# Mathematical Methods 2019 v1.2

Unit 1 sample marking scheme

March 2019

## Examination

This sample has been compiled by the QCAA to model one possible approach to allocating marks in an examination. It matches the examination mark allocations as specified in the syllabus (~ 60% simple familiar, ~ 20% complex familiar and ~ 20% complex unfamiliar) and ensures that all the objectives are assessed.

### Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 1 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 1 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 1 topics.

# Task

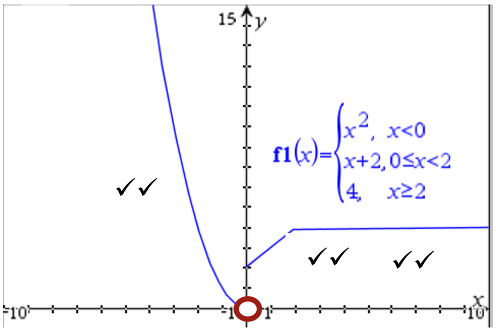
See the sample assessment instrument for Unit 1 Topics 1–5: Examination (available on the QCAA Portal).

# Sample marking scheme

The annotations are written descriptions of the expected response for each question and are related to the assessment objectives.

| | | |
|---|---|---|
| Note: ✓ = $\frac{1}{2}$ mark | **Marking scheme** | |
| **1a.**<br>comprehend (equate quadratic to zero)<br>select appropriate procedure (factorisation/ quadratic formula)<br>use appropriate procedure to determine intercepts<br>communicate using mathematical terminology and symbols<br>**1b.**<br>select appropriate procedure<br>use appropriate procedure to determine intercepts | **Paper 1 (technology-free)**<br>**Question 1 (3 marks) SF**<br>a. $x$ –intercept/s $(y = 0)$<br>$0 = 2x^2 - 4x - 6$ ✓<br>Using factorisation ✓<br>$0 = 2(x^2 - 2x - 3)$<br>$0 = 2(x - 3)(x + 1)$<br>$x = 3, -1$ ✓<br>$x$ –intercepts at $(3, 0)$ and $(-1, 0)$ ✓<br>b. $y$ – intercept $(x = 0)$<br>$y = 2 \times 0^2 - 4 \times 0 - 6$ ✓<br>$y$ – intercept $(0, -6)$ ✓ | |
| | **Question 2 (5 marks) SF**<br>a. Centre $(-2, 1)$ ✓<br>Radius 1 ✓<br>Equation of circle<br>$(x + 2)^2 + (y - 1)^2 = 1$ ✓✓<br>b. Domain $-3 \leq x \leq -1$ ✓✓<br>Range $0 \leq y \leq 2$ ✓✓<br>c. The circle shown is not a function. ✓<br>For some $x$ – values there is more than one corresponding $y$ – value. ✓ | **2a.**<br>select, recall and use:<br>• rules for locus of a circle<br>• equation of a circle<br>**2b.**<br>select, recall and use the definition for domain and range<br>**2c.**<br>select, recall and use the definition for function/relation<br>communicate using mathematical and everyday language |

## Question 3 (4 marks) SF




✓✓

**3.**

select and use the appropriate procedure to produce a sketch of parabola and two straight lines

communicate the sketch appropriately, attending to precision (graphical display, use of solid dot to 'include' and open dot to 'not include')

## Question 4 (11 marks) SF

a. $r = \frac{6}{b-1}$ or $\frac{b+4}{6}$ ✓✓

b. $\frac{6}{b-1} = \frac{b+4}{6}$ ✓✓

$36 = (b+4)(b-1)$ ✓✓

$b^2 + 3b - 40 = 0$ (given in question)

