Mathematical Methods 2019 v1.2

IA3: Sample assessment instrument

Examination (15%)

This sample has been compiled by the QCAA to assist and support teachers in planning and developing assessment instruments for individual school settings.

Student name

Student number

Teacher

Exam date

Marking summary

Criterion	Marks allocated	Provisional marks
Foundational knowledge and problem-solving	15	
Overall	**15**	

Conditions

Technique	Examination
Unit	Unit 4: Further functions and statistics
Topic/s	Topic 1: Further differentiation and applications 3 Topic 2: Trigonometric functions 2 Topic 3: Discrete random variables 2 Topic 4: Continuous random variables and the normal distribution Topic 5: Interval estimates for proportions
Time	2 hours + 5 minutes perusal
Seen/Unseen	Unseen questions
Other	Only the QCAA formula sheet must be provided. Notes are not permitted. Use of technology is required; schools must specify the technology used.

Instructions

- Paper 1 (technology-free): 45 minutes
- Paper 2 (technology-active): 75 minutes + 5 minutes perusal
- Show all working in the spaces provided.
- Write responses using black or blue pen.
- Unless otherwise instructed, all numerical answers should be given to 2 decimal places.
- This assessment instrument is comprised of two papers — Paper 1 is technology-free and Paper 2 is technology-active.
- Use of a non-CAS graphics calculator is required for Paper 2 (technology-active) only unless an analytic procedure is required.

Paper 1 (technology-free)

Question 1 (4 marks)

Calculate the expected value for a uniform probability density distribution $p(x) = 0.02$ defined on the interval $[20, 70]$.

Question 2 (6 marks)

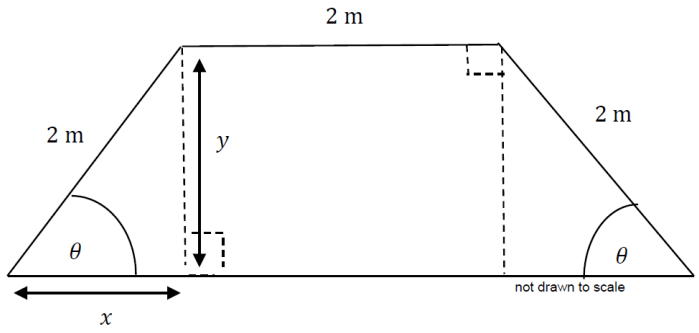
Show that there is only one point of inflection on the curve:

$h(x) = x^5 + 5x^4 + \frac{10x^3}{3} - 50x^2 + 5x + 2.$

Use mathematical reasoning to justify your response.

Question 3 (8 marks)

The diagram below shows a plan for a trapezium-shaped window.



Three sides of the window are 2 m long. The angle between the sloping sides of the window and the base is θ where $0 < \theta < \frac{\pi}{2}$.

a. Show that the area (A) of the trapezium is given by the function $A = 4\sin(\theta)\cos(\theta) + 4\sin(\theta)$.

b. Determine the value of θ that will maximise the area of the window.

Paper 2 (technology-active)

Question 4 (7 marks)

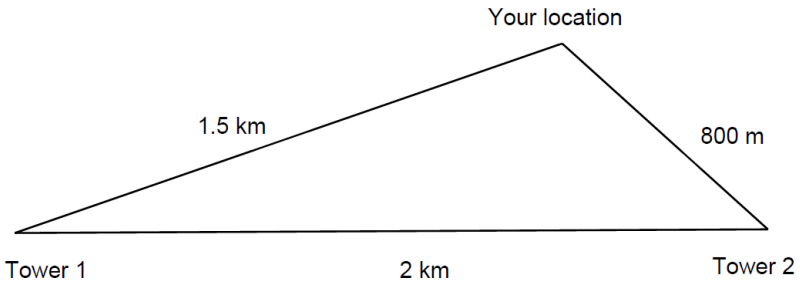
Two mobile phone towers in range of you are located 2 kilometres apart along a straight road that runs east to west. Your location is north of the road.

