Mathematical Methods 2019 v1.2

IA3 sample marking scheme

October 2023

Examination (15%)

This sample has been compiled by the QCAA to model one possible approach to allocating marks in an examination. It matches the examination mark allocations as specified in the syllabus (~ 60% simple familiar, ~ 20% complex familiar and ~ 20% complex unfamiliar) and ensures that a balance of the objectives are assessed.

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

Queensland Government
Queensland Curriculum & Assessment Authority
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Instrument-specific marking guide (ISMG)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
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4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4
	> 13%	3

• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.	> 7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.	> 0%	1
• does not satisfy any of the descriptors above.		0

Task

See IA3 sample assessment instrument: Examination (15%) (available on the QCAA Portal).

Sample marking scheme

Criterion	Marks allocated	Provisional marks
Foundational knowledge and problem-solving Assessment objectives 1, 2, 3, 4, 5 and 6	15	—
Total	**15**	**—**

The annotations are written descriptions of the expected response for each question and are related to the assessment objectives.

Note: ✓ = $\frac{1}{2}$ mark

Paper 1 (technology-free)

Question 1 (SF 4 marks)

$$E(X) = \int_{20}^{70} xp(x)\,dx \checkmark$$

$$= \int_{20}^{70} x \times 0.02\,dx \checkmark$$

$$= [0.01x^2]_{20}^{70} \checkmark\checkmark$$

$$= 0.01 \times 70^2 - 0.01 \times 20^2 \checkmark$$

$$E(x) = 49 - 4 = 45 \checkmark$$

✓✓

1.
comprehend information to identify relevant concepts and techniques to determine the expected value
select and use facts and procedures to:

- set up integral equation for a given probability density function
- integrate a polynomial function
- set up a definite integral

communicate using mathematical symbols

2.
recall and use definitions and procedures for determining:

- a point of inflection
- the derivative of a polynomial function

comprehend the purpose of given information (point of inflection at $x = 1$) is to identify relevant technique to use (factor theorem)
recall and use procedures for factorising a cubic polynomial
recall and use procedures for:

Question 2 (CU 6 marks)

Solve for $h''(x) = 0$ ✓

Given $h(x) = x^5 + 5x^4 + \frac{10x^3}{3} - 50x^2 + 5x + 2$

$h'(x) = 5x^4 + 20x^3 + 10x^2 - 100x + 5$ ✓

$h''(x) = 20x^3 + 60x^2 + 20x - 100$ ✓

$h''(x) = 0 = 20(x^3 + 3x^2 + x - 5)$

$x = 1$ identified as a zero of the function $x^3 + 3x^2 + x - 5$
$\therefore (x - 1)$ is a factor ✓✓

Using the factor theorem: ✓

- factorising a cubic polynomial
- determining the zeros of a quadratic

communicate change in curvature at the inflection point

evaluate reasonableness of solution to interpret their mathematical result using knowledge of given information

$h''(x) = 0 = 20(x-1)(x^2+4x+5)$ ✓✓

The discriminant of $x^2+4x+5<0$ ∴ no zeros ✓✓

∴ there is a possible point of inflection at the point $x = 1$

Checking the signs of the second derivative before and after $x = 1$ to determine concavity: ✓

x	0	1	2
$h''(x)$	–100	0	340

$h''(x)$ changes sign at $x = 1$ so it is a point of inflection. ✓

Question 3 (SF 2, CF 6 marks)

a.

$\sin(\theta) = \frac{y}{2} \to y = 2\sin(\theta)$ ✓

$\cos(\theta) = \frac{x}{2} \to x = 2\cos(\theta)$ ✓

Area of trapezium $= \frac{1}{2} \times ((2x+2)+2) \times y$ ✓

Substituting:

Area of trapezium $A = (2\cos(\theta) + 2) \times 2\sin(\theta)$

$A = 4\sin(\theta)\cos(\theta) + 4\sin(\theta)$ ✓

b.

