General Mathematics marking guide

External assessment 2022

Short response (95 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

For all Queensland schools

Purpose

This marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of '0' will be recorded.

Where no response to a question has been made, a mark of 'N' will be recorded.

Allow FT mark/s — refers to 'follow through', where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Paper 1 Multiple choice

Question	Response
1	B
2	C
3	A
4	D
5	B
6	C
7	C
8	A
9	B
10	D
11	D
12	C
13	D
14	A
15	A

Paper 1 Short response

Q	Sample response	The response:
16a)	$n = 42.6t + 55.4$	• correctly determines the equation of the least-squares line **[1 mark]**
16b)	Let $t = 21$ $n = 42.6(21) + 55.4$ $= 950$ The predicted number of sales is 950.	• substitutes into equation from Question 16a) **[1 mark]** • predicts number of sales **[1 mark]**

Q	Sample response	The response:
17a)	$r = 1 + \frac{i}{n}$ $1.00375 = 1 + \frac{i}{12}$ $0.00375 = \frac{i}{12}$ $i = 0.045$ Therefore, the annual interest rate is 4.5% p.a. compounding monthly.	• correctly substitutes into an appropriate rule **[1 mark]** • calculates annual interest rate **[1 mark]**
17b)	Method 1: Recursion $A_0 = 50\,000$ $A_1 = 50\,187.50$ $A_2 = 50\,375.70$ $A_3 = 50\,564.61$ $A_4 = 50\,754.23$ $A_5 = 50\,944.56$ $A_6 = 51\,135.60$ Therefore, the investment would exceed $51 000 at 6 months. Method 2: Compound interest rule $A = P(1 + i)^n$ $51\,000 = 50\,000 \times 1.00375^n$ Using trial and error: when $n = 5, A = 50\,944.56$ $n = 6, A = 51\,135.60$ Therefore, the investment will exceed $51 000 at 6 months.	• correctly uses an appropriate method **[1 mark]** • determines when the investment would exceed $51 000 **[1 mark]**

Q	Sample response	The response:
18	Arithmetic sequence $t_1 = 87$ $d = t_2 - t_1$ $= 209 - 87$ $= 122$ $t_n = t_1 + (n-1)d$ $\therefore t_n = 87 + 122(n-1)$ At 25 weeks, $n = 25$ $t_{25} = 87 + 122 \times 24$ $t_{25} = 3015$	• correctly determines t_1 **[1 mark]** • correctly determines d **[1 mark]** • uses an arithmetic sequence **[1 mark]** • predicts number of songs at 25 weeks **[1 mark]**

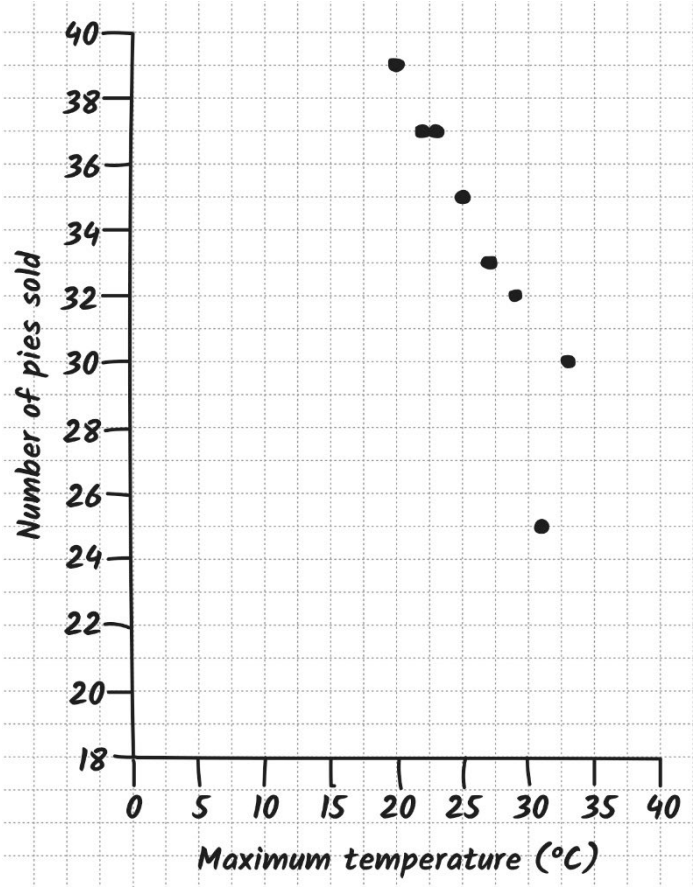
Q	Sample response	The response:
19a)	Trend — long term is positive because the amount of rainfall generally increases as time increases. Seasonality — The data is seasonal with a high 4th quarter every year.	• appropriately describes the long-term trend **[1 mark]** • appropriately describes the seasonality **[1 mark]**
19b)	y-intercept — The model predicts that 156.5 mm of rainfall was falling in the 4th quarter of 2015. Slope — On average an additional 1.763 mm of rainfall was precipitated each quarter.	• appropriately interprets the y-intercept **[1 mark]** • appropriately interprets the slope **[1 mark]**