Based on the signal strength, you have determined that you are 1.5 kilometres from Tower 1 and 800 metres from Tower 2.

Determine:

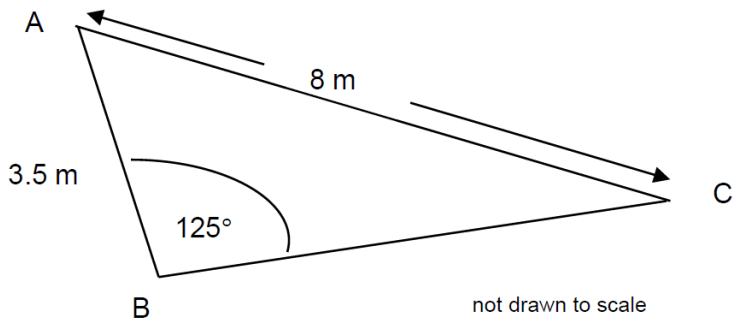
a. your direction relative to Tower 1

b. the time it would take you to get to the road if you walked due south at 5 kilometres per hour.



Question 5 (7 marks)

A homeowner is building a triangular garden bed, as shown in the diagram below.



a. Determine $\angle BAC$.

b. Determine the area of $\triangle BAC$

A layer of soil 15 centimetres deep is needed to fill the garden bed.

c. Determine the volume of soil that is needed.

Question 6 (4 marks)

Assume that 45% of people watch more than an hour of Facebook or YouTube videos a week.

Using the normal approximation, determine the approximate probability that, in a randomly selected sample of size 400, more than 50% of people have watched more than an hour of Facebook or YouTube videos in a week.

Question 7 (4 marks)

A multiple-choice test consists of 8 questions. Each question has four possible answers. Only one of the answers is correct. For each question, Tom randomly selects one of the four answers.

Determine the probability that Tom answers exactly three questions correctly.

Question 8 (6 marks)

In the shotput competition at a school athletics carnival, it is determined that the distance (x metres) that the shotput is thrown is a continuous variable X with a probability density function defined by:

$$f(x) = \begin{cases} \frac{1}{1152}(144 - x^2), & 0 \leq x \leq 12 \\ 0, & \text{otherwise} \end{cases}$$

a. Show that the mean distance thrown is 4.5 metres.

b. Determine the standard deviation for the distance thrown.

c. Determine $P(X \geq 2 | X \leq 4)$.

Question 9 (5 marks)

A random sample of recent graduates ($n = 120$) found that 15 are unemployed.

a. Calculate an approximate 90% confidence interval (to 4 decimal places) for the proportion of all graduates who fail to find a job. Interpret your result.

b. Determine the sample size required to achieve a margin of error of 2% for the approximate 90% confidence interval.

Question 10 (6 marks)

A radar unit is used to measure the speed of cars on a motorway. The speeds are normally distributed.

It is observed that 6.7% of the time cars travel less than 75 kilometres per hour, while there is a 15.87% chance that cars travel greater than 100 kilometres per hour.

Determine the mean and standard deviation for the speed of cars (round to the nearest whole number).

Question 11 (7 marks)

Suppose that 80% of all Australian teenagers use a smartphone.

If a random sample of 20 teenagers is chosen from this population, determine the probability that the sample proportion lies within one standard deviation of the population proportion.

Use mathematical reasoning to justify your response.

Examination marks summary

Paper 1 (technology-free)	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
1	4		
2			6
3	2	6	
Totals	**6**	**6**	**6**

Paper 2 (technology-active)	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
4	7		
5	7		
6	4		
7	4		
8	6		
9	5		
10		6	
11			7
Totals	**33**	**6**	**7**

Total Paper	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
Marks	39	12	13
Percentage	**60.93%**	**18.75%**	**20.31%**

Instrument-specific marking guide (IA3): Examination (15%)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4

• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.	> 13%	3
	> 7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.	> 0%	1
• does not satisfy any of the descriptors above.		0