Mathematical Methods 2019 v1.2

IA1: Mid-level annotated sample response

September 2021

Problem-solving and modelling task (20%)

This sample has been compiled by the QCAA to assist and support teachers to match evidence in student responses to the characteristics described in the instrument-specific marking guide (ISMG).

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3.

Queensland Government
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Instrument-specific marking guide (ISMG)

Criterion: Formulate

Assessment objectives

1. select, recall and use facts, rules definitions and procedures drawn from Unit 3 Topics 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• documentation of appropriate assumptions • accurate documentation of relevant observations • accurate translation of all aspects of the problem by identifying mathematical concepts and techniques.	3–4
• statement of some assumptions • statement of some observations • translation of simple aspects of the problem by identifying mathematical concepts and techniques.	1–2
• does not satisfy any of the descriptors above.	0

Criterion: Solve

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3

The student work has the following characteristics:	Marks
• accurate use of complex procedures to reach a valid solution • discerning application of mathematical concepts and techniques relevant to the task • accurate and appropriate use of technology.	6–7
• use of complex procedures to reach a reasonable solution • application of mathematical concepts and techniques relevant to the task • use of technology.	4–5
• use of simple procedures to make some progress towards a solution • simplistic application of mathematical concepts and techniques relevant to the task • superficial use of technology.	2–3
• inappropriate use of technology or procedures.	1
• does not satisfy any of the descriptors above.	0

Criterion: Evaluate and verify

Assessment objectives

4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model • justification of decisions made using mathematical reasoning.	4–5
• statements about the reasonableness of solutions by considering the context of the task • statements about relevant strengths and limitations of the solution and/or model. • statements about decisions made relevant to the context of the task.	2–3
• statement about a decision and/or the reasonableness of a solution.	1
• does not satisfy any of the descriptors above.	0

Criterion: Communicate

Assessment objective

3. communicate using mathematical, statistical and everyday language and conventions

The student work has the following characteristics:	Marks
• correct use of appropriate technical vocabulary, procedural vocabulary, and conventions to develop the response • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet.	3–4
• use of some appropriate language and conventions to develop the response • adequate organisation of the response.	1–2
• does not satisfy any of the descriptors above.	0

Task

You will formulate at least two mathematical models to describe the sprint of an elite sprinter in a 100 m, 200 m or 400 m sprint race. You should use non-polynomial models. From these models, you will select the best one and analyse it to make recommendations as to how the sprinter could improve their time.

The World Athletics website (https://worldathletics.org/about-iaaf/documents/research-centre) contains split times from a number of athletics world championships. You may use this, or another source approved by your teacher, upon which to base your model.

To complete this task, you must

- respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams
- provide a response to the context that highlights the real-life application of mathematics
- respond using a written report format that can be read and interpreted independently of the instrument task sheet
- develop a unique response.

See IA1 sample assessment instrument: Problem-solving and modelling task (20%) (available on the QCAA Portal).

Sample response

Criterion	Marks allocated	Provisional marks
Formulate Assessment objectives 1, 2 and 5	4	3
Solve Assessment objectives 1 and 6	7	4
Evaluate and verify Assessment objectives 4 and 5	5	2
Communicate Assessment objective 3	4	3
Overall	**20**	**12**

The annotations show the match to the instrument-specific marking guide (ISMG) performance-level descriptors.

	# Introduction
Communicate [3–4] coherent and concise organisation of the response … The introduction describes what the task is about and briefly outlines how the writer intends to complete the task. **Formulate [1–2]** Statement of some observations The writer has shown the observations in a table, but has not cited a source or given any other information about the observations.	The given task is to develop a model and analyse a mathematical model of a sprinter's performance in order to recommend ways that they can focus their training to improve. I chose Usain Bolt's 2017 world championship final. The displacement data is shown in Table 1.

Table 1. Usain Bolt's displacement data.

Displacement (m)	Time from start (s)
0 (reaction time)	0.183
10	1.96
20	2.98
30	3.88
40	4.76
50	5.64
60	6.49
70	7.34
80	8.20
90	9.06
100	9.95

I fitted different two functions to the data in Table 1.

