Mathematical Methods 2019 v1.2

IA1: High-level annotated sample response

November 2022

Problem-solving and modelling task (20%)

This sample has been compiled by the QCAA to assist and support teachers to match evidence in student responses to the characteristics described in the instrument-specific marking guide (ISMG).

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3.

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Instrument-specific marking guide (ISMG)

Criterion: Formulate

Assessment objectives

1. select, recall and use facts, rules definitions and procedures drawn from Unit 3 Topics 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• documentation of appropriate assumptions • accurate documentation of relevant observations • accurate translation of all aspects of the problem by identifying mathematical concepts and techniques.	3–4
• statement of some assumptions • statement of some observations • translation of simple aspects of the problem by identifying mathematical concepts and techniques.	1–2
• does not satisfy any of the descriptors above.	0

Criterion: Solve

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 2 and/or 3
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 2 and/or 3

The student work has the following characteristics:	Marks
• accurate use of complex procedures to reach a valid solution • discerning application of mathematical concepts and techniques relevant to the task • accurate and appropriate use of technology.	6–7
• use of complex procedures to reach a reasonable solution • application of mathematical concepts and techniques relevant to the task • use of technology.	4–5
• use of simple procedures to make some progress towards a solution • simplistic application of mathematical concepts and techniques relevant to the task • superficial use of technology.	2–3
• inappropriate use of technology or procedures.	1
• does not satisfy any of the descriptors above.	0

Criterion: Evaluate and verify

Assessment objectives

4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning

The student work has the following characteristics:	Marks
• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model. • justification of decisions made using mathematical reasoning.	4–5
• statements about the reasonableness of solutions by considering the context of the task • statements of relevant strengths and limitations of the solution and/or model • statements about decisions made relevant to the context of the task.	2–3
• statement about a decision and/or the reasonableness of a solution.	1
• does not satisfy any of the descriptors above.	0

Criterion: Communicate

Assessment objective

3. communicate using mathematical, statistical and everyday language and conventions

The student work has the following characteristics:	Marks
• correct use of appropriate technical vocabulary, procedural vocabulary, and conventions to develop the response • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion, which can be read independently of the task sheet.	3–4
• use of some appropriate language and conventions to develop the response • adequate organisation of the response.	1–2
• does not satisfy any of the descriptors above.	0

Task

You will formulate at least two mathematical models to describe the sprint of an elite sprinter in a 100 m, 200 m or 400 m sprint race. You should use non-polynomial models. From these models, you will select the best one and analyse it to make recommendations as to how the sprinter could improve their time.

The World Athletics website (https://worldathletics.org/about-iaaf/documents/research-centre) contains split times from a number of athletics world championships. You may use this, or another source approved by your teacher, upon which to base your model.

To complete this task, you must

- respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams
- provide a response to the context that highlights the real-life application of mathematics
- respond using a written report format that can be read and interpreted independently of the instrument task sheet
- develop a unique response.

See IA1 sample assessment instrument: Problem-solving and modelling task (20%) (available on the QCAA Portal).

Sample response

Criterion	Marks allocated	Provisional marks
Formulate Assessment objectives 1, 2 and 5	4	4
Solve Assessment objectives 1 and 6	7	7
Evaluate and verify Assessment objectives 4 and 5	5	4
Communicate Assessment objective 3	4	4
Overall	**20**	**19**

The annotations show the match to the instrument-specific marking guide (ISMG) performance-level descriptors.

