General Mathematics marking guide

External assessment 2023

Paper 1 (57 marks)

Paper 2 (38 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

For all Queensland schools

Purpose

This marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of '0' will be recorded.

Where no response to a question has been made, a mark of 'N' will be recorded.

Allow FT mark/s — refers to 'follow through', where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Paper 1: Multiple choice

Question	Response
1	A
2	C
3	B
4	C
5	B
6	B
7	D
8	A
9	B
10	C
11	C
12	D
13	C
14	B
15	B

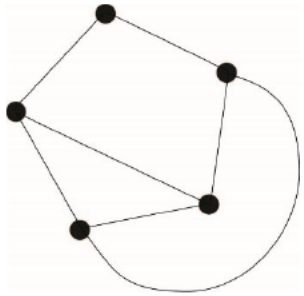
Paper 1: Short response

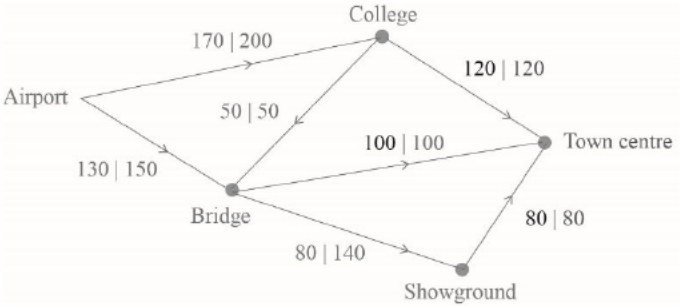
Q	Sample response	The response:
16	**Method 1** Angular difference = 90° + 120° = **210°** Time difference = $\frac{210°}{15°/\text{h}}$ = **14 hours** Town B is east of town A, so town B is 14 hours ahead of town A. Local time in town B = 2:00 am + 14 hours = **4:00 pm** **Method 2** Town B's longitude is east, so its time is ahead of UTC. Time difference = $\frac{120°}{15°/\text{h}}$ = **8 hours** ($\text{UTC}^{+}8$) Town A's longitude is west, so its time is behind UTC. Time difference = $\frac{90°}{15°/\text{h}}$ = **6 hours** ($\text{UTC}^{-}6$) Time difference = $^{+}8 - {}^{-}6$ = **14 hours** Town B is east of town A, so town B is 14 hours ahead of town A. Local time in town B = 2:00 am + 14 hours = **4:00 pm**	• correctly determines the angular difference **[1 mark]** • determines absolute time difference between town A and town B **[1 mark]** • determines local time in town B **[1 mark]** • correctly determines the time difference for each of town A and town B compared to 0° **[1 mark]** • determines absolute time difference between town A and town B **[1 mark]** • determines local time in town B **[1 mark]**

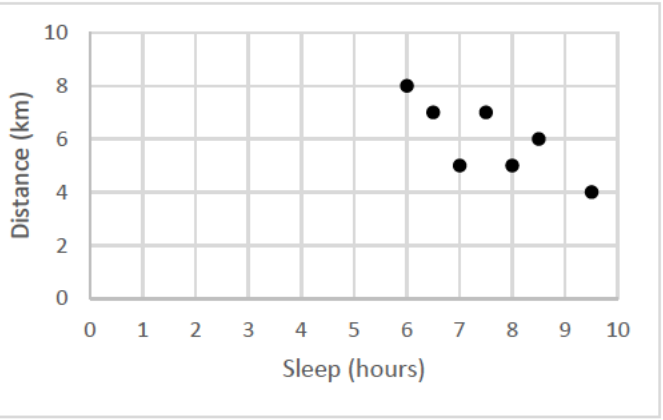
Q	Sample response	The response:
17	$i = \frac{6.6}{12 \times 100}$ $= 0.0055$ $n = 25 \times 12$ $= 300$	• correctly determines the i and n values **[1 mark]**
	Amount borrowed, $A = 570\,000 - 50\,000$ $= 520\,000$	• correctly determines the amount borrowed **[1 mark]**
	$A = M\left(\frac{1-(1+i)^{-n}}{i}\right)$ $520\,000 = M\left(\frac{1-(1+0.0055)^{-300}}{0.0055}\right)$	• substitutes into appropriate annuity rule **[1 mark]**
	$M = \frac{520\,000}{\left(\frac{1-(1+0.0055)^{-300}}{0.0055}\right)}$ $= 3543.64$ Monthly repayment is $3543.64	• determines monthly repayment **[1 mark]**