Q	Sample response	The response:
20	Method 1 $\begin{matrix} & A & B & C & \text{Row reduction} \\ \text{Store 1} & 19 & 17 & 24 & -17 \\ \text{Store 2} & 15 & 14 & 22 & -14 \\ \text{Store 3} & 23 & 16 & 40 & -16 \end{matrix}$ $\begin{matrix} & \begin{bmatrix} 2 & 0 & 7 \\ 1 & 0 & 8 \\ 7 & 0 & 24 \end{bmatrix} \\ \text{Column reduction} & -1 \quad 0 \quad -7 \end{matrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 6 & 0 & 17 \end{bmatrix}$ Therefore for minimum distance, Store 1 must deliver to C, Store 2 must deliver to A and Store 3 must deliver to B. Minimum total distance = 24 + 15 + 16 = 55 km Method 2 $\begin{matrix} & A & B & C \\ \text{Store 1} & 19 & 17 & 24 \\ \text{Store 2} & 15 & 14 & 22 \\ \text{Store 3} & 23 & 16 & 40 \\ \text{Column reduction} & -15 & -14 & -22 \end{matrix}$ $\begin{matrix} & & \text{Row reduction} \\ \begin{bmatrix} 4 & 3 & 2 \\ 0 & 0 & 0 \\ 8 & 2 & 18 \end{bmatrix} & & \begin{matrix} -2 \\ 0 \\ -2 \end{matrix} \end{matrix}$ $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 6 & 0 & 16 \end{bmatrix}$ Therefore for minimum distance, Store 1 must deliver to C, Store 2 must deliver to A and Store 3 must deliver to B. Minimum total distance = 24 + 15 + 16 = 55 km	• correctly reduces each row **[1 mark]** • correctly reduces each column **[1 mark]** • identifies which store delivers to which location **[1 mark]** • determines minimum total distance **[1 mark]** • correctly reduces each column **[1 mark]** • correctly reduces each row **[1 mark]** • identifies which store delivers to which location **[1 mark]** • determines minimum total distance **[1 mark]**

Q	Sample response	The response:
21a)	RWPDR	• correctly identifies one cycle **[1 mark]**
21b)	The graph is semi-Eulerian because it has two odd degree vertices and the remaining vertices are even degree.	• correctly identifies the graph as semi-Eulerian **[1 mark]** • justifies the decision **[1 mark]**
21c)	$\begin{array}{c} \\ B \\ C \\ D \\ P \\ R \\ W \end{array} \begin{array}{c} \begin{array}{cccccc} B & C & D & P & R & W \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 & 2 & 0 \end{bmatrix} \end{array}$	• correctly constructs a valid adjacency matrix with same horizontal and vertical labels **[1 mark]** • correctly determines values in the adjacency matrix **[1 mark]**

Q	Sample response	The response:
22a)	Marovoay 16.1° S 46.6° E Iakora 23.1° S 46.6° E	• correctly determines the latitudes for both locations within ±0.2° **[1 mark]**
22b)	Angular distance = 23.1 – 16.1 = 7 Distance = 111.2 x angular distance = 111.2 x 7 = 778.4 Marovoay is approximately 778 km north of Iakora.	• determines angular distance **[1 mark]** • substitutes into appropriate distance formula **[1 mark]** • determines distance, including units **[1 mark]**

Q	Sample response	The response:
23a)	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{40-20}{7-2}$ $= \frac{20}{5}$ $= 4$ $p - p_1 = m(n - n_1)$ $p - 20 = 4(n - 2)$ $p - 20 = 4n - 8$ $p = 4n + 12$	• correctly determines the slope **[1 mark]** • determines equation of least-squares line **[1 mark]**
23b)	$p = 4(15) + 12$ $p = 72$ A person with 15 years experience could expect an hourly pay of $72.	• substitutes into equation from Question 23a) **[1 mark]** • predicts hourly pay, including units **[1 mark]**

