Mathematical Methods 2019 v1.2

Units 1 and 2 sample assessment instrument

September 2018

Examination — short response

This sample has been compiled by the QCAA to assist and support teachers in planning and developing assessment instruments for individual school settings.

Schools develop internal assessments for each senior subject, based on the learning described in Units 1 and 2 of the subject syllabus. Each unit objective must be assessed at least once.

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 1 and 2
2. comprehend mathematical concepts and techniques drawn from Units 1 and 2
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 1 and 2.

Queensland Government
Queensland Curriculum & Assessment Authority
181295

Subject	Mathematical Methods
Technique	Examination
Unit	1: Algebra, statistics and functions 2: Calculus and further functions
Topic	Representatively sample subject matter from topics in Units 1 and 2

Conditions			
Response type	Short response		
Time	Paper 1 (technology-free): 50 minutes Paper 2 (technology-active): 70 minutes	**Perusal**	5 minutes (Paper 2)
Other	• QCAA formula sheet must be provided • Notes are not permitted • Approved non-CAS graphics calculator		

Instructions

- Show all working in the spaces provided.
- Write responses using black or blue pen.
- Unless otherwise instructed, give answers to **two decimal places**.
- Use of a non-CAS graphics calculator is permitted in Paper 2 (technology-active) only.
- A scientific calculator may also be used in Paper 2 (technology-active).

Feedback

Paper 1 (technology-free) — total marks: 55

Question 1 (4 marks)

Consider the function $f(x) = 3\sin(2x)$.

a. State the amplitude of $f(x)$.

b. Determine the period of $f(x)$.

c. Sketch the graph of $f(x)$, for $0 \le x \le 2\pi$.

Question 2 (4 marks)

Solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$.

Paper 1 (technology-free) — total marks: 55

Question 3 (11 marks)

Determine the derivatives of the following functions. Do not simplify.

a. $2x^4 - 3x^2 + \frac{5}{x} + 6$

b. $(x^3 + x)^5$

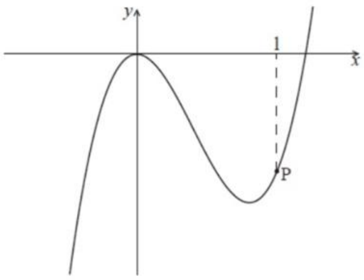
c. $(2x^2 + 3)(\sqrt{x - 1})$

d. $\frac{1-x}{x^3-3}$

Paper 1 (technology-free) — total marks: 55

Question 4 (4 marks)

Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.



The point P lies on the graph of $f(x)$. At point P, $x = 1$.
The graph of $f(x)$ has a gradient of 3 at point P. Determine the value of a.

Question 5 (5 marks)

The following table shows the probability distribution of a discrete random variable X.

x	0	2	5	9
$P(X = x)$	0.3	a	$2a$	0.1

a. Determine the value of a.
b. Determine $E(X)$.

Paper 1 (technology-free) — total marks: 55

Question 6 (8 marks)

Determine solutions for the following equations in the interval $0 \leq x \leq 2\pi$.

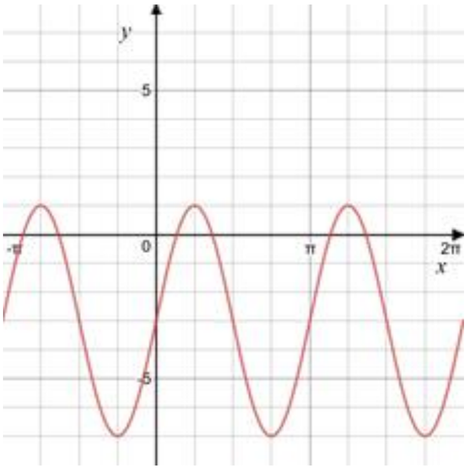
a. $2\sin(x) - 1 = 0$

b. $2\cos^2(x) - \cos(x) = 0$

Paper 1 (technology-free) — total marks: 55

Question 7 (6 marks)

Determine the function for the graph below.



Use mathematical reasoning to justify your response.