Q	Sample response	The response:
18a)	Site C	• correctly names the site **[1 mark]**
18b)	Western Australia	• correctly names the state **[1 mark]**
18c)	Sites B and C are in the same standard time zone because they have the same longitude.	• correctly determines sites B and C are in the same standard time zone **[1 mark]** • correctly explains using longitude **[1 mark]**

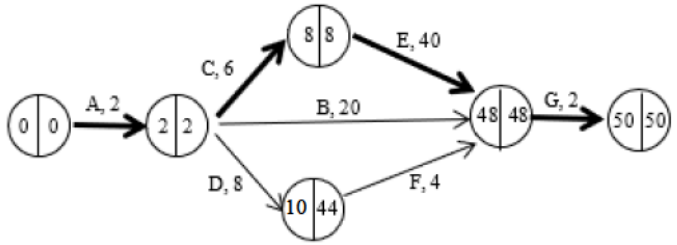
Q	Sample response	The response:
19	Option A: $i = 0.056, n = 12$ $i_{\text{effective}} = \left(1+\frac{i}{n}\right)^n - 1$ $= \left(1+\frac{0.056}{12}\right)^{12} - 1$ $\approx 0.05745...$ Option B: $i = 0.0562, n = 4$ $i_{\text{effective}} = \left(1+\frac{i}{n}\right)^n - 1$ $= \left(1+\frac{0.0562}{4}\right)^{4} - 1$ $\approx 0.05739...$ $0.05745 > 0.05739$ Ngarra's decision is reasonable because option A has a higher effective interest rate.	• correctly substitutes into appropriate rule for either option **[1 mark]** • calculates effective interest rate for option A **[1 mark]** • calculates effective interest rate for option B **[1 mark]** • provides a statement of reasonableness linked to effective interest rate **[1 mark]**

Q	Sample response	The response:
20a)	$v = 4$ $f = 3$ $e = 5$ $v + f - e = 4 + 3 - 5 = 2$	• correctly identifies the number of vertices, faces and edges for graph 1 **[1 mark]** • applies Euler's formula to graph 1 **[1 mark]**
20b)	Show no crossing edges.	• correctly identifies the feature to be changed **[1 mark]**
20c)		• correctly draws graph 2 as a simple connected graph with seven edges that do not cross and five vertices (one degree 2 vertex, four degree 3 vertices) **[1 mark]**

Q	Sample response	The response:
21a)	Capacity of cut X = 200 + 100 + 140 = **440** vehicles per hour	• correctly determines the capacity of cut X **[1 mark]**
21b)		
	Maximum flow = **120 + 100 + 80**	• correctly identifies an appropriate method **[1 mark]**
	= **300** vehicles per hour	• identifies maximum flow **[1 mark]**
21c)	Maximum flow during weather emergency = **120** vehicles per hour	• correctly determines the maximum flow **[1 mark]**

Q	Sample response	The response:
22a)		• correctly identifies the axis for the response variable **[1 mark]** • formats scatterplot with appropriate scaling and labelling for both axes **[1 mark]** • accurately plots all points **[1 mark]**
22b)	The association is **negative**.	• identifies direction **[1 mark]**

Q	Sample response	The response:
23a)	Degree = 4	• correctly states the degree **[1 mark]**
23b)	2 edges	• correctly states the number of edges **[1 mark]**
23c)	$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 1 & 1 & 0 & 2 & 0 \end{bmatrix} \end{array}$	• correctly completes entries for one row or one column in a 5 x 5 matrix **[1 mark]** • correctly enters 1 for number of edges joining A to A **[1 mark]** • completes adjacency matrix **[1 mark]**

Q	Sample response	The response:
24a)		• correctly constructs a network diagram showing the appropriate sequence for all tasks **[1 mark]** • labels all tasks and durations on network diagram **[1 mark]** • shows earliest and latest starting times for all tasks **[1 mark]**
24b)	Critical activities: A, C, E, G. Minimum completion time = 2 + 6 + 40 + 2 = **50 minutes**	• determines critical activities **[1 mark]** • determines minimum completion time, including units **[1 mark]**