Q	Sample response	The response:
24a)		• correctly identifies the explanatory and response variables **[1 mark]** • accurately plots points **[1 mark]** • formats scatterplot with appropriate scaling and labelling of axes **[1 mark]**
24b)	The association is a strong, negative, linear relationship	• describes association as – strong **[1 mark]** – negative **[1 mark]**

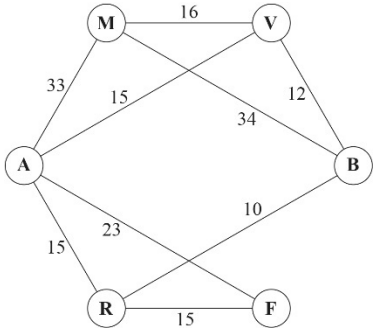
Q	Sample response	The response:
25a)	$i = \frac{2.4}{1200}$ $= 0.002$ $n = 15 \times 12$ $= 180$ $M = 993.14$ $A = M\left(\frac{1-(1+i)^{-n}}{i}\right)$ $= 993.14\left(\frac{1-(1+i)^{-180}}{i}\right)$ $= 150\,000.29$ They borrowed \$150 000.	• correctly determines the i, n and M values **[1 mark]** • substitutes into the appropriate annuity formula **[1 mark]** • determines amount of money borrowed, including units **[1 mark]**
25b)	$A_{n+1} = rA_n - R$ $A_{n+1} = \left(1 + \frac{2.4}{1200}\right)A_n - 993.14$ $A_{n+1} = 1.002A_n - 993.14$	• correctly selects the appropriate formula **[1 mark]** • determines recurrence relation **[1 mark]**

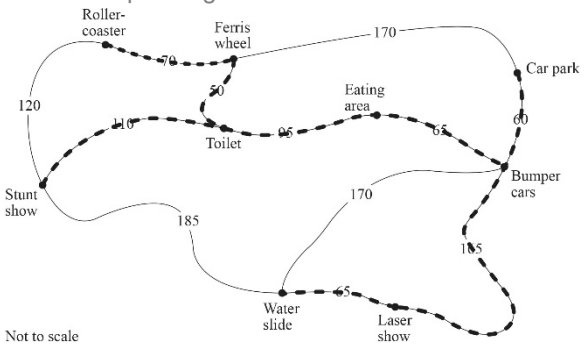
Paper 2 Short response

Q	Sample response	The response:
1	Let $x =$ autumn's seasonal index Total of seasonal indices: $1.11 + 1.42 + 0.62 + x = 4$	• correctly identifies the sum of all the seasonal indices **[1 mark]**
	$\therefore x = 0.85$	• correctly determines autumn's seasonal index **[1 mark]**
	Actual value for autumn $\text{actual value} = \text{deseasonalised value} \times \text{seasonal index}$ $= 36.4 \times 0.85$ $= 30.94$	• uses an appropriate method for determining actual value **[1 mark]**
	In autumn they had actual sales of 30 940 swimsuits.	• determines actual sales for autumn **[1 mark]**

Q	Sample response	The response:
2	Monthly amount $A = M\left(\frac{(1+i)^n - 1}{i}\right)$ $51\,343.85 = M\left(\frac{\left(1+\frac{0.086}{12}\right)^{48} - 1}{\frac{0.086}{12}}\right)$ $51\,343.85 = M \times 57.0487$ $\therefore M = 900$ Fortnightly annuity balance $A = M\left(\frac{(1+i)^n - 1}{i}\right)$ $A = 450\left(\frac{\left(1+\frac{0.079}{26}\right)^{104} - 1}{\frac{0.079}{26}}\right)$ $= 54\,941.61$ $\text{Diff} = 54\,941.61 - 51\,343.85$ $= 3597.76$ The advice that she would have been at least $3000 better off is reasonable as $3597.76 > $3000.	• correctly substitutes parameters into the appropriate annuity rule **[1 mark]** • correctly determines the monthly amount **[1 mark]** • determines value of fortnightly annuity **[1 mark]** • determines difference in annuity balances **[1 mark]** • compares values to evaluate the reasonableness of the advice **[1 mark]**