Communicate [3–4] coherent and concise organisation of the response … The introduction describes what the task is about and briefly outlines how the writer to completed the task. A body is also clearly evident. The body has been divided into appropriate sections.	# Introduction Usain Bolt was a dominant competitor in the 100 m sprint taking three consecutive Olympic gold medals and breaking the world record twice (Wikipedia, 2020). The objective of this report is to develop a mathematical model of Bolt's 2017 World Championship 100 m final performance with an aim to identifying how he can improve his performance through amending his race strategy. I fit two different models to Bolt's 10 m split times and identified an exponential model of velocity to be the most appropriate because of its relatively good fit to observations. I then analysed model parameters to examine how Bolt's sprint time was affected by changing different parameters affecting his reaction time, acceleration and maximum velocity. I found that an increase in the maximum speed of his sprint gave the greatest decrease in time to run the race.
Formulate [3–4] accurate documentation of relevant observations The writer has documented the observations by citing their source and summarising relevant information about the data.	# Observations and assumptions The sprint performance I chose to analyse is Usain Bolt's men's 100 m sprint final performance at the 2017 International Association of Athletic Federations world championship in London. Bolt came third to Justin Gatlin by 0.03 s. I obtained reaction time and 10 m split times from Table 2.1 of (Bissas, et al., n.d.) and are summarised in Table 1. **Table 1. Data for Usain Bolt's 100m 2017 World Championship sprint final** <table><tr><th>Displacement (m)</th><th>Time from start (s)</th></tr><tr><td>0 (reaction time)</td><td>0.183</td></tr><tr><td>10</td><td>1.96</td></tr><tr><td>20</td><td>2.98</td></tr><tr><td>30</td><td>3.88</td></tr><tr><td>40</td><td>4.76</td></tr><tr><td>50</td><td>5.64</td></tr><tr><td>60</td><td>6.49</td></tr><tr><td>70</td><td>7.34</td></tr><tr><td>80</td><td>8.20</td></tr><tr><td>90</td><td>9.06</td></tr><tr><td>100</td><td>9.95</td></tr></table> The split data was collected using video analysis of from five different cameras, offering sufficient accuracy for my purposes. During this race, there was a 0.9 m/s headwind (Bissas, et al., n.d.). Using Excel, I graphed the data (see Figure 1).

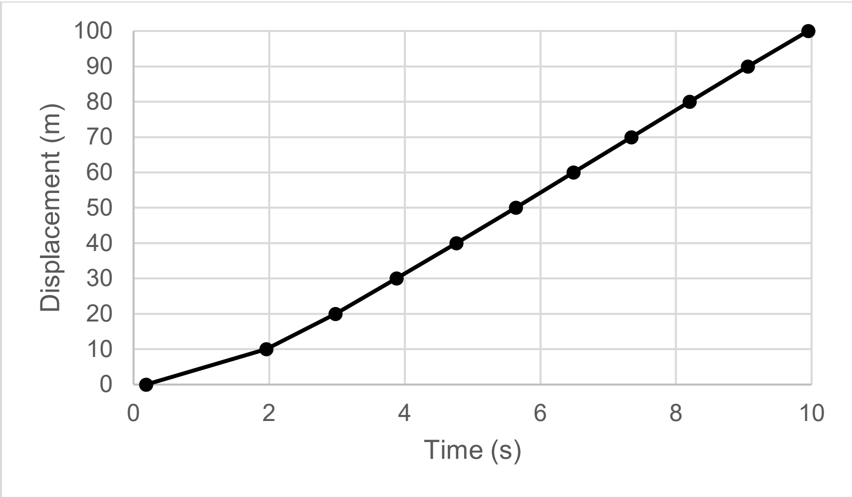

Figure 1. The displacement-time graph for Bolt's 2017 World Championship final.

Formulate [3–4]
documentation of appropriate assumptions
The writer has stated assumptions and documented them by justifying them and/or discussing their effect on the model.

To formulate the mathematical models of Bolt's sprint race, I made the following assumptions

1. Bolt's displacement and velocity was zero for $t \leq 0.183$. In a sprint race, competitors start from a stationary position. There is a time-lag between when the race starts and when a competitor starts moving (known as the reaction time).
2. The wind does not affect the overall results of the model. Wind affects the time it takes for a sprinter to run 100 m, however, it does not fundamentally change the overall description of a race. Since the purpose of this task is to formulate a model to help a sprinter improve their race technique, it is the shape of the functions, rather than their absolute values, that are important.
3. Bolt's displacement as a function of time is not described by a polynomial. A specification of the task is that only nonpolynomial functions are to be considered, so, I have not considered any polynomial functions.