Solve $A' = 0$

$A' = 4\sin(\theta) \times -\sin(\theta) + \cos(\theta) \times 4\cos(\theta) + 4\cos(\theta)$

$A' = -4\sin^2(\theta) + 4\cos^2(\theta) + 4\cos(\theta)$ ✓✓

Using Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$

$A' = -4(1-\cos^2(\theta)) + 4\cos^2(\theta) + 4\cos(\theta)$ ✓

$A' = 8\cos^2(\theta) + 4\cos(\theta) - 4$ ✓

Solving for θ $(A' = 0)$ ✓

$0 = 4(2\cos^2(\theta) + \cos(\theta) - 1)$

Factorising:

$0 = 4(2\cos(\theta) - 1)(\cos(\theta) + 1)$ ✓

3a.
recall and use definitions and procedures for determining expression for y and x in terms of θ

use rule for trapezium to generate given area function (opportunity to evaluate reasonableness of result)

comprehend information to identify optimisation problem

3b.
recall and use definitions and procedures for:

- determining the derivative of a product
- determining the derivative of trigonometric functions
- generating a disguised quadratic equation
- solving quadratic equations
- solving trigonometric equations

	Using null factor theorem: ✓ $2\cos(\theta)-1=0$ ✓and $\cos(\theta)+1=0$ ✓ $\cos(\theta)=\frac{1}{2}$ $\cos(\theta)=-1$ $\theta=\frac{\pi}{3}$ $\theta=\pi$	
recall exact values of $\sin\theta$ and $\cos\theta$	For the trapezium, $\theta=\frac{\pi}{3}$ is the only reasonable option✓✓	
use the second derivative to state and show value is maximum communicate findings	To determine if the area is a maximum: $A''(x) = -16\cos(\theta)\sin(\theta) - 4\sin(\theta)$ ✓ $A''\left(\frac{\pi}{3}\right) = -16\cos(\frac{\pi}{3})\sin(\frac{\pi}{3}) - 4\sin(\frac{\pi}{3})$ $A''\left(\frac{\pi}{3}\right) = -16\times\frac{1}{2}\times\frac{\sqrt{3}}{2} - 4\times\frac{\sqrt{3}}{2} = -8\sqrt{3} < 0$ ✓ ∴ the area of the trapezium is maximum when $\theta = \frac{\pi}{3}$. ✓ *(Note: first derivative test may also be used)*	

4a. use cosine rule to set up equation to solve use technology to solve for angle near Tower 1 recall and use definition for bearings to state position relative to Tower 1	# Paper 2 (technology-active) ## Question 4 (SF 7 marks) a. Using cosine rule (3 lengths of a triangle are given) ✓ $0.8^2 = 1.5^2 + 2^2 - 2\times 1.5\times 2\times\cos(A)$ (A is the angle at Tower 1) ✓ $\cos(A) = \frac{0.8^2 - 1.5^2 - 2^2}{-2\times 1.5\times 2}$ $A = 20.77°$ ✓✓ Therefore, you are situated at a position 20.77° north of east from Tower 1 ✓✓	

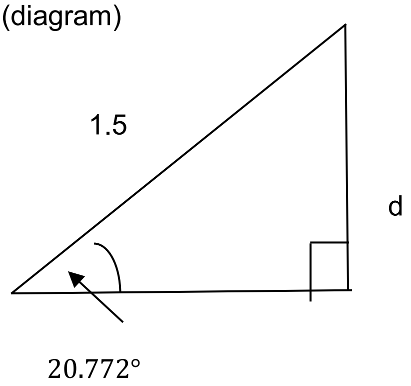
4b.
comprehend that critical element to determine distance requires right-angle triangle

use and recall appropriate trigonometric ratio to use to generate an equation

use technology appropriately to solve for distance and time

b.

