $f''(x) = \frac{1}{2x^2} - \frac{4}{(x+1)^3}$	correctly determines the first derivative **[1 mark]** correctly determines the quadratic equation to identify the stationary point/s **[1 mark]** determines valid and non-valid solutions of k **[1 mark]** determines x-ordinate of stationary point **[1 mark]**

Q	Sample response	The response:
	$f''(1) = \frac{1}{2} - \frac{4}{8}$ $f''(1) = 0$ For each k value, $x = 1$ is the x-ordinate of both a stationary point ($f'(x) = 0$) and a point of inflection ($f''(x) = 0$) There is a point of horizontal inflection at $x = 1$ when $k = \pm 2$	determines values of second derivative for both values of k **[1 mark]** shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
	Method 2 $f'(x) = \frac{1}{kx} - \left(\frac{k(x+1) - kx}{(x+1)^2}\right)$	correctly determines the first derivative **[1 mark]**
	Stationary points $f'(x) = 0$ $0 = \frac{1}{kx} - \left(\frac{k(x+1) - kx}{(x+1)^2}\right)$ $\frac{1}{kx} = \frac{k}{(x+1)^2}$ $k^2 x = (x+1)^2$ $k^2 = \frac{(x+1)^2}{x}$ (i) $k = \pm\sqrt{\frac{(x+1)^2}{x}}$ (ii)	correctly establishes expression for k^2 in terms of x **[1 mark]**
	Point of inflection $f''(x) = 0$ $\therefore 0 = \frac{-1}{kx^2} + \frac{2k}{(x+1)^3}$ $k^2 = \frac{(x+1)^3}{2x^2}$ sub into (i) $\frac{(x+1)^2}{x} = \frac{(x+1)^3}{2x^2}$ $2x^2(x+1)^2 - x(x+1)^3 = 0$ $x(x+1)^2\big(2x - (x+1)\big) = 0$ $x(x+1)^2(x-1) = 0$ $x = 0, -1, 1$	determines x values **[1 mark]**
	sub into (ii) $x = 0, -1$ non-valid solutions Using $x = 1$	determines valid and non-valid solutions of x **[1 mark]**

Q	Sample response	The response:
	$k = \pm\sqrt{\frac{(1+1)^2}{1}} = \pm 2$ For each k value $x = 1$ is the x-ordinate of both a stationary point $(f'(x) = 0)$ and a point of inflection $(f''(x) = 0)$ There is a point of horizontal inflection at $x = 1$ when $k = \pm 2$	determines k values **[1 mark]** shows logical organisation communicating key steps **[1 mark]**

<table>
<tr><th>Q</th><th>Sample response</th><th>The response:</th></tr>
<tr><td>20</td><td>Let $R(t)$ = radius of the tree trunk
$R(t) = \int 1.5 + \sin\left(\frac{\pi t}{5}\right) dt$
$= 1.5t - \frac{5}{\pi}\cos\left(\frac{\pi t}{5}\right) + c$
Given initial radius is 15 cm (end of first stage/beginning of second stage)
$15 = -\frac{5}{\pi} + c$
$c = 15 + \frac{5}{\pi}$
$R(t) = 1.5t - \frac{5}{\pi}\cos\left(\frac{\pi t}{5}\right) + 15 + \frac{5}{\pi}$

Volume of tree trunk $= 500\pi(R(t))^2$
Volume of tree trunk at $t = 10$
$V_{10} = \pi \times (R(10))^2 \times 500$
Volume of tree trunk at $t = 0$
$V_0 = \pi \times (R(0))^2 \times 500$
$\rightarrow V_{10} = 450\,000\pi\ \text{cm}^3$ and $V_0 = 112\,500\pi\ \text{cm}^3$
The mass of the tree is the density times the volume, and since the density is 1 g/cm^3
∴ ratio of the tree trunk at $t = 0$ and $t = 10$ years
$= \frac{450\,000\pi}{112\,500\pi}$
$= 4$
The tree has quadrupled in mass over this time.</td><td>correctly establishes an integrated expression for the radius of the tree [1 mark]

determines radius of the tree [1 mark]

identifies use of formula and radius to determine volume of tree trunk [1 mark]

determines volumes of tree trunk initially and at 10 years [1 mark]

determines ratio of mass [1 mark]

shows logical organisation communicating key steps [1 mark]</td></tr>
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External assessment marking guide