Formulate [3–4] accurate translation of all aspects of the problem … The task has been accurately translated by identifying the relevant mathematical concepts and techniques to be used.	# Formulate I chose two functions to fit to the given data and used the R^2 value for each fit to select the most appropriate one to analyse. To analyse the model, I used calculus to determine the velocity and acceleration as these are important factors to know about to improve a sprint race performance.
Formulate [3–4] documentation of appropriate assumptions Appropriate assumptions have been stated and justified.	## Assumptions To formulate my models I made the following assumptions: 1. Bolt was stationary for the first 0.183 s of the race. This is known as the reaction time, and is the time between when the race starts and an athlete starts moving. 2. Bolt's displacement is a nonpolynomial function. The task specified to only consider nonpolynomial functions, so, I have considered an exponential and a sinusoidal function.
Solve [6–7]	# Solve I proposed two possible models of Bolt's displacement data, an exponential model which has the general form $f(t) = Ae^{kt} + c$ and a sine function, which has the general form $f(t) = a\sin(2\pi f - b) + c$

accurate and appropriate use of technology Use of technology to determine parameter values, generate statistics and plot the data and functions. **Communicate [3–4]**	I input the data from Table 1 into Desmos (Desmos, Inc, 2020) and used the curve fitting functionality to determine the values of the parameters that give the best fit for each proposed model. The observations and curves of best fit are shown in Figure 1.

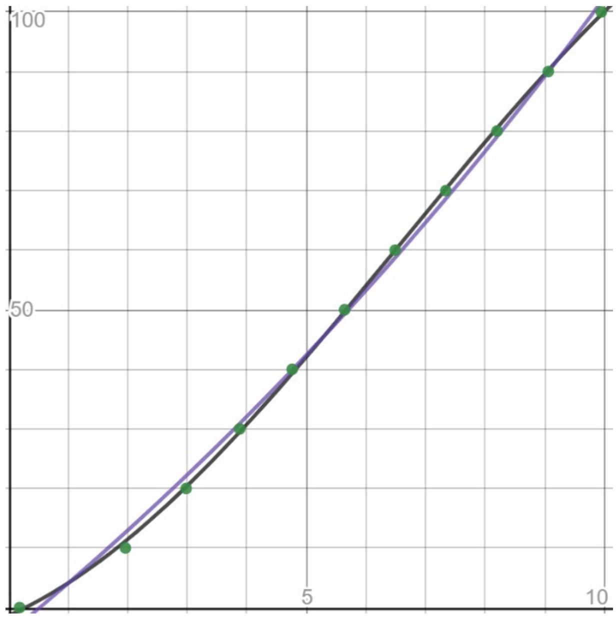

Figure 1. Observations (green dots) and proposed models (black is the sine model and purple is the exponential model)

A screen shot of Desmos that includes the parameter and R^2 values is show in Figure 2.

Since the R^2 of the sine model is greater than the exponential model, I selected that as the model to analyse as it best describes the observations. So, the displacement function is given by:

$$s(t) = 66.8466\sin(2\pi \times 0.028995t - 1.19493) + 60.8291, t > 0.183$$

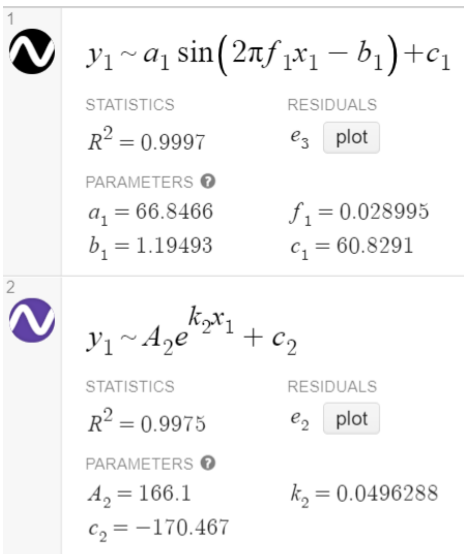

Figure 2. Statistics and parameter values for each model from Desmos.