✓✓ (diagram)



$$\sin 20.772° = \frac{d}{1.5} ✓✓$$

$d = 0.532\ km$ ✓✓

Time to walk this distance is $\frac{0.532}{5} = 0.1064$ hours (or 6.38 minutes). ✓✓

Question 5 (SF 7 marks)

a. Using sine rule:

$\frac{8}{\sin 125°} = \frac{3.5}{\sin ACB°}$ ✓✓

$ACB = \sin^{-1}\left(\frac{3.5\sin(125°)}{8}\right)$

$ACB = 21°$ ✓✓

$\therefore BAC = 34°$ ✓✓

b. Area of $\Delta BAC = \frac{1}{2} \times 3.5 \times 8 \times \sin 34° = 7.8287$ ✓✓

c. Volume of soil (m^3) = Area of ΔBAC × height of prism ✓✓

Volume of soil $(m^3) = 7.8287 \times .15$ ✓

Volume of soil (m^3 ✓✓) $= 1.17$ ✓

5a.
use sine rule to set up equation to solve
use technology to solve for angle ACB and BAC

5b.
use rule to determine area of BAC

5c.
recall rule for volume of a regular prism to determine volume of soil required
communicate using correct units

6.
comprehend that sample proportion techniques are required
use rule for standard deviation of a sample proportion
use technology to determine probability
communicate mathematical set-up that was considered when determining the solution

Question 6 (SF 4 marks)

$n = 400$ ✓ $mean = p = 0.45$ ✓

Standard deviation = $\sqrt{\frac{0.45(1-0.45)}{400}} = 0.024875$ ✓✓

∴ the probability that more than 50% of people have watched more than an hour of Facebook or YouTube videos in a week:

$P(\hat{P} > 0.50) = 0.02$ ✓✓

(using normCdf (0.5, 1, 0.45, 0.024875)) ✓✓

Question 7 (SF 4 marks)

Binomial experiment ✓

$n = 8$ ✓

p represents the probability the correct answer is chosen

$p = \frac{1}{4}$ ✓

q is the probability the incorrect answer is chosen $q = \frac{3}{4}$

Determine the term in p^3 in the expansion $(p+q)^8$ ✓

P(3 correct answers) = $\binom{8}{3} p^3 q^5$ ✓✓

$= 56 \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^5$

$= 0.21$ ✓✓

or determine this value using binomPdf (8, 0.25, 3) on graphics calculator; the working must include the mathematical set-up that is considered when determining the value

7.
comprehend context used to model binomial random variable
recall and use the probabilities associated with the binomial distribution with parameters $n = 8$ and $p = \frac{1}{5}$
use procedure to determine probability for 3 successes
communicate mathematical set-up that was considered when calculating the solution

Question 8 (SF 6 marks)

a. Mean distance =

$$\int_0^{12} x.f(x)dx = \int_0^{12} \frac{x}{1152}(144 - x^2)dx$$

$$= \frac{1}{1152}\int_0^{12} 144x - x^3 dx \text{ ✓✓}$$

$$= \frac{1}{1152}[72x^2 - \frac{x^4}{4}]_0^{12}$$

$$= \frac{1}{1152}\left(72 \times 12^2 - \frac{12^4}{4}\right)$$

8a.
use rule for determining the mean of a continuous random variable
communicate mathematical set-up that was considered when calculating the solution