Formulate [3–4]
accurate translation of all aspects of the problem by identifying mathematical concepts and techniques
The writer has identified relevant concepts and techniques required to solve the problem.

Mathematical concepts and procedures

To analyse Bolt's performance, I created models of his velocity during the race. I then used calculus methods to determine functions to describe his displacement and acceleration. I estimated velocity, v, using the displacement data in Table 1 and the following formula

$$v = \frac{\Delta s}{\Delta \mathrm{t}}$$

Where $\Delta s = 10$ m is the change in displacement and Δt is the change in time. The estimated velocity is shown in Figure 2 (the raw data and details of calculations is shown in Appendix 1).

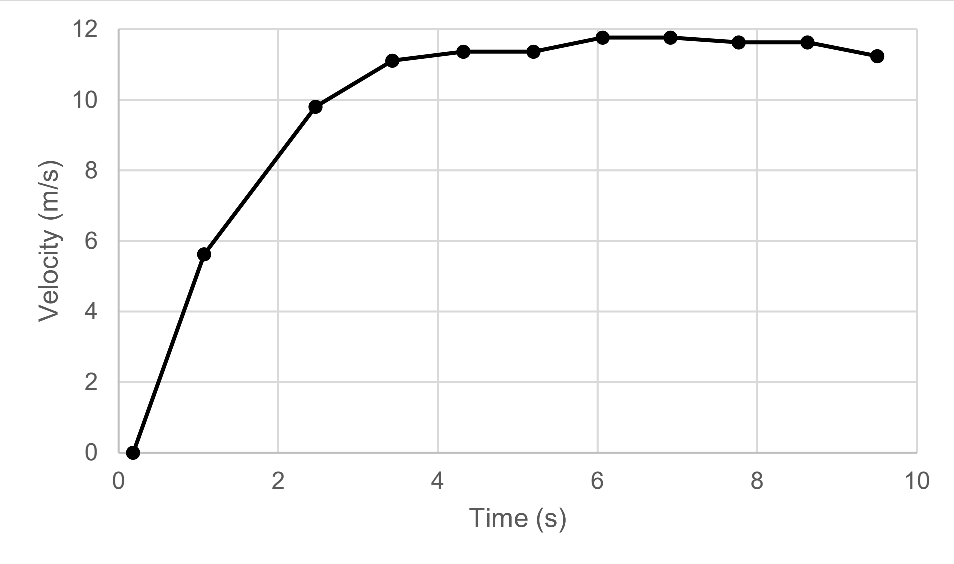

Figure 2. Bolt's velocity, estimated from the displacement data.

The shape of the velocity data has a rapid rate increase over the first ~ 4 s and then remains approximately constant. There are therefore several possible candidate functions to model Bolt's velocity, including:

1. logarithmic function
2. exponential function

I used a graphics calculator and the Scipy Python package to determine best fit parameters to each of these functions.

Determining the models

Model 1: logarithmic function

A natural logarithm function has the general form

$$v(t) = a\ln(t - b) + c$$

Solve [6–7]
accurate and appropriate use of technology
The writer has documented their appropriate use of technology. Supporting material in the appendices gives details as to the accuracy of its use.

The graphics calculator has curve fitting functionality for a natural logarithm function, however, it does not allow for horizontal translation. Therefore, I used the curve fit function in the Scipy module of Python (Virtanen, et al., 2020). The Python code used for curve fitting is shown in Appendix 2. The Scipy curve fitting function requires initial guesses of each parameter. The general shape looks like a log function that has been translated to the left slightly and up, so, my initial guesses were $a = 1, b = -1, c = 10$. The results of the curve fitting were:

$$a = 2.3429$$
$$b = 0.13717$$
$$c = 7.1877$$

Model 2: exponential function

An exponential decay function has the general form

$$v(t) = Ae^{-kt} + c$$

As with the logarithmic function, I used the Scipy curve fitting function. The general shape is of an exponential decay function that has been reflected in the x-axis and translated up. Therefore, my initial guesses were $A = -1, k = 1, b = 13$. The results of the curve fitting were:

$$A = -13.616$$
$$k = 0.79435$$
$$c = 11.6862$$

Communicate [3–4]
coherent and concise organisation of the response …
The use of graphs and tables to convey complex information makes the response concise

Comparing models

The model results are graphed against the observed velocity in Figure 3. A visual inspection indicates that model 2 gives the best fit.