Paper 2

Multiple choice

Question	Response
1	C
2	B
3	B
4	C
5	C
6	A
7	B
8	A
9	B
10	D

Short response

Q	Sample response	The response:
11	a) $5\% \times 20 = 1$	correctly determines the expected number of the underweight packets **[1 mark]**
	b) $\text{Variance} = np(1-p)$ $= 0.95$	correctly determines the variance of the underweight packets **[1 mark]**
	c) $n = 20$ $\text{success} = p = 0.05$ Number of successes $= 0$ or 1 Using GDC $P(\text{at most 1 are underweight}) = 0.7358$	correctly uses an appropriate mathematical representation to communicate approach **[1 mark]** correctly determines the probability **[1 mark]**
12	a) $A(t) = 45\ln(t+1) + c$ $B(t) = 3500e^{0.03t} + c$ Given $A(0) = 5000, B(0) = 3500$ $A(t) = 45\ln(t+1) + 5000$ $B(t) = 3500e^{0.03t}$	correctly determines the general population functions for City A **[1 mark]** correctly determines the general population functions for City B **[1 mark]** determines population functions for the two cities **[1 mark]**
	b) $B(10) = 3500e^{0.3}$ $B(10) = 4724$	determines population of City B in 2028 **[1 mark]**

Q	Sample response	The response:
	c) $45\ln(t+1) + 5000 = 3500e^{0.03t}$ Using GDC $t = 12.66$ $\therefore$ during the year 2030 the populations will be the same.	uses an appropriate mathematical representation to communicate approach **[1 mark]** determines time **[1 mark]** determines the required time **[1 mark]**
13	a) $\hat{p} = \frac{102}{121} = 0.84$ b) Using normal distribution $\mu = 0.9$ $\sigma = \sqrt{\frac{0.9 \times 0.1}{121}}$ Using GDC $P(\hat{p} < 0.85)$ $= 0.0334$ c) There is 3.34% chance of shipping less than 85% of orders when the population parameter is 0.90. Observing a sample proportion of 0.843 (or even lower) would have occurred by chance less than 3.34% of the time if the retailer's claim is true. Therefore, we suspect that the retailer's claim is dubious.	correctly determines the sample proportion **[1 mark]** correctly identifies the mean of the population **[1 mark]** determines expression for standard deviation of the normal distribution **[1 mark]** determines probability of producing sample proportion or less **[1 mark]** identifies meaning of the probability of producing the sample proportion **[1 mark]** evaluates reasonableness of the retailer's claim **[1 mark]**

Q14 Method 1: Interpreting the given equation as a CDF

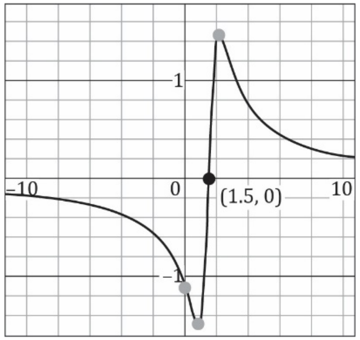
Sample response	The response:
a) $\text{pdf}\left(f(x)\right) = \frac{d}{dx}\left(\frac{-10}{x}\right)$	correctly identifies the use of the derivative of the cumulative distribution function **[1 mark]**
$f(x) = \begin{cases} \frac{10}{x^2}, & 5 \le x \le 10 \\ 0, & \text{otherwise} \end{cases}$	correctly determines the derivative of the cumulative distribution function (pdf) **[1 mark]** uses appropriate convention to communicate the probability density function as a piecewise function with given domains **[1 mark]**
b) $F(7) - F(5) = \frac{-10}{7} - \frac{-10}{5}$ $= \frac{4}{7}$	correctly determines the probability that there are 5 to 7 minutes between train arrivals **[1 mark]**
c) $\text{Mean} = \int_5^{10} \frac{10}{x}\,dx$ Using GDC	provides a statement identifying the use of expected value for a continuous random variable **[1 mark]**
$\text{Mean} = 6.93$ minutes	determines mean time between arrivals **[1 mark]**