Paper 1 (technology-free) — total marks: 55

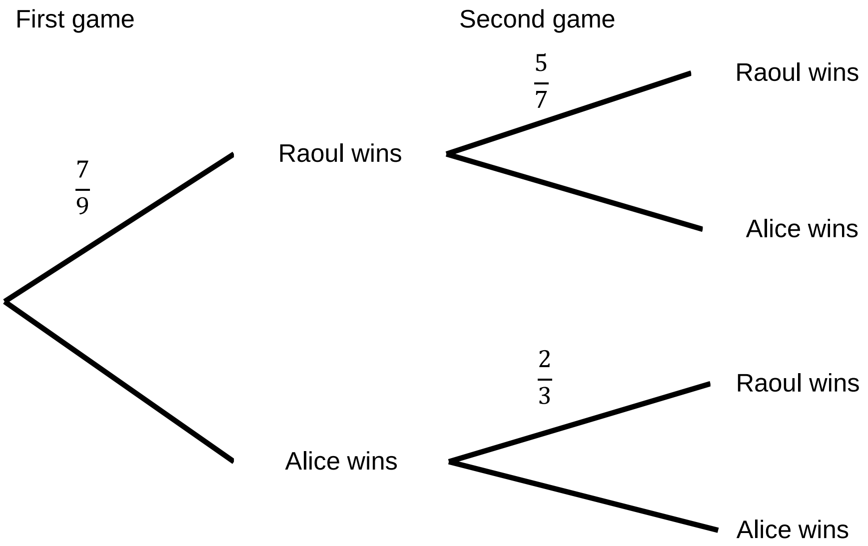
Question 8 (7 marks)

Raoul and Alice play two games of tennis. The probability that Raoul wins the first game is $\frac{7}{9}$.

If Raoul wins the first game, the probability that he wins the second game is $\frac{5}{7}$.

If Raoul loses the first game, the probability that he wins the second game is $\frac{2}{3}$.

a. Complete the following tree diagram.



b. Calculate the probability that Raoul wins the first game and Alice wins the second game.

c. Given that Raoul wins at least one game, calculate the probability that he wins both games.

Paper 1 (technology-free) — total marks: 55

Question 9 (6 marks)

A curve has the function $y = x^4 + 8x^2 - 20x + 8$.

Show that the only stationary point for this function is a minimum.

Use mathematical reasoning to justify your response.

Paper 2 (technology-active) — total marks: 42

Question 1 (2 marks)

Let C and D be independent events, with $P(C) = 2k$ and $P(D) = 3k^2$, where $0 < k < 0.5$.
Calculate k if $P(C \cap D) = 0.162$.

Question 2 (9 marks)

The running cost of an electric bike is $(2 + .001v^3)$ dollars per hour when the speed is v kilometres per hour.

a. Show that the cost C of a trip of k kilometres is given by $C = 2kv^{-1} + .001kv^2$.
b. Verify that the speed that minimises the running cost is independent of k.

Use mathematical reasoning to justify your response.

Paper 2 (technology-active) — total marks: 42

Question 3 (4 marks)

Consider the following income statistics for businesses in a particular suburb.

Year's earnings	Percentage
$40 000 loss	10%
$0	35%
$40 000 profit	45%
$200 000 profit	10%

Determine the expected value and the standard deviation.

Paper 2 (technology-active) — total marks: 42

Question 4 (5 marks)

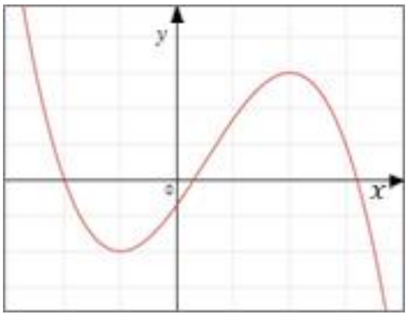
Consider the infinite geometric sequence 2500, −625, 156.25, −39.0626 …
Determine:

a. the common ratio
b. the tenth term
c. the sum of the infinite sequence.

Paper 2 (technology-active) — total marks: 42

Question 5 (6 marks)

The following diagram shows part of the graph of $f(x) = -\frac{10}{27}x^3 + \frac{5}{9}x^2 + \frac{20}{9}x - \frac{19}{27}$.



The graph of f is translated to the graph of g by a movement of k units vertically.

Determine all the values of k so that $g(x) = 0$ has exactly **one** solution.

Paper 2 (technology-active) — total marks: 42

Question 6 (9 marks)

The function $f(x)$ has a stationary point at $(1, -2)$.
Given that $f(x) = (x + p)(x^2 + q)$, where p and q are constant, determine the values of p and q.
Evaluate the reasonableness of the solution and include a written record of the method that was used.

Paper 2 (technology-active) — total marks: 42

Question 7 (7 marks)

The outside temperature during a particular day can be modelled as a periodic function.

The temperature is 10°C at midnight, and the high and low temperature during the day are 14°C and 6°C, respectively.

Given that t is the number of hours since midnight, determine the probability that the temperature will be 8°C or lower on this day.

Explain all mathematical reasoning and include the model used to determine your solution.

Student results summary

Paper 1 (technology-free)	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
1	4		
2	4		
3	11		
4		4	
5	5		
6	8		
7	6		
8	7		
9			6
Totals	**45**	**4**	**6**

Paper 2 (technology-active)	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
1	2		
2	3		6
3	4		
4	5		
5		6	
6		9	
7			7
Totals	**14**	**15**	**13**