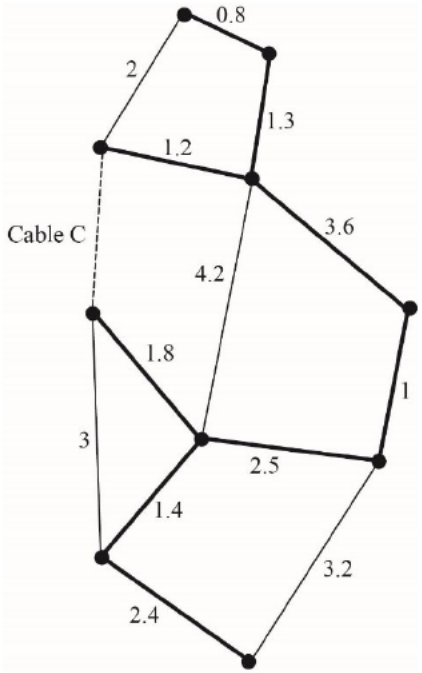
Q	Sample response	The response:
25a)	Latitude and longitude of X = 3° S 141° E	• correctly determines the latitude and longitude of X to the nearest degree **[1 mark]**
25b)	Latitude and longitude of Y = 9° S 141° E	• correctly determines the latitude and longitude of Y to the nearest degree **[1 mark]**
25c)	Angular distance $= 9° - 3°$ $= 6°$ $D = 111.2 \times$ angular distance $= 111.2 \times 6°$ ≈ 667.2 km The distance between X and Y is 667.2 km.	• determines angular distance **[1 mark]** • substitutes into appropriate rule **[1 mark]** • determines distance **[1 mark]**

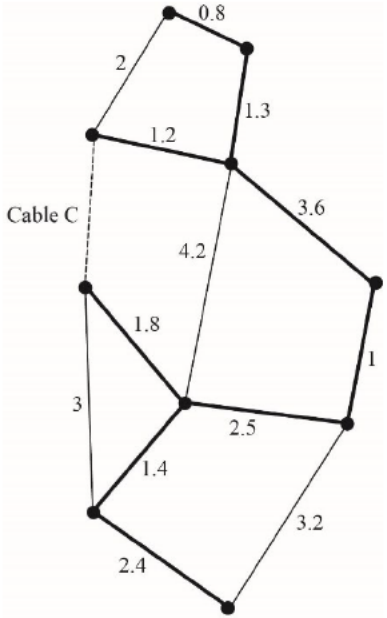
Paper 2: Short response

Q	Sample response	The response:
1	**Method 1** $\begin{array}{r} \\ \text{J} \\ \text{K} \\ \text{L} \\ \text{column reduction} \end{array}\begin{array}{ccc} \text{S} & \text{C} & \text{R} \\ 40 & 56 & 66 \\ 36 & 60 & 72 \\ 25 & 48 & 78 \\ -25 & -48 & -66 \end{array}$ $\begin{bmatrix} 15 & 8 & 0 \\ 11 & 12 & 6 \\ 0 & 0 & 12 \end{bmatrix}$ row reduction -0, -6, -0 $\begin{bmatrix} 15 & 8 & 0 \\ 5 & 6 & 0 \\ 0 & 0 & 12 \end{bmatrix}$ Number of lines needed to cover all zeroes < number of tasks 2 < 3, so continue algorithm steps. Smallest uncovered number is 5. Subtract 5 from all uncovered numbers and add 5 to number covered twice. $\begin{bmatrix} 10 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 17 \end{bmatrix}$ Number of lines needed to cover all zeroes = number of tasks 3 = 3, so assign tasks. To minimise the total relay time, assign **Jane to run**, **Knox to swim** and **Levi to cycle**. Predicted minimum total relay time $= 66 + 36 + 48$ $= 150 \text{ min}$ $= 2 \text{ h } 30 \text{ min}$	• correctly reduces each column **[1 mark]** • reduces each row **[1 mark]** • continues algorithm steps until number of lines needed to cover all zeroes equals number of tasks **[1 mark]** • assigns each athlete to complete one section **[1 mark]** • predicts minimum total relay time including units **[1 mark]**