Q14 Method 2: Interpreting the given equation as a PDF

Sample response	The response:
a) $F(x) = \int_{-\infty}^{x} f(t)\,dt$	
$= \int_{5}^{x} 2 - \frac{10}{t}\,dt$ for $5 \leq x \leq 10$	correctly identifies the use of the integral of the probability distribution function **[1 mark]**
$\int_{5}^{x} 2 - \frac{10}{t}\,dt = [2t - 10\ln(t)]_{5}^{x}$	
$= 2\left(x - 5\ln\left(\frac{x}{5}\right) - 5\right)$	correctly determines the integral of the probability distribution function (CDF) **[1 mark]**
$\therefore F(x) = \begin{cases} 0 & x < 5 \\ 2\left(x - 5\ln\left(\frac{x}{5}\right) - 5\right) & 5 \leq x \leq 10 \\ 1 & x > 10 \end{cases}$	uses appropriate convention to communicate the cumulative distribution function as a piecewise function with given domains **[1 mark]**
b) $F(7) - F(5) = 2\left(7 - 5\ln\left(\frac{7}{5}\right) - 5\right) - 2\left(5 - 5\ln\left(\frac{5}{5}\right) - 5\right)$	
$= 0.6353$	correctly determines the probability that there are 5 to 7 minutes between train arrivals **[1 mark]**
c) $\text{Mean} = \int_{5}^{10} 2x - 10\,dx$	provides a statement identifying the use of expected value for a continuous random variable **[1 mark]**
Using GDC	
$\text{Mean} = 25$ minutes	determines mean time between arrivals **[1 mark]**

Q	Sample response	The response:
15	a) Using trapezoidal rule: $11.12 = \frac{1+3}{2} + \frac{3+2.6}{2} + \frac{2.6+2.2}{2} + \frac{2.2+x}{2} + \frac{x+1.76}{2}$ $3.92 = 1.98 + x$ $x = 1.94$ m b) $\int_0^5 \frac{4x}{x^2+1} + 1 \quad dx$ $= 11.5162$ m^2	correctly establishes equation in x **[1 mark]** determines x **[1 mark]** correctly determines the area **[1 mark]**
16	Given $\mu = 591$ Using GDC z-score associated with 65th percentile $= 0.38532$ $0.38532 = \frac{593 - 591}{\sigma}$ $\sigma = 5.1905$ Using GDC z-score associated with 20% rejection region = −0.841621 To determine the smallest volume that will be accepted (x) $-0.841\,621 = \frac{x - 591}{5.1905}$ $x = 587$ mL	correctly determines the z-score associated with the 65th percentile **[1 mark]** determines σ **[1 mark]** correctly determines the z-score associated with the 20% rejection region **[1 mark]** determines the smallest volume **[1 mark]**

Q	Sample response	The response:
17	$\hat{p} = \frac{303}{326}$ Using confidence interval formula (margin of error) $0.02 = 1.96 \times \sqrt{\frac{\frac{303}{326} \times \frac{23}{326}}{n}}$ Using GDC $n = 629.778$ $\therefore$ a sample of 630 lecturers would be required.	correctly determines the sample proportion **[1 mark]** establishes equation in n **[1 mark]** determines n **[1 mark]** determines reasonable value of n **[1 mark]**
18	Constructing the line BD Using the cosine rule $BD^2 = 4.1^2 + 7.6^2 - 2 \times 4.1 \times 7.6 \times \cos 117°$ $BD = 10.142$ cm Using the cosine rule $10.142^2 = 5.4^2 + DC^2 - 2 \times 5.4 \times DC \cos 62°$ Using solve application GDC $DC = 11.4865$ Perimeter of $ABCD$ $= 11.4865 + 5.4 + 4.1 + 7.6$ $= 28.5865$ cm	 correctly determines BD **[1 mark]** establishes equation in DC **[1 mark]** determines DC **[1 mark]** determines perimeter **[1 mark]**

