Mathematical Methods 2019 v1.2

IA2: Sample assessment instrument

Examination (15%)

This sample has been compiled by the QCAA to assist and support teachers in planning and developing assessment instruments for individual school settings.

Student name

Student number

Teacher

Exam date

Marking summary

Criterion	Marks allocated	Provisional marks
Foundational knowledge and problem-solving	15	
Overall	**15**	

Conditions

Technique	Examination
Unit	Unit 3: Further calculus
Topic/s	Topic 1: The logarithmic function 2 Topic 2: Further differentiation and applications 2 Topic 3: Integrals
Time	2 hours + 5 minutes perusal
Seen/Unseen	Unseen questions
Other	Only the QCAA formula sheet must be provided Notes are not permitted Use of technology is required; schools must specify the technology used

Instructions

- Show all working in the space provided.
- Use a black or blue pen.
- Use of a non-CAS graphics calculator is permitted in the technology-active paper only.

Paper 1 (technology-free)

Question 1 (7 marks)

Solve each equation below:

a. $x = 2\log_6(3) + \log_6(4) - \log_6(1)$

b. $\left(e^x - 2\right)\left(e^x - 3\right) = 0$

Question 2 (13 marks)

Determine the derivative of the following functions:

a. $f\left(x\right)=e^{x}+\sin\left(2x\right)$

b. $f\left(x\right)=e^{\sin(x)}$

c. $f\left(x\right)=\cos^{3}\left(x\right)$

d. $f\left(x\right)=x+x\ln\left(x\right)$ (give solution in simplest form)

Question 3 (7 marks)

Determine the exact value for each of the following definite integrals (give solutions in simplest form):

e. $\int_1^3 4x^2\,dx$

f. $\int_0^2 6e^{2t} + t\,dt$

Question 4 (6 marks)

A particle moves along the x-axis with position at time t given by $x(t) = e^t \sin(t)$ for $0 \leq t \leq 2\pi$.

The rate of change of position with respect to time is called velocity. Determine all possible values of t when the particle is at rest.

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Question 5 (6 marks)

Determine the coordinates of the stationary point of the function $f(x) = \dfrac{\ln(2x)}{x}, x > 0$.

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Question 6 (5 marks)

The spread of a flu in a certain school is modelled by the equation $P(t) = \frac{100}{1+e^{b-t}}, t \geq 0$ where $P(t)$ is the total number of students infected after t days.

Given that the number of students infected after day 3 is 50, determine the rate the flu is spreading at this time.

Paper 2 (technology-active)

Question 7 (9 marks)

Use $f(x) = \ln(3x - 2)$ and $g(x) = -4\cos(0.5x) + 2$.

a. Sketch $f(x)$ clearly labelling any intercepts and asymptotes.

b. Determine $f'(x)$.

c. Determine $g'(x)$.

d. Determine when $f(x)$ and $g(x)$ have the same gradients over the domain $1 \leq x \leq 8$.

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Question 8 (8 marks)

The number of rabbits increases according to the model $n(t) = Ae^{0.55t}, t \geq 0$, where t is time in years, $n(t)$ is the population size at time t, and A is the initial size of the population.

Rabbits were introduced to a small island eight years ago. The current rabbit population on the island is estimated to be 4200.

a. What was the initial size of the rabbit population?

b. Estimate the population 12 years after they were introduced.

c. Determine when the population is increasing at a rate of 250 000 rabbits per year.

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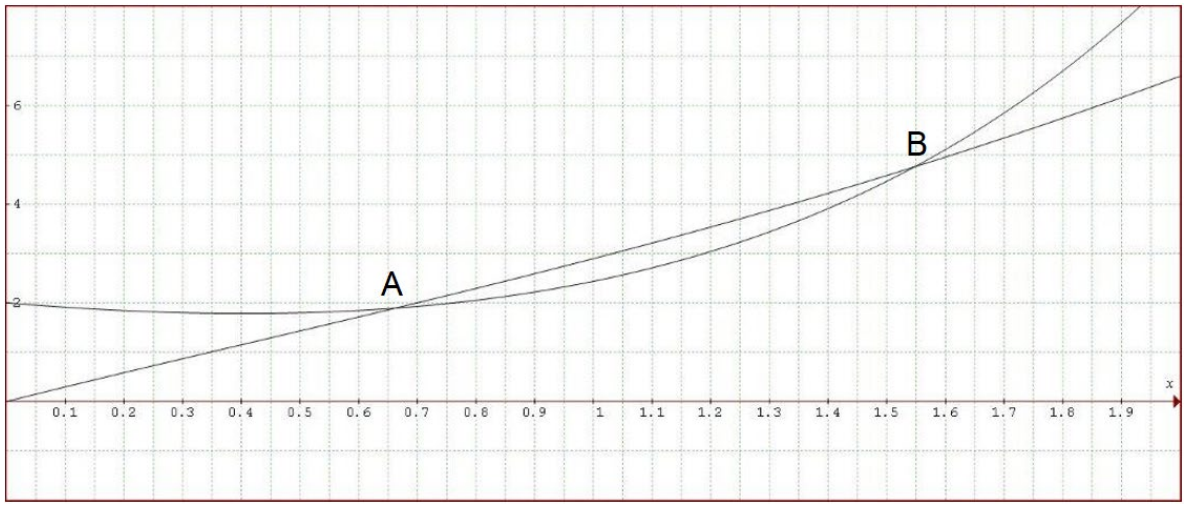
Question 9 (4 marks)

In the figure below, $f(x)$ and $g(x)$ intersect at A and B.

If $f(x) = 2e^x - 3x$ and $g(x) = -3e^{-x} + x^2 + 3$, find the area of the region bounded by $f(x)$ and $g(x)$.



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Question 10 (8 marks)

Determine the line $y = mx$ that divides the area under the curve $y = 2x(2-x)$ over $[0,2]$ into two regions of equal area.

Justify all procedures and decisions by explaining mathematical reasoning.

Evaluate the reasonableness of your solution.

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Question 11 (7 marks)

Terry is an avid skier. The equation that best models his favourite run is given by $H = 1.8e^{-x} + 0.43$ where H is the height in kilometres above sea level and x represents the cross-sectional width of the run in kilometres.

The run terminates at a place that is 1 km above sea level. The cover of snow on the mountain is 2 m (assume this cover is constant across the entire run). If the run is 300 m wide, calculate the volume of snow on the run.

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Examination marks summary

Paper 1 (technology-free)	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
1	7		
2	13		
3	7		
4		6	
5		6	
6		5	
Totals	**27**	**17**	**0**

Paper 2 (technology-active)	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
7	9		
8	8		
9	4		
10			8
11			7
Totals	**21**	**0**	**15**

Total paper	Simple familiar (SF)	Complex familiar (CF)	Complex unfamiliar (CU)
Marks	48	17	15
Percentage	**60%**	**21.25%**	**18.75%**

Instrument-specific marking guide (IA2): Examination (15%)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 3 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4

The student work has the following characteristics:	Cut-off	Marks
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.	> 13%	3
	> 7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.	> 0%	1
• does not satisfy any of the descriptors above.		0