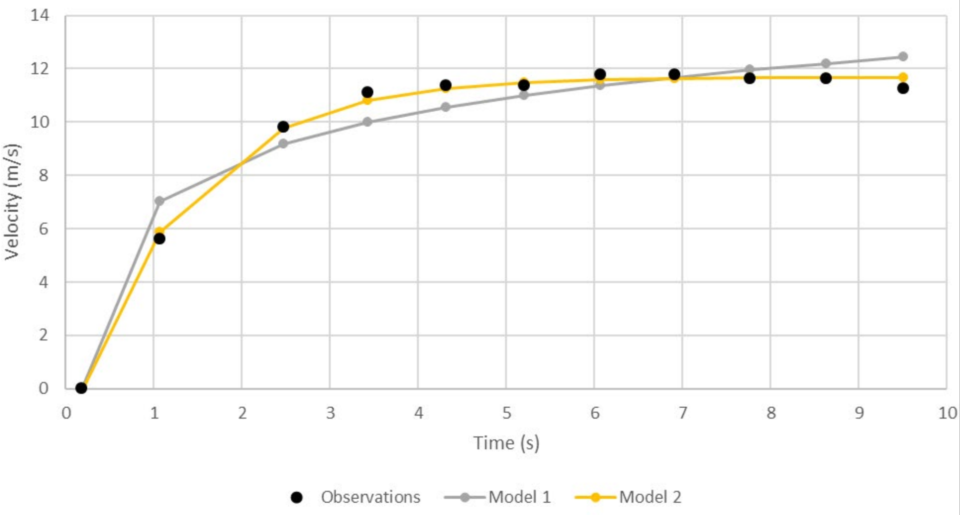

Figure 3. Modelled and observed velocities of Bolt's 100m sprint final.

Solve [6–7]
discerning application of mathematical concepts and techniques relevant to the task
The writer has documented their choice of one mathematical technique over another

A standard way to quantitatively assess the goodness of fit of a line is to use the coefficient of determination, R^2, which describes the proportion of explained variance. However, the coefficient of determination is not appropriate for nonlinear fits because the explained variance and error variance do not sum to the total variance (Frost, 2020).

Thus, to compare the models, I used the sum of the squared error

$$SSE = \sum \left(v_i - f(t_i)\right)^2$$

The squared error and sums are shown in Appendix 3 and summarised in Table 2. The standard error analysis confirms the discussion above, namely that model 2 provides the best fit.

Table 2. Sum of squared error for each model.

Model	SSE
Model 1 (logarithmic)	6.49
Model 2 (exponential)	0.45

Thus, the model for velocity is given by

$$v(t) = \begin{cases} 0, & t \le 0.183 \\ -13.616e^{-0.79435t} + 11.6862, & t > 0.183 \end{cases}$$

Communicate [3–4] correct use of appropriate technical vocabulary, procedural vocabulary, and conventions... The writer has correctly used vocabulary and notation relevant to calculus.	

