Mathematical Methods 2019 v1.2

Unit 1 sample assessment instrument

September 2019

Examination

This sample has been compiled by the QCAA to assist and support teachers in planning and developing assessment instruments for individual school settings.

Schools develop internal assessments for each senior subject, based on the learning described in Units 1 and 2 of the subject syllabus. Each unit objective must be assessed at least once.

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 1 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 1 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 1 topics.

Subject	Mathematical Methods
Technique	Examination
Unit	1: Algebra, statistics and functions
Topic	1: Arithmetic and geometric sequences and series 1 2: Functions and graphs 3: Counting and probability 4: Exponential functions 1 5: Arithmetic and geometric sequences and series 2

Conditions			
Response type	Short response		
Time	Paper 1: 60 minutes Paper 2: 60 minutes	**Perusal**	5 minutes (Paper 2)
Other	• QCAA formula sheet must be provided • Notes are not permitted • Approved non-CAS graphics calculator		

Instructions

- Show all working in the spaces provided.
- Write responses using black or blue pen.
- Unless otherwise instructed, give answers to two decimal places.
- Use of a non-CAS graphics calculator is permitted in Paper 2 (technology-active) only.

Criterion	Marks allocated	Result
Foundational knowledge and problem-solving Assessment objectives 1,2,3,4,5 and 6		
Total		

Paper 1 (technology-free) — total marks: 54

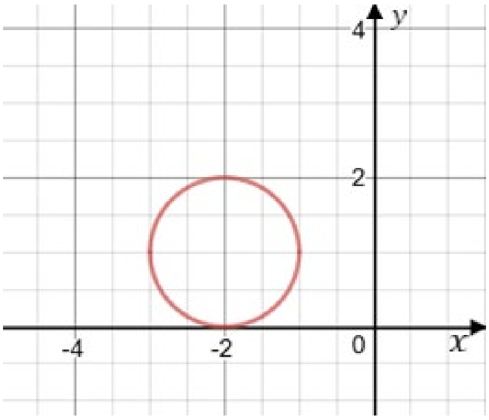
Question 1 (3 marks)

For the function $y = 2x^2 - 4x - 6$, determine:

a. the x-intercept/s
b. the y-intercept.

Question 2 (5 marks)

Consider the following graph:



a. State the equation of the circle.
b. State the domain and range of the circle.
c. Is the circle shown a function? Justify your decision.

Paper 1 (technology-free) — total marks: 54

Question 3 (4 marks)

Sketch a graph of the following piece-wise function:

$$y = \begin{cases} x^2, & x < 0 \\ x + 2, & 0 \leq x < 2 \\ 4. & x \geq 2 \end{cases}$$

Paper 1 (technology-free) — total marks: 54

Question 4 (11 marks)

The first three terms of an infinite geometric sequence are $b-1, 6, \ b+4,$ where $b \in \mathbb{Z}$.

a. State an expression for the common ratio r.
b. Show that b satisfies the equation $b^2+3b-40=0$.
c. Determine the possible values of b.
d. Determine the possible values of r.
e. State which value of r leads to a finite sum, justifying your decision. Calculate the sum of this sequence.

Paper 1 (technology-free) — total marks: 54

Question 5 (4 marks)

Determine the fourth term of the expansion $(2a + b)^5$.

Question 6 (6 marks)

Two events A and B are such that $P(A) = 0.3$ and $P(A \cup B) = 0.6$.

a. Given that A and B are mutually exclusive, determine $P(B)$.
b. Given that A and B are independent, determine $P(B)$.

Question 7 (4 marks)

a. Expand and simplify the following factors:
   $(\mathrm{x} + 4)(2\mathrm{x} - 3)(\mathrm{x} + 6)$.
b. State the x and y intercepts for the function $y = (x + 4)(2x - 3)(x + 6)$.

Paper 1 (technology-free) — total marks: 54

Question 8 (7 marks)

Solve the following simultaneous equations:

$y^2 = x$

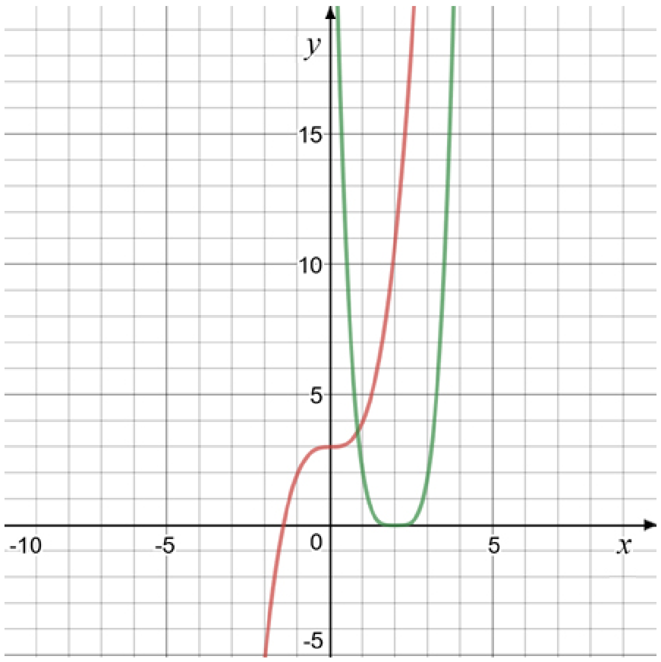
$y = x - 2$

Use a graphical display to evaluate the reasonableness of the solutions.