Q	Sample response	The response:
3	x parameters x = 1, 2, …, 10 $\bar{x} = 5.5$ $s_x = 3.02765$ Given $\bar{y} = 9660$ $s_y = 3010$ $r = 0.9987$ Least-squares line parameters $b = r\frac{s_y}{s_x}$ $= 0.9987 \times \frac{3010}{3.02765}$ $= 992.878$ $a = \bar{y} - b\bar{x}$ $= 9660 - 992.878 \times 5.5$ $= 4199.17$ Profit in the 11th year $y = a + bx$ $= 4199.17 + 992.878(11)$ $= 15\,120.83$ $= \$15\,121$ Predicted profit in the 11th year is $15 121.	• correctly determines $\bar{x}$ and s_x **[1 mark]** • determines b **[1 mark]** • determines a **[1 mark]** • determines 11th year profit to the nearest dollar **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
4	Network diagram  Shortest path = M-V-B-R-F = 16+12+10+15 = 53 New road length = 53 – 4 = 49 The new road will be 49 km long.	• correctly represents the connected towns as a network **[1 mark]** • correctly includes lengths on the network **[1 mark]** • determines shortest current path **[1 mark]** • identifies length of shortest path **[1 mark]** • determines new road length **[1 mark]**

Q	Sample response	The response:
5	Minimum spanning tree  Total length of removed paths $= 120 + 185 + 170 + 170$ $= 645$ Annual savings $= 645 \times 214$ $= 138\,030$ ∴ The manager is correct and they will save more than \$138 000 if they remove the unnecessary paths to all nine key locations.	• correctly identifies minimum spanning tree **[1 mark]** • correctly calculates total length of the removed paths **[1 mark]** • calculates annual savings **[1 mark]** • compares values to evaluate the reasonableness of the belief **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
6	Slope sequence $-0.8, 0.4, -0.2, \ldots$ This forms a geometric sequence with $t_1 = -0.8$ and $r = -0.5$. $\therefore t_n = -0.8 \times (-0.5)^{(n-1)}$ y-intercept sequence 1.2, 2.7, 4.2, … This forms an arithmetic sequence with $t_1 = 1.2$ and $d = 1.5$. $\therefore t_n = 1.2 + (n-1) \times 1.5$ The equation for Line 5 $m = -0.8 \times (-0.5)^4$ $= -0.05$ $c = 1.2 + 4 \times 1.5$ $= 7.2$ $\therefore y_5 = -0.05x + 7.2$ Solve simultaneously $y_1 = y_5$ $\therefore -0.8x + 1.2 = -0.05x + 7.2$ $\therefore -0.75x = 6$ $\therefore x = -8$ sub into y_1 $\therefore y = -0.8(-8) + 1.2$ $\therefore y = 7.6$ The intersection point is $(-8, 7.6)$.	• correctly determines the geometric sequence parameters for the slopes **[1 mark]** • correctly determines the arithmetic sequence parameters for the y-intercepts **[1 mark]** • determines slope for Line 5 **[1 mark]** • determines y-intercept for Line 5 **[1 mark]** • determines x-coordinate of intersection point **[1 mark]** • determines y-coordinate of intersection point **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

Q	Sample response	The response:
7	Ship's travel time from X to Tarawa $\text{speed} = \frac{\text{distance}}{\text{time}}$ $50 = \frac{1350}{\text{time}}$ $\therefore \text{time} = 27 \text{ hours}$ Ship's travel time is 27 hours.	• correctly substitutes into an appropriate rule **[1 mark]** • correctly calculates ship's travel time of 27 hours **[1 mark]**
	Time difference between Tarawa (GMT +12) and X (GMT –12) +12 – (–12) = 24 hours ∴Tarawa is 24 hours ahead of X.	• correctly determines the time difference between Tarawa and X **[1 mark]**
	Time difference between Queensland (GMT +10) and Tarawa (GMT +12) +10 – (+12) = -2 hours ∴ Queensland is 2 hours behind Tarawa.	• correctly determines the time difference between Queensland and Tarawa **[1 mark]**
	Tarawa time at time of message = 6:12 am Wednesday + 24 hours = 6:12 am Thursday Tarawa time when ship arrives in Tarawa = 6:12 am Thursday + 27 hours = 9:12 am Friday Queensland time when ship arrives in Tarawa = 9:12 am Friday – 2 hours	• appropriately applies ship's travel time and both time differences to 6:12 am Wednesday **[1 mark]**
	= 7:12 am Friday	• determines time and day in Queensland at time of ship's arrival in Tarawa **[1 mark]** • shows logical organisation, communicating key steps **[1 mark]**

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