✓✓

c. Solving quadratic (using quadratic formula) ✓

$b = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times -40}}{2}$ ✓

$b = -8, 5$ ✓✓

d. Substitute $b$ into $r = \frac{6}{b-1}$. ✓✓

$When\ b = -8, r = \frac{6}{-9} = \frac{-2}{3}$ and

$When\ b = 5, r = \frac{6}{4} = \frac{3}{2}$ ✓✓

e. $r = \frac{-2}{3}$ will result in a finite sum as

$|r| < 1$ ✓✓

$S_\infty = \frac{a}{1-r} = \frac{-8-1}{(1-\frac{-2}{3})}$ ✓✓

$= \frac{-9}{\frac{5}{3}} = \frac{-27}{5} = -5.4$ ✓✓

**4a.**

select and use rule for common ratio in a geometric progression

**4b.**

select and use rule for common ratio in a geometric progression

comprehend how to generate an equation

select and use rules for rearranging an equation

clearly communicate how the equation was generated

**4c.**

select and use the appropriate procedure

determine the zeros of the equation

**4d.**

use substitution to determine $r$

**4e.**

justify the decision

select and use the rule for $S_\infty$ to determine the finite sum

## Question 5 (4 marks) SF

Using Pascal's triangle the coefficient of the term is 10 ✓✓

$(2a)$ is to the power of 2 ✓✓

∴ $b$ is to the power of 3 ✓

fourth term $= 10 \times (2a)^2 \times b^3$ ✓✓

fourth term $= 40a^2b^3$ ✓

(*or expansion may be used and fourth term identified*)

**5.**

recall and use Pascal's triangle to determine coefficient

recall binomial expansion rule to determine power for $a$ and $b$

use index laws

communicate term

## Question 6 (2, 4 marks) SF, CF

**a.**

select and use rule for mutually exclusive events

use substitution procedure and algebraic skills to determine $P(B)$

a.

$$P(A) + P(B) = P(A \cup B) ✓✓$$
$$P(B) = 0.6 - 0.3 ✓$$
$$= 0.3 ✓$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) ✓✓$$

b. Given that $A$ and $B$ are independent:

$$P(A \cup B) = P(A) + P(B) - P(A) \times P(B) ✓✓$$
$$0.6 = 0.3 + P(B) - 0.3P(B) ✓$$
$$0.3 = 0.7P(B) ✓$$
$$P(B) = \frac{3}{7} ✓✓$$

**6b.**

recall and use rule for a combined event

select and use rule for independent events

use substitution procedure

use procedure for adding like terms

use algebraic skills to determine $P(B)$

## Question 7 (4 marks) SF

**7a.**

recall and use rules for expansion

a. $(x+4)(2x-3)(x+6)$
$= (2x^2 + 5x - 12)(x+6)$ ✓✓
$= 2x^3 + 17x^2 + 18x - 72$ ✓✓

**7b.**

recall and use rules for determining $x$-intercepts given function in factorised form

recall and use rule for determining $y$-intercept given expanded version of a function

b. $x -$ intercepts $(-4, 0)$ $\left(\frac{3}{2}, 0\right)$ $(-6, 0)$ ✓✓✓

$y -$ intercept $(0, -72)$ ✓

## Question 8 (7 marks) CF

Using substitution

$x = (x-2)^2$ ✓

$x = x^2 - 4x + 4$ ✓

$0 = x^2 - 5x + 4$ ✓

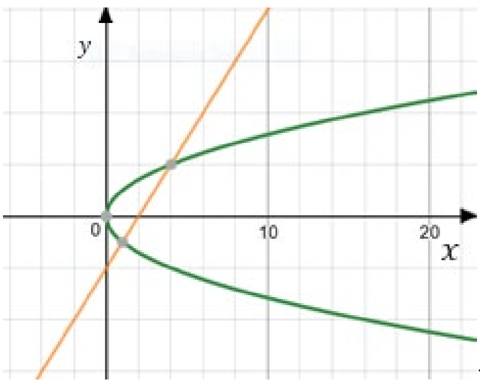
Factorising

$0 = (x-4)(x-1)$ ✓

$x = 4, 1$ ✓

Substituting into $y = x - 2$

$y = 2, -1$ ✓




✓✓✓✓ (sketches)

Points of intersection (4, 2) ✓ (1, –1) ✓

The points of intersection between the parabola and the straight line correspond with the simultaneous solution. ✓✓

**8.**

comprehend solution to two equations to two unknowns is required

recall and use:

- procedure for solving simultaneously
- rules for expanding and rearranging

comprehend solution to a quadratic is required

recall and use rules for solving a quadratic equation to determine solutions

recall shapes of graphs and sketch

comprehend points of intersection indicate simultaneous solution

evaluate the reasonableness of results

## Question 9 (5 marks) CU

LHS of equation:

$(x-2)^3+1$

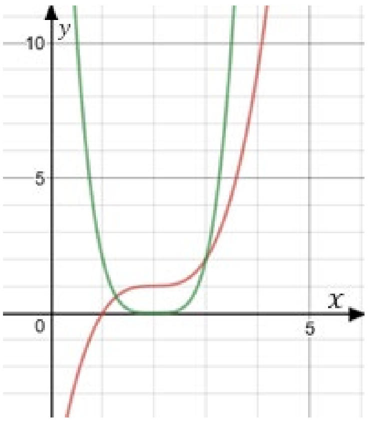
Function $y=x^3+3$ has been transformed 2 units to the right and 2 units down vertically ✓✓

RHS of equation:

$2(x-2)^4$

$y=2(x-2)^4$ has not been transformed. ✓

No change to sketch of quartic function.