Model analysis

To use the model to analyse Bolt's performance, I needed to determine the modelled displacement and acceleration. Displacement, s, is the integral of velocity,

$$s(t) = \int v(t)\,dt$$
$$s(t) = \int Ae^{-kt} + c\,dt$$
$$s(t) = \frac{Ae^{-kt}}{-k} + ct + d$$
$$s(t) = 17.147e^{-0.7943t} + 11.686t + d$$

It follows from assumption 1 that we must have $s(0.183) = 0$, therefore

$$d = 0 - 17.147e^{-0.7943\times0.183} - 11.686 \times 0.183$$
$$d = -16.966$$

Thus, the model for displacement is

$$s(t) = \begin{cases} 0, & t \leq 0.183 \\ 17.147e^{-0.7943t} + 11.686t - 16.966, & t > 0.183 \end{cases}$$

Acceleration, a, is the derivative of velocity,

$$a(t) = \frac{dv}{dt}$$
$$a(t) = \frac{d}{dt}(Ae^{-kt} + c)$$
$$a(t) = -Ake^{-kt}$$
$$a(t) = 10.82e^{-0.7943t}$$

Evaluate and verify [4–5]
evaluation of the reasonableness of solutions by considering the results, assumptions and observations.
The writer has evaluated the reasonableness of the solution by comparing the modelled race time with the observed race time. The writer did not consider their assumptions in their evaluation.

The modelled displacement is graphed with the observations in Figure 4 and appears to compare very well. I used a graphics calculator to determine that $t = 10.01$ for $s(t) = 100$, which is 0.06 s slower than the actual race result. Therefore, for the purposes of the model analysis, the reference time to improve is 10.01 s.

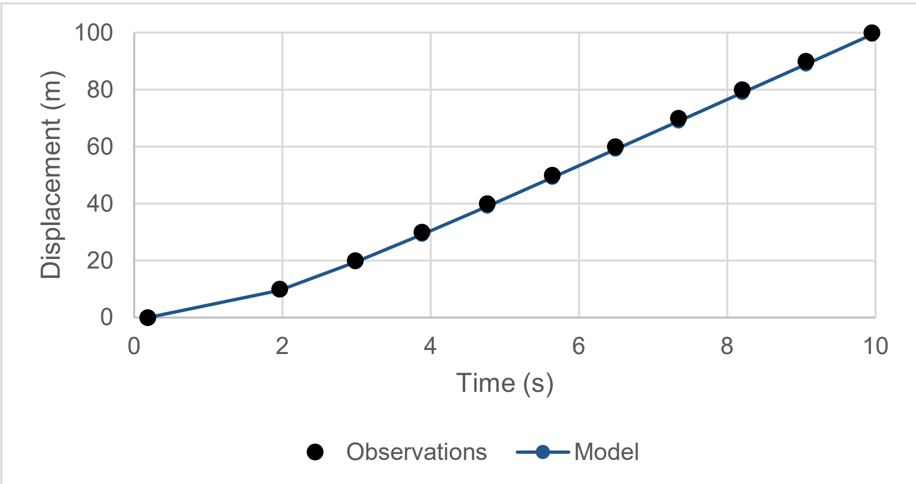

Figure 4. Modelled and observed displacement of Bolt's 100 m sprint final.

There are four distinct phases of a 100 m sprint race (Percia, 2020):

1. start phase – when the athlete is in contact with the starting blocks
2. acceleration phase – when the athlete is accelerating to their maximum velocity
3. constant speed phase - when the athlete is moving at a constant velocity
4. deceleration phase – when the athlete is decelerating from maximum velocity

In the exponential model of velocity, the

- length of the start phase is determined by a horizontal translation, which is mostly controlled by the coefficient A
- acceleration phase is determined by the rate of change, which is mostly controlled by the coefficient k and
- constant speed phase is determined by the vertical translation, which is mostly controlled by the parameter c.

Evaluate and verify [4-5]
justification of decisions made using mathematical reasoning
The writer has used mathematical reasoning to justify their approach to solving the problem

As Bolt came third by 0.03 seconds there are only small gains that would have been required to make up the deficit. The effect on the modelled race time of changing each of these parameters is considered next.