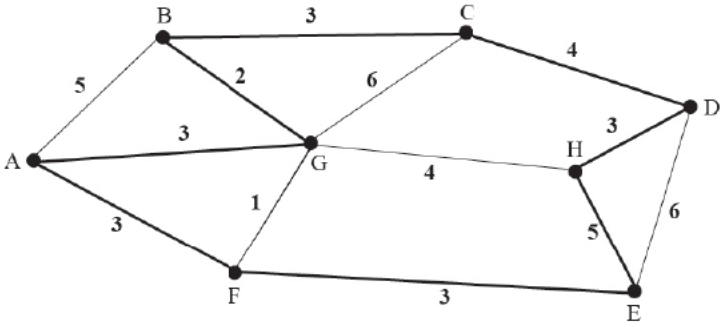
Q	Sample response	The response:
	Method 2 $$\begin{array}{cccc} & S & C & R & \text{row reduction} \\ J & 40 & 56 & 66 & -40 \\ K & 36 & 60 & 72 & -36 \\ L & 25 & 48 & 78 & -25 \end{array}$$ $$\begin{bmatrix} 0 & 16 & 26 \\ 0 & 24 & 36 \\ 0 & 23 & 53 \end{bmatrix}$$ column reduction $-0 \quad -16 \quad -26$ $$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 8 & 10 \\ 0 & 7 & 27 \end{bmatrix}$$ Number of lines needed to cover all zeroes < number of tasks 2 < 3, so continue algorithm steps. Smallest uncovered number is 7. Subtract 7 from all uncovered numbers and add 7 to number covered twice. $$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 20 \end{bmatrix}$$ Number of lines needed to cover all zeroes = number of tasks 3 = 3, so assign tasks. To minimise the total relay time, assign **Jane to run**, **Knox to swim** and **Levi to cycle**. Predicted minimum total relay time $= 66 + 36 + 48$ $= 150$ min $= 2$ h 30 min	• correctly reduces each row **[1 mark]** • reduces each column **[1 mark]** • continues algorithm steps until number of lines needed to cover all zeroes equals number of tasks **[1 mark]** • assigns athlete to complete one section **[1 mark]** • predicts minimum total relay time including units **[1 mark]**

Q	Sample response							The response:
2	Year	Season	Number of skin wounds	Yearly average	$\frac{\text{Number}}{\text{Yearly average}}$	Seasonal indices	Deseseasonalised number	• correctly determines the yearly averages **[1 mark]** • determines number/yearly average values **[1 mark]** • determines seasonal indices **[1 mark]** • determines deseasonalised numbers **[1 mark]**
	2021	Autumn	285	242	1.1776 ...	1.2307...	232	
		Winter	28		0.1157 ...	0.1090...	257	
		Spring	195		0.8057 ...	0.7982...	244	
		Summer	460		1.9008 ...	1.8620...	247	
	2022	Autumn	276	215	1.2837 ...	1.2307...	224	
		Winter	22		0.1023 ...	0.1090...	202	
		Spring	170		0.7906 ...	0.7982...	213	
		Summer	392		1.8232 ...	1.8620...	211	

Q	Sample response	The response:
3	**Method 1** Minimum spanning tree without cable C  Minimum total cable length without cable C = 0.8 + 1.3 + 1.2 + 3.6 + 1 + 2.5 + 1.8 + 1.4 + 2.4 = **16** km Minimum total cable length with cable C = 16 – 1 = **15** km Length of cable C = 15 – (0.8 + 1.3 + 1.2 + 1 + 2.5 + 1.8 + 1.4 + 2.4) = **2.6** km	• correctly identifies minimum spanning tree without cable C **[1 mark]** • correctly determines minimum total cable length without cable C **[1 mark]** • determines minimum total cable length with cable C **[1 mark]** • determines length of cable C **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
	Method 2 Minimum spanning tree without cable C  For all stations to be connected by cables, the **3.6 km cable is not required** when cable C reopens. Length of cable C = 3.6 – 1 **= 2.6** km Minimum total cable length with cable C = 0.8 + 1.3 + 1.2 + 2.6 + 1 + 2.5 + 1.8 + 1.4 + 2.4 = **15** km	• correctly identifies minimum spanning tree without cable C **[1 mark]** • correctly identifies the 3.6 km cable is not required when cable C reopens **[1 mark]** • determines length of cable C **[1 mark]** • determines minimum total cable length with cable C **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
4	Calculate correlation coefficient for each dataset. **Dataset** | **Correlation coefficient, *r*** $t_{vs}\ f$ | 0.3 $p_{vs}\ f$ | 0.8 0.8 > 0.3 The **explanatory variable** for the stronger linear association **is** ***p***, number of people fishing. $y = a + bx$ Using calculator, $a = -130$, $b = 11$ Equation in terms of given variables is $f = -130 + 11p$ $= -130 + 11 \times 50$ $= 420$ It is predicted that 420 fish will be caught.	• correctly calculates correlation coefficient for each dataset **[1 mark]** • identifies explanatory variable for stronger linear association **[1 mark]** • determines least-squares line equation for dataset with stronger linear association **[1 mark]** • substitutes value for relevant explanatory variable **[1 mark]** • predicts number of fish caught **[1 mark]**