✓✓

The points of intersection represent the solution to the given equation. ✓

Solutions are approximately $x=1.3$ and $x=3$ (accept approximations that are reasonable). ✓✓✓✓

**9.**

justify procedures and decisions by:

- communicating techniques that must be used to develop a solution (transformation of curves)
- recalling shape of curves (identify cubic and/or quartic).

use rules for transforming curve to produce a graphical display

recall procedure for visually solving for a solution to an equation (point of intersection)

use procedure to determine the solution to the given equation

## Question 10 (5 marks) CU

Expanding RHS

$p(x)=x^3-3x^2+20$✓✓

Using trial and error:

$x=-2$ is a root of the function ∴ $(x+2)$ is a factor ✓✓

$p(x)=(x+2)(x^2-5x+10)$ ✓✓

$\boldsymbol{x^2-5x+10}$ has no real roots $(\boldsymbol{b^2-4ac<0})$ ✓✓

∴ the only root for this function is $x=-2$ and therefore the functions cuts the $x$-axis at only one point. ✓✓

**10.**

justify procedures and decisions by identifying cubic form of the function to then use factor theorem

select, recall and use:

- factor theorem
- the role of the discriminant

communicate findings

# Paper 2 (technology-active)

## Question 1 (7 marks) SF

**1a.**

select and use rule

**1b.**

select and use rule

**1c.**

select and use rule

justify decision

communicate information logically

a. $P(F) = \frac{7}{30}$ ✓✓ (accept $\frac{35}{150}$)

b. $P(Year\ 11\ French\ student) = \frac{15}{150} = \frac{1}{10}$ ✓✓

c. If the events are independent then

$P(F \cap Y) = P(F) \times P(Y)$✓✓

$LHS = P(F \cap Y) = \frac{1}{10}$ ✓✓ (from b. above)

$RHS = P(F) \times P(Y) = \frac{35}{150} \times \frac{70}{150} \approx 0.109$ ✓✓

$LHS \neq RHS$ $\therefore$ $F$ and $Y$ are not independent ✓✓

✓✓

## Question 2 (3 marks) SF

**2.**

understand relevant technique to use

communicate appropriately (everyday language or notation)

substitute into formula to determine solution

Determine $P(Student\ takes\ Geography\ |Student\ takes\ MM)$ ✓

$= \frac{P(Geog \cap MM)}{P(MM)}$✓✓

$= \frac{0.095}{0.64}$✓✓

$= 0.1484$✓

## Question 3 (7 marks) SF

**3a.**

select, recall and use rule

**3b.**

select, recall and use rule

**3c.**

understand critical element (to set up the inequality and solve for $n$

use GDC to determine $n$ (graphically, using algebraic function, trial and error)

comprehend findings to determine $n$

communicate findings logically

a. $r = \frac{2.24}{0.56} = 4$✓✓

b. $t_{10} = 0.56 \times 4^{(10-1)}$
$= 146800.64$✓✓

c. solve $0.56 \times \frac{4^n - 1}{(4-1)} > 195000$✓✓

$\text{solve}\left(0.56 \cdot \frac{4^n - 1}{4-1} = 195000, n\right)$ $\quad n = 9.99729$

✓✓✓✓

When $n$ = 9.99, $S_n = 195000$
$\therefore$ the least value of $n$ is 10✓✓
✓✓

## Question 4 (3 marks) SF

**4.**

select and use rule

determine value of investment (may use technology function on GDC)

Using the annuity formula ✓✓

$= \$1000 \times \frac{(1.05^5 - 1)}{0.05}$ ✓✓

$= \$1000 \times 5.52563$

$= \$5525.63$ ✓✓

## Question 5 (7 marks) SF

a. $t_3 = 11 = t_1 + 2d\ (i)$ ✓✓

$t_5 = 19 = t_1 + 4d\ (ii)$ ✓✓

$(ii) - (i)$

$8 = 2d$

$d = 4$ ✓✓

b. Substitute $d = 4$ into $(i)$

$t_1 = 3$ ✓✓

c. $S_{20} = \frac{20}{2}(2 \times 3 + 19 \times 4)$✓✓

$S_{20} = 820$✓✓

✓✓

**5a.**

comprehend information to generate equations

select and use rule/procedures (may use technology to solve simultaneously)