Start phase

The start phase has to do with reaction time. Any reaction time less than 0.1 s is considered a false start by World Athletics (World Athletics, 2020), so, the maximum gain that can be made by Bolt is 0.083 s. If Bolt improved his reaction time by 0.04 s to be 0.143 s without changing any other aspect of his race, this would act as a translation to the model. This is a reasonable gain to make as an analysis of world championship reaction times found that most reaction times were between 0.13 and 0.15 s (Duffy, 2002). To determine how the coefficient is affected, I rewrote the first term of the model

Solve [6-7]
accurate use of complex procedures to reach a valid solution
Integration procedures are evident. The solution involves a combination of parts that are interconnected.

$$Ae^{-kt} = -|A|e^{-kt}$$
$$Ae^{-kt} = -e^{-k(t-b)}$$
$$Ae^{-kt} = -e^{-kt}e^{kb}$$
$$A = -e^{kb}$$
$$b = \frac{\ln(-A)}{k}$$
$$b = \frac{\ln(13.616)}{0.7943}$$
$$b = 3.2879$$

Therefore, to decrease the reaction time by 0.04 s, I decreased b by that amount yielding

$$A = -e^{0.7943\times 3.2479} = -13.196$$

Integrating the velocity function with the new value of A (since these are repeated calculations, the details have been put in Appendix 4) gives

$$s(t) = 16.612e^{-0.7943t} + 11.686t - 16.499, t > 0.143$$

Using the intersection function of the graphics calculator, I determined that the time to run 100 m to be approximately 9.97 s. As would be expected intuitively, if Bolt is able to shorten his reaction time by 0.04 s, that corresponds to a proportional shortening of his race time.

Acceleration phase

From the model of acceleration, Bolt's initial acceleration is

$$a(0.183) = 10.82e^{-0.7943\times 0.183} = 9.356$$

Given Bolt is at the pinnacle of the sport, large gains in acceleration are unlikely, so I assumed a modest increase of 2%. I used the graphics calculator to solve the acceleration model for k

$$-13.616 \times -ke^{-k\times 0.183} = 9.356 \times 1.02$$
$$ke^{-k\times 0.183} = 0.70088$$
$$k = 0.81337$$

This gives a velocity function of

$$v(t) = -13.616e^{-0.81337t} + 11.6862, t > 0.183$$

	Integrating (see Appendix 4) gives $$s(t) = 16.740e^{-0.81337t} + 11.6862t - 16.563, t > 0.183$$ Using the intersection function of the graphics calculator, I determined that the time to run 100 m to be 9.97 s, which is approximately 0.04 s faster than the original model. So, for Bolt to improve his time by 0.04 s, he could increase his acceleration by 2%. **Constant speed phase** The parameter that controls the maximum speed is c. I assumed an increase in maximum speed of 2%, thus $$v(t) = -13.616e^{-0.81337t} + 11.9199, t > 0.183$$ Integrating (see Appendix 4) gives $$s(t) = 17.147e^{-0.79435t} + 11.9199t - 16.657, t > 0.183$$ Using the graphics calculator, I determined that the time to run 100 m to be 9.79 s, which is approximately 0.22 s faster than the original model. This is a very significant decrease in time taken to run the race. Thus, increasing his maximum speed is the area of where Bolt could obtain the greatest improvement.
Evaluate and verify [4–5] documentation of relevant strengths and limitations of the solution and/or model The writer has stated relevant strengths and limitations about the model and documented these by documenting their impacts on the model and/or results. **Communicate [3–4]** coherent and concise organisation of the response … The conclusion summarises the report, giving information about the problem that had to be solved, and discussion about the results, including	**Evaluation** There are several strengths and limitations of the model to consider when interpreting its results Strengths of the selected model include that • it fits the observations well (see the Comparing models section) • there are parameters that are clearly linked to different phases of the sprint race which I was able to analyse and make recommendations about where Bolt could improve his race strategy (see the Model analysis section) Limitations of the selected model include that the • modelled time to run the race is 0.06 s slower than the time it took Bolt to run the race. This would have placed him in equal fourth place. This limitation does not affect the overall use of the model as a tool to identify areas of improvement, however, it means that the model should be used with extreme caution to predict accurate 100 m sprint times. • parameters, whilst clearly linked to different phases of the sprint, are not completely independent of each other, for example, a change to the value of A necessitates some small changes to other parameters to satisfy initial conditions • model does not incorporate the deceleration phase of the sprint. On this last limitation, the model could be refined to incorporate the deceleration through additional terms in the velocity equation, e.g. $v(t) = Ae^{-kt} - mt + c$ which would assume that the deceleration is linear. **Conclusion** The objective of this report was to analyse the performance of Usain Bolt's 2017 100 m sprint final performance, with an aim to identify where improvements could be made. I achieved this by creating a mathematical model of his race and examined how changes in different parameters affected the different phases of the sprint and the overall time. Bolt came third to Justin Gatlin by 0.03 s. I found that a decrease in reaction time of 0.04 s or an increase in acceleration of 2% would have been sufficient to reduce Bolt's race time by 0.04 s and make up the deficit. However, I found that the greatest effect was an increase in maximum velocity by 2%, which decreased Bolt's race time by 0.22 s. This is a huge decrease in time and is likely unachievable, however, it