	$= \frac{5184}{1152} = 4.5 \text{ m}$ ✓✓ *(or determine this value using 'Calculus — Numerical integral' on graphics calculator)* b. Variance = $\int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$ ✓ $= \int_0^{12} \frac{(x-4.5)^2(144-x^2)}{1152dx}$ ✓✓ $= 8.55$ *(using graphics calculator 'Calculus — Numerical integral')* standard deviation = $\sqrt{8.55}$ = 2.924 ✓ c. $P(X \geq 2/X \leq 4) = \frac{P(X\geq 2 \cap X \leq 4)}{P(X \leq 4)}$ ✓ $= \frac{\int_2^4 \frac{1}{1152}(144-x^2)dx}{\int_0^4 \frac{1}{1152}(144-x^2)dx}$ ✓ *(using graphics calculator 'Calculus — Numerical integral')* $= \frac{0.233796}{0.481481}$ ✓ $= 0.49$ ✓	**8b.** use rule for determining the variance of a continuous random variable recall rule to determine standard deviation **8c.** use rule for conditional probability select, recall and use rules for continuous random variables given by integrals use technology to determine probability communicate mathematical set-up that was considered when calculating the solution
9a. recall and use rule to determine the sampling proportion use rule for approximate confidence interval to determine interval communicate understanding of the meaning of the interval	**Question 9 (SF 5 marks)** a. $\hat{p} = \frac{15}{120} = 0.125$ ✓ 90% confidence interval: $p \pm z\sqrt{\frac{p(1-p)}{n}}$ $= 0.125 \pm 1.65\sqrt{\frac{0.125 \times 0.875}{120}}$ ✓ $(0.0752, 0.1748)$ ✓✓ Based on a sample size of 120 and a sample estimate of 0.125, I am fairly certain (90% certain) that the interval from 0.0752 to 0.1748 contains the true population proportion p. ✓✓	

9b. use rule for margin of errors use technology (or otherwise) to determine sample size communicate mathematical set-up that was considered when determining a solution evaluate reasonableness of solution	b. $0.02 = 1.65\sqrt{\frac{0.125 \times 0.875}{n}}$ ✓ *(using nsolve)* ✓ $n = 744.434$ ✓ ∴ to achieve a margin of error of 2% a sample size of 744 is required. ✓ *Note: Answers may vary if z-score to more than 2 decimal places is used.*	
	Question 10 (CF 6 marks) $P(X < 75) = 0.067;\ P(X > 100) = 0.1587$ ✓✓ $P(z < z_1) = 0.067;\ z_1 = -1.49851$ ✓ $P(z > z_2) = 0.1587;\ z_2 = 0.999815$ ✓ $\frac{75 - \mu}{\sigma} = -1.49851$ ✓ Rearranging: $-1.49851\sigma + \mu = 75\ (i)$ ✓ $\frac{100-\mu}{\sigma} = 0.999815$ ✓ Rearranging: $0.999815\sigma + \mu = 100\ (ii)$ ✓ *(using Solve system of linear equations on graphics calculator)* ✓✓ $\sigma = 10$ ✓ $\mu = 90$ ✓ *(Note: Some students may use analytic approach to solve for the mean and standard deviation.)*	**10.** comprehend use of standardised normal variables use technology appropriately to determine z-scores to compare samples recall and use procedures to generate simultaneous equations use technology (or otherwise) to determine the mean and standard deviation communicate mathematical set-up that was considered when determining the solution to nearest whole number

11. recall that the mean of the sampling distribution is the same as the value of the population proportion use rule for standard deviation of a sample proportion use procedure to determine one standard deviation above and below the mean comprehend sampling distribution is for a small sample size mathematical reasoning provided for determining the number of 'successes' to determine the probabilities associated with the binomial distribution communicate mathematical set-up that was considered when determining the solution (using probabilities associated with the binomial distribution or technology)	**Question 11 (CU 7 marks)** $n = 20$ ✓ Mean $= p = 0.8$ ✓ Standard deviation $= \sqrt{\frac{0.8 \times (1-0.8)}{20}} = 0.08944$ ✓✓ One standard deviation above and below: $0.8 - 0.08944 = 0.71056$ ✓ and $0.8 + 0.08944 = 0.88944$ ✓ $P\left(0.71056 \leq \hat{P} \leq 0.88944\right) = P(14.2 \leq X \leq 17.788)$ ✓✓ $= P(15 \leq X \leq 17)$ since X is discrete ✓✓ $= \binom{20}{15} 0.8^{15} \times 0.2^5 + \binom{20}{16} 0.8^{16} \times 0.2^4 + \binom{20}{17} 0.8^{17} \times 0.2^3$ or ✓✓ *(using binomCdf (20, 0.8, 15, 17) on graphics calculator)* $= 0.598$ ✓✓	