Q	Sample response	The response:
19	Downhill drive will correspond to domain where derivative is negative. Graphing $D'(x)$  The function is decreasing $-10 \leq x < 1.5$ The distance driving downhill $\int_{-10}^{1.5} \sqrt{1 + (D'(x))^2}\, dx$ Using GDC $= 11.5$ kilometres* Time to travel 11.5 kilometres driving at 40 km/hour $t = \frac{11.5}{40} = 0.29$ hours* * Note: An erroneous answer of 0.33 hours, corresponding to a distance of 13.25 km, was obtained when students failed to ensure that x and D(x) were measured in the same units. This was awarded full marks as the length of a curve formula was unfamiliar to students.	correctly identifies values of x associated with downhill drive **[1 mark]** correctly uses an appropriate mathematical representation **[1 mark]** determines decreasing interval **[1 mark]** establishes integral expression for the total distance travelled downhill **[1 mark]** determines the distance travelled downhill **[1 mark]** determines time travelling downhill **[1 mark]** shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
20	**Method 1** $p_1 =$ proportion of Town A who prefer to drink tea $p_2 =$ proportion of Town B who prefer to drink tea The sample proportions are: $\hat{p}_1 = \frac{111}{216}$ $\hat{p}_2 = \frac{150}{257}$ Using the 99% confidence interval for the difference of two proportions $\left(\frac{111}{216} - \frac{150}{257} - 2.576\sqrt{\frac{\frac{111}{216}\left(1-\frac{111}{216}\right)}{216} + \frac{\frac{150}{257}\left(1-\frac{150}{257}\right)}{257}}, \frac{111}{216} - \frac{150}{257} + 2.576\sqrt{\frac{\frac{111}{216}\left(1-\frac{111}{216}\right)}{216} + \frac{\frac{150}{257}\left(1-\frac{150}{257}\right)}{257}}\right)$ $= (-0.188,\ 0.048\ )$ This interval contains zero; therefore, there is no evidence in the data to say that the two proportions are different, i.e. preference to drink tea does not depend on where the person lives.	correctly determines the sample proportions **[1 mark]** establishes confidence interval for the difference of two proportions **[1 mark]** determines 99% confidence interval **[1 mark]** interprets 99% confidence interval to determine equality of proportions **[1 mark]** shows logical organisation communicating key steps **[1 mark]**
20	**Method 2** $p_1 =$ proportion of Town A who prefer to drink coffee $p_2 =$ proportion of Town B who prefer to drink coffee The sample proportions are:	

Q	Sample response	The response:
	$\hat{p}_1 = \frac{105}{216}$ $\hat{p}_2 = \frac{107}{257}$	correctly determines the sample proportions **[1 mark]**
	Using the 99% confidence interval for the difference of two proportions $\left(\left(\frac{105}{216} - \frac{107}{257} - 2.576\sqrt{\frac{\frac{105}{216}\left(1-\frac{105}{216}\right)}{216} + \frac{\frac{107}{257}\left(1-\frac{107}{257}\right)}{257}}, \frac{105}{216} - \frac{107}{257} + 2.576\sqrt{\frac{\frac{105}{216}\left(1-\frac{105}{216}\right)}{216} + \frac{\frac{107}{257}\left(1-\frac{107}{257}\right)}{257}}\right)\right)$	establishes confidence interval for the difference of two proportions **[1 mark]**
	$= (-0.048,\ 0.188\ )$	determines 99% confidence interval **[1 mark]**
	This interval contains zero; therefore, there is no evidence in the data to say that the two proportions are different, i.e. the preference to drink coffee does not depend on where the person lives.	interprets 99% confidence interval to determine equality of proportions **[1 mark]**
		shows logical organisation communicating key steps **[1 mark]**