conclusions drawn from the information.	indicates that a small increase in maximum velocity is the most effective strategy and this is where training should be focussed. **References** Bissas, A. et al., n.d.. *Biomechanical report for the IAAF World Championships London 2017 100m Men's,* s.l.: International Association of Athletics Federations. Duffy, K., 2002. *Reaction times and sprint false starts.* [Online] Available at: https://condellpark.com/kd/reactiontime.htm#:~:text=Most%20elite%20sprinters%20have%20a,that's%20one%20part%20in%20100. [Accessed 17 December 2020]. Frost, J., 2020. *R-squared Is Not Valid for Nonlinear Regression.* [Online] Available at: https://statisticsbyjim.com/regression/r-squared-invalid-nonlinear-regression/ [Accessed 11 December 2020]. Percia, M., 2020. *Phases of Sprinting.* [Online] Available at: https://www.livestrong.com/article/475175-phases-of-sprinting/ [Accessed 11 December 2020]. Virtanen, P. et al., 2020. SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python. *Nature Methods,* 17(3), pp. 261-272. World Athletics, 2020. *100 metres.* [Online] Available at: https://www.worldathletics.org/disciplines/sprint-event/100-metres [Accessed 11 December 2020].

Appendix 1

Table 2 shows the velocity data, calculated from the displacement data in Table 1. The first row follows from assumption 1 (that Bolt's velocity is zero for $t \leq 0.183$ s. The other rows are calculated by taking differences between the displacement and time to calculate velocity as

$$v = \frac{\Delta \mathrm{s}}{\Delta \mathrm{t}}$$

An example calculation for the second row is as follows

$$\Delta s = 10 - 0 = 10$$
$$\Delta t = 1.96 - 0.183 = 1.777$$
$$v = \frac{10}{1.777} = 5.63$$

The time, t, is taken as the mid-point of the two times to calculate Δt,

$$t = \frac{0.183 + 1.96}{2} = 1.0715$$

Table 3. Velocity-time data for Bolt's 2017 World Championship 100 m sprint.

Δs	Δt	v	t
		0	0.183
10	1.777	5.63	1.0715
10	1.02	9.80	2.47
10	0.9	11.11	3.43
10	0.88	11.36	4.32
10	0.88	11.36	5.2
10	0.85	11.76	6.065
10	0.85	11.76	6.915
10	0.86	11.63	7.77
10	0.86	11.63	8.63
10	0.89	11.24	9.505

Appendix 2

Below shows the Python code used for the nonlinear curve fitting.

```
import numpy as np
import scipy.optimize as opt

#Define the data
v=np.array([0,5.63,9.80,11.11,11.36,11.36,11.76,11.76,11.63,11.63,11.24])
t=np.array([0.183,1.0715,2.47,3.43,4.32,5.2,6.065,6.915,7.77,8.63,9.505])

###Natural log###
def ln(x,a,b,c): return a*np.log(x-b)+c
p0=[1,-1,10] #Initial guess at parameters a,b,c,d
poptln,pcov=opt.curve_fit(ln,t,v,p0=p0)
print('ln',poptln)

###Exponential function###
def exp(x,A,k,b): return A*np.exp(k*x)+b
p0=[-1,-1,13] #initial guess at parameters A,k,d
poptexp,pcov=opt.curve_fit(exp,t,v,p0=p0)
print('exp',poptexp)
```