Q	Sample response	The response:
5	Hamiltonian cycle starting and ending at A (by trial and error).  Shortest distance = 3 + 2 + 3 + 4 + 3 + 5 + 3 + 3 = **26** km Time to drive the shortest distance $t = \frac{d}{s}$ $= \frac{26}{40}$ $= 0.65 \text{ h}$ $= 39 \text{ min}$ Time to check eight stations = 8 x 15 = **120 min** Total time for patrol = 39 + 120 = **159 min** = 2h 39 min Time when guard finishes patrol = 9:00 am + 2 h 39 min = **11:39 am**	• correctly applies arithmetic sequence to determine all road lengths **[1 mark]** • identifies Hamiltonian cycle beginning at A **[1 mark]** • calculates shortest total distance **[1 mark]** • calculates total time to drive shortest distance **[1 mark]** • calculates total time spent at eight sites **[1 mark]** • calculates total time for patrol **[1 mark]** • determines time of day **[1 mark]**

Q	Sample response	The response:
6	Compare R^2 values: 0.95 < 0.96. So, **age explains a higher percentage of the account balance variation** for the **industry B** dataset. Linear model for industry A: Let x = age, y = account balance $y = bx + a$ Using calculator, b = 7910 and a = –205 520 $y = 7910x + -205\,520$ Linear model for industry B: Let x = age, y = account balance $y = bx + a$ Using calculator, b = 9570 and a = –243 440 $y = 9570x + -243\,440$ 40-year-old Leigh works in industry B; substitute x = 40 y = 9570 x **40** + –243 440 = **139 360** Tony's age = 40 + 10 = 50 Tony works in industry A; substitute x = 50 y = 7910 x **50** + –205 520 = **189 980** Difference = 189 980 – 139 360 = **50 620** The difference in account balances for Leigh and Tony is predicted to be $50 620.	• correctly identifies dataset for which age explains a higher percentage of the account balance variation **[1 mark]** • correctly determines linear model for age vs account balance for industry A data **[1 mark]** • correctly determines linear model for age vs account balance for industry B data **[1 mark]** • substitutes x = 40 into appropriate equation and calculates Leigh's current account balance **[1 mark]** • substitutes x = 50 into appropriate equation and calculates Tony's current account balance **[1 mark]** • calculates difference in current account balances for Leigh and Tony **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
7	**Method 1** Compound interest investment $A = P(1+i)^n$ $= 100\,000\left(1+\frac{3.8}{12\times 100}\right)^{5\times 12}$ $= 120\,888.66$ The balance of the investment account is $120 888.66. Perpetuity $M = A \times i$ $6000 = A \times 0.04$ $A = \frac{6000}{0.04}$ $= 150\,000$ The present value of the perpetuity needs to be $150 000. 120 888.66 < 150 000 The **compound interest investment will not provide enough money** to finance the perpetuity.	• correctly substitutes into an appropriate rule for compound interest investment **[1 mark]** • determines balance of investment account **[1 mark]** • correctly substitutes into an appropriate rule for perpetuity **[1 mark]** • determines present value of perpetuity **[1 mark]** • determines if the compound interest investment is large enough to finance the perpetuity **[1 mark]**

Q	Sample response	The response:
	Method 2 Perpetuity $M = A \times i$ $6000 = A \times 0.04$ $A = \frac{6000}{0.04}$ $= 150\,000$ The present value of the perpetuity needs to be \$150 000. Compound interest investment Find principal, P, for balance needing to be at least \$150 000. $A = P(1+i)^n$ $150\,000 = P\left(1 + \frac{3.8}{12 \times 100}\right)^{5 \times 12}$ $P = \frac{150\,000}{\left(1 + \frac{3.8}{12 \times 100}\right)^{5 \times 12}}$ $= 124\,081.11$ The principal needs to be \$124 081.11. 100 000 < 124 081.11 The **compound interest investment will not provide enough money** to finance the perpetuity.	• correctly substitutes into an appropriate rule for perpetuity **[1 mark]** • determines present value of perpetuity **[1 mark]** • correctly substitutes into an appropriate rule for compound interest investment **[1 mark]** • determines required principal for investment account **[1 mark]** • determines if the compound interest investment is large enough to finance the perpetuity **[1 mark]**