correct use of appropriate technical vocabulary … Calculus and technical vocabulary are used appropriately. **Solve [4–5]** application of mathematical concepts relevant to the task Shows understanding of calculus concepts and principles relevant to an aspect of the problem. **Evaluate and verify [4–5]** evaluation of the reasonableness of solutions by considering the results… Analytic results have been verified by using an alternative method involving technology. **Solve [2–3]** use of simple procedures to make some progress towards a solution … The writer has used simple procedures but has not developed a full solution as they have not adequately related the model results to areas of improvement.	To analyse Bolt's performance, I differentiated the model to get the velocity, $v(t) = \frac{d}{dt}s(t)$ $v(t) = 2\pi \times 0.028995 \times 66.8466 \cos(2\pi \times 0.028995t - 1.19493)$ $v(t) = 12.1782 \cos(2\pi \times 0.028995t - 1.19493)$ and then again to get the acceleration, $a(t) = \frac{d}{dt}v(t)$ $a(t) = -2\pi \times 0.028995 \times 12.1782 \sin(2\pi \times 0.028995t - 1.19493)$ $a(t) = -2.21863 \sin(2\pi \times 0.028995t - 1.19493)$ These calculations were verified by using Desmos to differentiate and double-differentiate the displacement function and then graph them together. The functions differentiated by Desmos were exactly the same as those above; therefore, the procedure is correct.

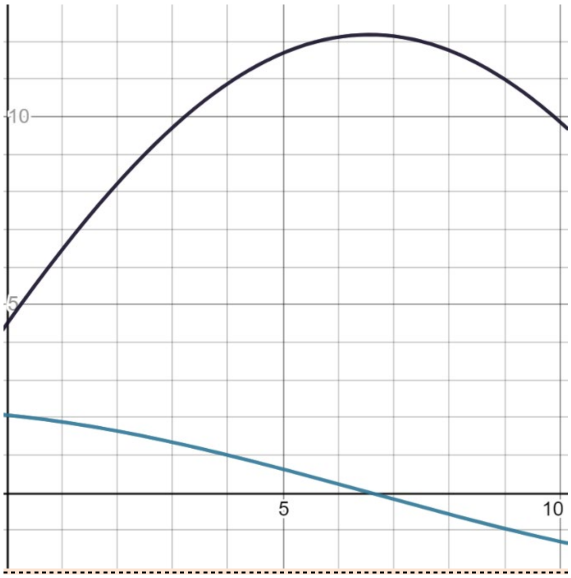

Figure 3. The velocity (black) and acceleration (blue) functions graphed. The differentiated Desmos functions are hidden by these curves.

In the 2017 world championship, Justin Gatlin beat Bolt by 0.03 s. For Bolt to have beaten Gatlin by 0.01 s, he would have needed to have been $\frac{0.04}{9.95} \times 100 =$ 0.4% faster. Multiplying each of the split times in Table 1 of Bolt by 0.996 gives new splits, shown in Table 2.

Table 2. New times for Bolt if he were 0.4% faster.

Displacement (m)	Time from start (s)
0 (reaction time)	0.182
10	1.95
20	2.97
30	3.86
40	4.74
50	5.62
60	6.46
70	7.31
80	8.17
90	9.02

	<table><tr><td>100</td><td>9.91</td></tr></table> Fitting a sine function to these new times in Desmos gives the following parameter values $a = 66.8982$ $f = 0.029085$ $b = 1.19415$ $c = 60.8574$ which corresponds to a small increase for a and small decreases for f, b and c. This indicates that Bolt should work on decreasing his reaction time and increasing his speed.
Evaluate and verify [2–3] statement about the reasonableness of solutions by considering the context … Consideration of the model fit, related to the context. **Evaluate and verify [2–3]** statement about relevant strengths and limitations … Some relevant strengths and limitations are stated but no documentation is provided. **Communicate [1–2]** Adequate organisation of the response The writer has written a recognisable conclusion; however, it is very brief and does not adequately synthesise the information contained in the report. There is also no in-text referencing or reference list.	**Evaluation** The model of Bolt's race fits the data very well since $R^2 = 0.9997$. A close fit to the observations is needed to be able to accurately model Bolt's race to figure out ways to improve his performance. The strengths of the model are that it fits the displacement data very closely and is easy to formulate. A limitation is that the modelled velocity should be zero for $t \leq 0.183$, however, as can be seen in Figure 3 it is not. **Conclusion** To improve his performance, calculus techniques used on the model of Bolt's race shows that his training should focus on decreasing his reaction time and increasing his speed.