Appendix 3

The below table shows the time, observed velocity, modelled velocity, and squared error. The squared error (SE) is calculated as

$$SE = \left(v - f(t)\right)^2$$

The sum of squared error is the sum of the squared error.

Time	v	**Model 1**		**Model 2**	
		$f(t)$	**SE**	$f(t)$	**SE**
0.183	0	-0.04	0.00	-0.09	0.01
1.0715	5.63	7.03	1.96	5.87	0.06
2.47	9.80	9.17	0.40	9.77	0.00
3.43	11.11	9.98	1.28	10.79	0.10
4.32	11.36	10.54	0.68	11.25	0.01
5.2	11.36	10.99	0.14	11.47	0.01
6.065	11.76	11.36	0.17	11.58	0.04
6.915	11.76	11.67	0.01	11.63	0.02
7.77	11.63	11.95	0.10	11.66	0.00
8.63	11.63	12.20	0.33	11.67	0.00
9.505	11.24	12.43	1.43	11.68	0.20
		SSE	6.49		0.45

Appendix 4

Start phase calculations

$$s(t) = \int v(t)\,dt$$
$$s(t) = \int Ae^{-kt} + c\,dt$$
$$s(t) = \frac{Ae^{-kt}}{-k} + ct + d$$
$$s(t) = \frac{-13.196e^{-0.7943t}}{-0.7943} + 11.686t + d$$
$$s(t) = 16.612e^{-0.7943t} + 11.686t + d$$

It follows from assumption 1 that we must have $s(0.143) = 0$, therefore

$$d = 0 - 16.612e^{-0.7943\times 0.143} - 11.686 \times 0.143$$
$$d = -16.499$$

Thus, the model for displacement is

$$s(t) = \begin{cases} 0, & t \le 0.143 \\ 16.612e^{-0.7943t} + 11.686t - 16.499, & t > 0.143 \end{cases}$$

Acceleration phase calculations

$$s(t) = \int v(t)\,dt$$
$$s(t) = \int Ae^{-kt} + c\,dt$$
$$s(t) = \frac{Ae^{-kt}}{-k} + ct + d$$
$$s(t) = \frac{-13.616e^{-0.81337t}}{-0.81337} + 11.686t + d$$
$$s(t) = 16.740e^{-0.81337t} + 11.686t + d$$

It follows from assumption 1 that we must have $s(0.183) = 0$, therefore

$$d = 0 - 16.740e^{-0.81337\times 0.183} - 11.686 \times 0.183$$
$$d = -16.563$$

Thus, the model for displacement is

$$s(t) = \begin{cases} 0, & t \le 0.183 \\ 16.740e^{-0.81337t} + 11.686t - 16.563, & t > 0.183 \end{cases}$$

Constant speed phase calculations

$$s(t) = \int v(t)\,dt$$
$$s(t) = \int Ae^{-kt} + c\,dt$$
$$s(t) = \frac{Ae^{-kt}}{-k} + ct + d$$
$$s(t) = \frac{-13.616e^{-0.79435t}}{-0.79435} + 11.9199t + d$$
$$s(t) = 17.147e^{-0.79435t} + 11.9199t + d$$

It follows from assumption 1 that we must have $s(0.183) = 0$, therefore

$$d = 0 - 17.147e^{-0.79435\times 0.183} - 11.9199 \times 0.183$$
$$d = -16.657$$

Thus, the model for displacement is

$$s(t) = \begin{cases} 0, & t \le 0.183 \\ 17.147e^{-0.79435t} + 11.9199t - 16.657, & t > 0.183 \end{cases}$$