# Mathematical Methods 2019 v1.2

Unit 1 Topic 2 sample assessment instrument

September 2018

## Problem-solving and modelling task

This sample has been compiled by the QCAA to assist and support teachers in planning and developing assessment instruments for individual school settings.

Schools develop internal assessments for each senior subject, based on the learning described in Units 1 and 2 of the subject syllabus. Each unit objective must be assessed at least once.

### Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 1 Topic 2
2. comprehend mathematical concepts and techniques drawn from Unit 1 Topic 2
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 1 Topic 2.

| | |
|---|---|
| **Subject** | Mathematical Methods |
| **Technique** | Problem-solving and modelling task |
| **Unit** | 1: Algebra, statistics and functions |
| **Topic** | 2: Functions and graphs |

| **Conditions** | | | |
|---|---|---|---|
| **Duration** | 4 weeks, including 3 hours of class time | | |
| **Mode** | Written report | **Length** | - up to 10 pages<br>- maximum 2000 words, excluding appendixes |
| **Individual/ group** | Individual | **Other** | — |
| **Resources available** | The use of technology is required, e.g.<br>• graphics calculator<br>• spreadsheet program<br>• other mathematical software. | | |

**Context**

Computer-generated images in video games and film and TV special effects are created using a number of mathematical concepts and techniques, including basic arithmetic, geometry, trigonometry, linear algebra and calculus.

A computer animation studio is interested in developing its own 'physics engine'. They have asked you to work on one aspect of the engine — how free-hanging objects act under the influence of gravity. It has been suggested that as a proof of concept, a function can be used to model the shape of a hanging chain.

**Task**

Consider a flexible chain of length, $l$, that hangs freely. The ends of the chain are secured at two fixed points that are at the same height and are distance, $d$, apart.

Develop a function that models the shape of the hanging chain, and then produce a report that explains how you developed and refined your model.

You must consider:

- a polynomial function
- the sum of a number of even-degree polynomials described as:
  $y = a_0 + b_2(x - a_2)^2 + b_4(x - a_4)^4 + \cdots + b_{2n}(x - a_{2n})^{2n} + \cdots$.

**To complete this task, you must:**

- use the problem-solving and mathematical modelling approach to develop your response
- respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams
- provide a response that highlights the real-life application of mathematics
- respond using a written report format that can be read and interpreted independently of the instrument task sheet
- develop a unique response
- use technology.

### Stimulus

Further reading

- *The Verge*, ‘Pixar’s senior scientist explains how maths makes the moves and games we love, www.theverge.com/2013/3/7/4074956/pixar-senior-scientist-derose-explains-how-math-makes-movies-games
- *TED-Ed*, ‘Pixar: The math behind the movies — Tony DeRose’, https://youtu.be/_IZMVMf4NQ0
- *+plus magazine*, ‘Maths goes to the movies’, https://plus.maths.org/content/maths-goes-movies

### Checkpoints

☐ One week after issue date: Students email evidence of their progress to their teacher.

☐ Two weeks after issue date: Students email a draft for feedback. General feedback on drafts is provided to the class, but no individual corrections are made.

☐ Three weeks after issue date: Students email evidence of their progress to their teacher.

☐ Four weeks after issue date: Students submit final response.

### Feedback

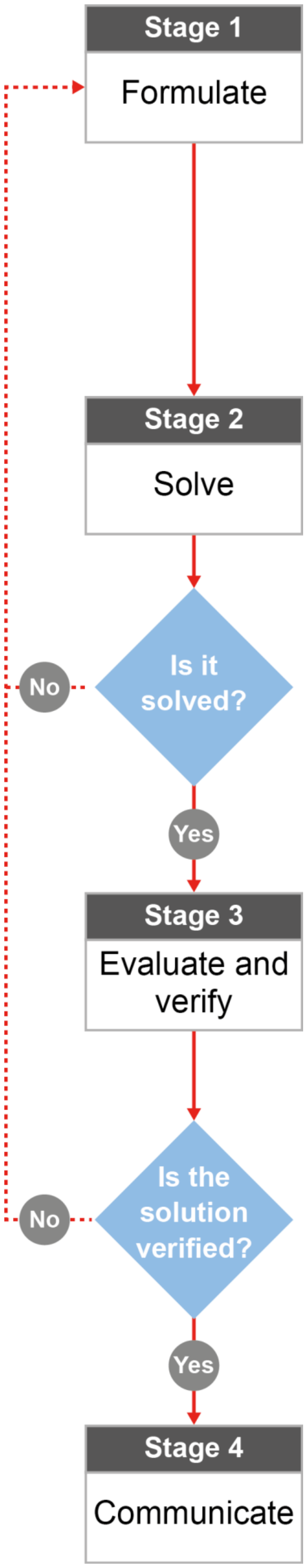
### Authentication strategies

- The teacher will provide class time for task completion.
- Students will each produce a unique response by using different lengths of chain and/or by securing the chain at fixed points that are varying distances apart.
- Students will provide documentation of their progress at indicated checkpoints.
- Students must submit a declaration of authenticity.
- The teacher will ensure class cross-marking occurs.

### Scaffolding

The approach to problem-solving and modelling must be used (see next page).

## Approach to problem-solving and modelling




Once you understand what the problem is asking, you must design a plan to solve the problem. Translate the problem into a mathematically purposeful representation by first determining the applicable mathematical and/or statistical principles, concepts, techniques and technology that are required to make progress with the problem, then list the mathematical techniques and procedures you will use to develop the response. Consider how you will determine the data values that lie on the chain and what methods you will use to generate the model. Appropriate assumptions, variables and observations must be identified and documented, based on the logic of a proposed solution and/or model.

In mathematical modelling, formulating a model involves the process of mathematisation – moving from the real world to the mathematical world.

Select and apply mathematical and/or statistical procedures, concepts and techniques previously learnt to solve the mathematical problem to be addressed through your model. Possible approaches are wide-ranging and include synthesising and refining the polynomial model, and generating and testing the feasibility of the sum of even-powered terms polynomial, as well as using standard mathematical techniques.

Solutions can be found using algebraic, graphic and technological methods.

Once a possible solution has been achieved, you need to consider the reasonableness of the solution and/or the utility of the model in terms of the problem. Evaluate your results and make a judgment about the solution/s to the problem in relation to the original issue, statement or question.

This involves exploring the strengths and limitations of your solution and/or model. Where necessary, this will require going back through the process to further refine your solution and/or model. In mathematical modelling, you must check that the output of your model provides a valid solution to the real-world problem it has been designed to address.

Use both a residual analysis and the correlation coefficient to interpret the results of the mathematics compared with the original task.

The development of solutions and models to abstract and real-world problems must be capable of being evaluated and used by others and so need to be communicated clearly and fully. Communicate your findings systematically and concisely using mathematical, statistical and everyday language. Draw conclusions, discussing the key results and the strengths and limitations of the solution and/or model. You could offer further explanation, justification, and/or recommendations, framed in the context of the initial problem.