Paper 1 (technology-free) — total marks: 54

Question 9 (5 marks)

The functions $y = x^3 + 3$ and $y = 2(x-2)^4$ are graphed below.



Using this information, determine an approximate solution (or solutions) to the following equation:

$$(x-2)^3 + 1 = 2(x-2)^4, 0 \leq x \leq 4$$

Use mathematical reasoning to justify your response.

Paper 1 (technology-free) — total marks: 54

Question 10 (5 marks)

Consider the polynomial $p(x) = x^2(x - 3) + 20$.
Show that the function cuts the x-axis at only one point.
Use mathematical reasoning to justify your response.

Paper 2 (technology-active) — total marks: 48

Question 1 (7 marks)

The table below shows subjects studied by 150 students at a school.

	Year 10	Year 11	Total
French	20	15	35
English	40	35	75
Visual Art	20	20	40
Totals	80	70	150

A student from the school is selected at random.

Let F be the event the student studies French.

Let Y be the event the student is in Year 11.

a. Determine $P(F)$.

b. Determine the probability that the student is a Year 11 French student.

c. Are the events F and Y independent? Justify your decision.

Question 2 (3 marks)

At a particular school, the probability that a student takes both Mathematical Methods and Geography is 0.095. The probability that a student takes Mathematical Methods is 0.64.

Determine the probability that a student takes Geography given that the student is taking Mathematical Methods.

Paper 2 (technology-active) — total marks: 48

Question 3 (7 marks)

The first three terms of a geometric sequence are $t_1 = 0.56$, $t_2 = 2.24$, and $t_3 = 8.96$.
Determine:

a. the value of r
b. the value of t_{10}
c. the least value of n such that $S_n > 195000$.

Question 4 (3 marks)

A person invests $1000 at the end of each year for five years. All the money invested earns 5% p.a. compound interest, payable yearly.
Calculate the total value of the investment after the 5 years.

Question 5 (7 marks)

In an arithmetic sequence, the third term is 11 and the fifth term is 19.
Determine:

a. the common difference
b. the first term
c. the sum of the first 20 terms of the sequence.

Paper 2 (technology-active) — total marks: 48

Question 6 (5 marks)

The graph of $y = x^4$ was stretched vertically by a factor of 4, reflected across the x-axis, and translated 1 unit to the left and p units vertically upward. If the transformed graph cuts the y-axis at (0, 2), determine the equation of the curve.

Explain all mathematical reasoning.

Question 7 (4 marks)

Let $f(x) = p + \frac{5}{x+a}$, for $x \neq -a$. The line $x = -2$ is a vertical asymptote to the graph of f.

The graph of f has a y-intercept at (0, 5).

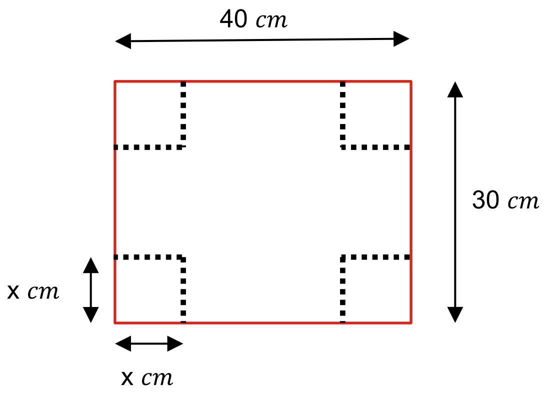
Determine the equation of the horizontal asymptote of the graph of f.

Paper 2 (technology-active) — total marks: 48

Question 8 (7 marks)

A 40 cm x 30 cm sheet of metal is used to make a cake tin.

Squares (side length of x cm) are cut from its corners and the metal is then folded upwards. Edges are fixed together to form the open rectangular tin (see template diagram below).



Explain:

a. how the volume changes as x changes
b. the value/s of x that produce the cake tin with maximum volume
c. the relevant domain for the model.

Use mathematical reasoning to justify your response.

Paper 2 (technology-active) — total marks: 48

Question 9 (5 marks)

Consider the expansion of $\left(\frac{x^3}{2}+\frac{a}{x}\right)^8$. The constant term is 5103.

Determine the possible value/s of a.

Justify procedures and decisions by explaining mathematical reasoning.

Student results summary

Paper 1 (technology-free)

Question number	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
1	3		
2	5		
3	4		
4	11		
5	4		
6a	2		
6b		4	
7	4		
8		7	
9			5
10			5
Total	**33**	**11**	**10**

Paper 2 (technology-active)

Question number	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
1	7		
2	3		
3	7		
4	3		
5	7		
6		5	
7		4	
8			7
9			5
Total	**27**	**9**	**12**