**5b.**

use substitution procedure

**5c.**

select and use rule to determine $S_{20}$

communicate clearly (including correct use of notation)

## Question 6 (5 marks) CF

Equation of curve is:

✓✓ ✓✓ ✓✓

$y = -4(x+1)^4 + p$

Given (0, 2) lies on the curve

Substitute point (0, 2)✓

$2 = -4(0+1)^4 + p$✓

$p = 6$✓

∴ equation of curve is $y = -4(x+1)^4 + 6$✓

**6.**

select, recall and use rules:

- vertical stretch
- reflection in $x$-axis
- vertical and horizontal translation

justify decisions by explaining mathematical reasoning

use procedure to determine $p$

communicate equation

## Question 7 (4 marks) CF

Given $x = -2$ is a vertical asymptote

Vertical asymptote $x + a = 0 \rightarrow x = -a$ so $a = 2$

$f(x) = p + \frac{5}{x+2}$ ✓✓

Given $y -$ intercept at $(0, 5)$

$5 = p + \frac{5}{0+2}$ ✓✓

$p = \frac{5}{2}$ ✓✓

Horizontal asymptote at $y = \frac{5}{2}$ (students may use technology to determine this)✓✓

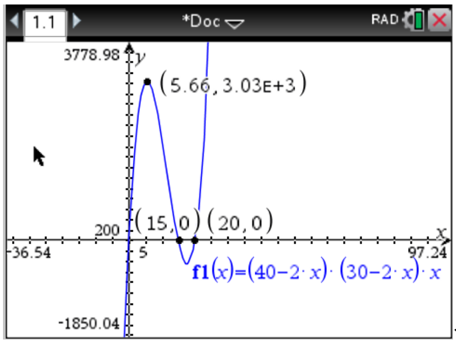
**7.**

recall and use rule for asymptote

use given point to generate an equation

recall and use rules to determine $p$

identify horizontal asymptote

| | |
|---|---|
| | ## Question 8 (7 marks) CU |
| **8.**<br>comprehend information given in template to generate volume function<br>use rule for volume of a rectangular prism | Using the given information the volume of the cake dish is:<br>$V(x) = (40-2x)(30-2x)x$, where $x$ is the height of the tin. ✓✓ |
| justify procedures and decisions:<br>• decide on method of solution<br>• communicate graph | Use technology to visualise relationship (graphing) or produce table of values✓✓<br>✓✓ |
| recall facts/definitions to identify intervals, maximum, domain | a. The function is increasing up to $x = 5.66$ and decreasing to $x = 15$ ✓✓<br>b. Maximum volume of 3030 $\text{cm}^3$ when $x = 5.66$ cm ✓✓<br>c. Domain $0 < x < 15$ ✓✓ |
| justify procedures and decisions | $x$ values outside this interval produce negative dimension. ✓✓ |
| | ## Question 9 (5 marks) CU |
| **9.**<br>select and use the rule for binomial expansion | General term is:<br>$\binom{8}{r}\left(\frac{x^3}{2}\right)^{8-r}\left(\frac{a}{x}\right)^r$ (1) ✓ |
| recall and use index laws | $= \binom{8}{r}\frac{x^{24-3r}}{2^{8-r}} \times \frac{a^r}{x^r}$✓✓ |
| explain mathematical reasoning ($x^0$ is the term independent of $x$) | constant is the term independent of $x$ (index is zero)✓✓ |
| recall and use index laws | term in $x \to \frac{x^{24-3r}}{x^r} = x^{24-4r}$✓<br>$\therefore 24 - 4r = 0$ |
| use algebraic skills | $r = 6$✓ substitute into (1) |
| comprehend how to use the solution and substitute to create equation | $\binom{8}{6}\left(\frac{x^3}{2}\right)^{8-6}\left(\frac{a}{x}\right)^6 \to \frac{28a^6}{4} = 5103$✓<br>$a^6 = 729$ |
| recall procedure to solve for unknown (may use GDC) | $a = \pm 3$✓✓ |