General Mathematics subject report

2021 cohort

February 2022

Queensland Government
QCAA Queensland Curriculum & Assessment Authority
220029
For all Queensland schools

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Introduction

Despite the challenges brought about by the COVID-19 pandemic, Queensland's education community can look back on 2021 with satisfaction at having implemented the first full assessment cycle in the new Queensland Certificate of Education (QCE) system. That meant delivering three internal assessments and one external assessment in each General subject.

This report analyses that cycle — from endorsing summative internal assessment instruments to confirming internal assessment marks, and designing and marking external assessment. It also gives readers information about:

- applying syllabus objectives in the design and marking of internal and external assessments
- patterns of student achievement.

The report promotes continuous improvement by:

- identifying effective practices in the design and marking of valid, accessible and reliable assessments
- recommending where and how to enhance the design and marking of valid, accessible and reliable assessment instruments
- providing examples of best practice where relevant, possible and appropriate.

Audience and use

This report should be read by school leaders, subject leaders and teachers to:

- inform teaching and learning and assessment preparation
- assist in assessment design practice
- assist in making assessment decisions
- help prepare students for external assessment.

The report is publicly available to promote transparency and accountability. Students, parents, community members and other education stakeholders can learn about the assessment practices and outcomes for General subjects (including alternative sequences (AS) and Senior External Examination (SEE) subjects, where relevant) and General (Extension) subjects.

Report preparation

The report includes analyses of data and other information from endorsement, confirmation and external assessment processes. It also includes advice from the chief confirmer, chief endorser and chief marker, developed in consultation with and support from QCAA subject matter experts.

Subject data summary

Subject completion

The following data includes students who completed the General subject or AS.

For the purposes of this report, while the 2021 summative units for the AS are AS units 1 and 2, this information will be included with the General summative Units 3 and 4.

Note: All data is correct as at 17 December 2021. Where percentages are provided, these are rounded to two decimal places and, therefore, may not add up to 100%.

Number of schools that offered the subject: 443.

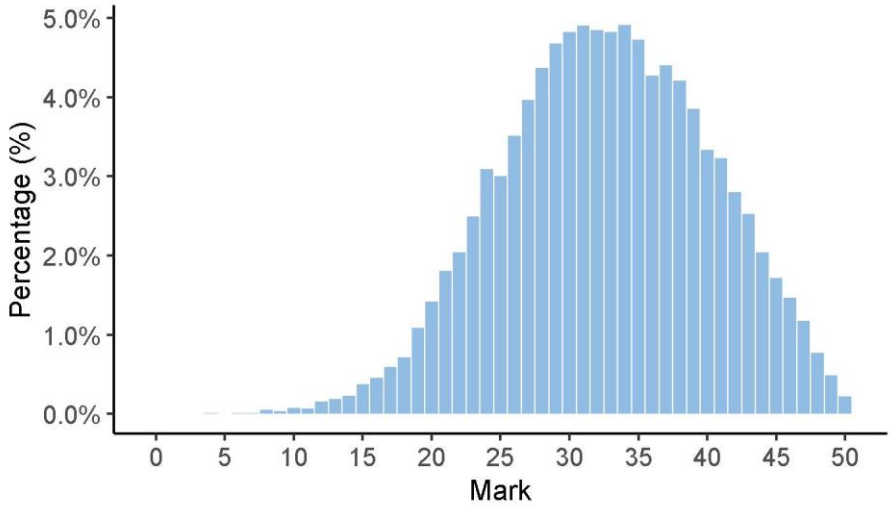
Completion of units	Unit 1	Unit 2	Units 3 and 4
Number of students completed	22777	22109	20280

Units 1 and 2 results

Number of students	Satisfactory	Unsatisfactory
Unit 1	20262	2515
Unit 2	19253	2856

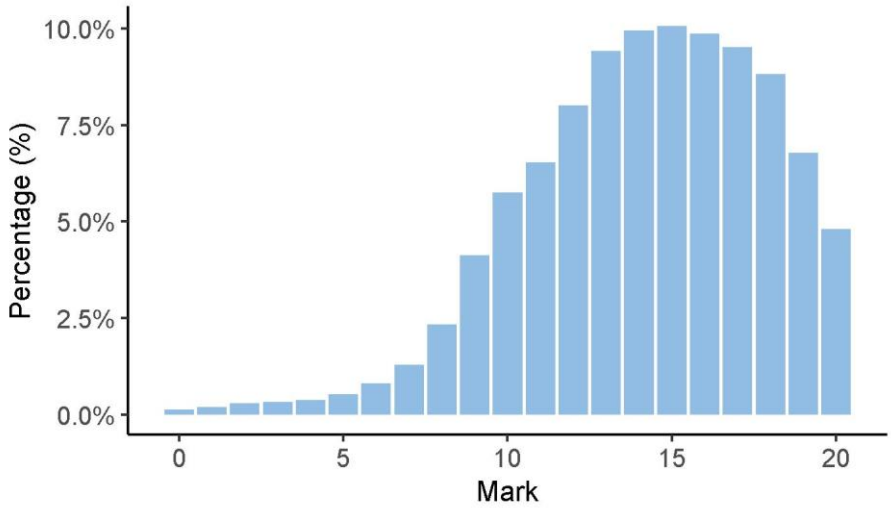
Units 3 and 4 internal assessment (IA) results

Total marks for IA

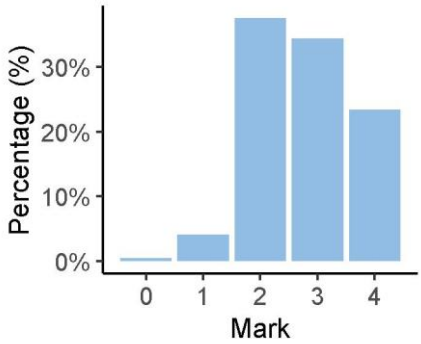


IA1 marks

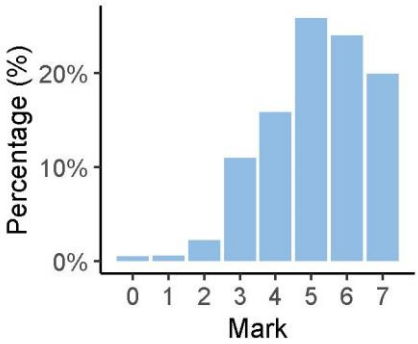
IA1 total



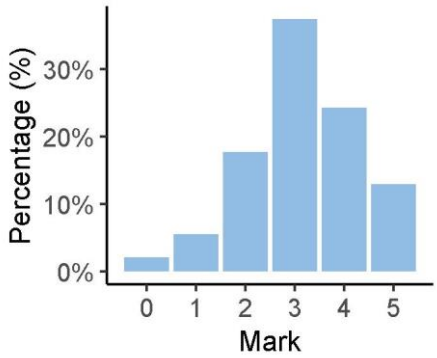
IA1 Criterion: Formulate



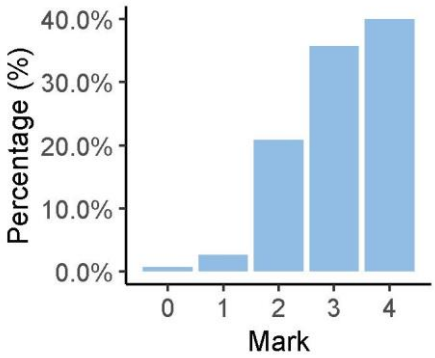
IA1 Criterion: Solve



IA1 Criterion: Evaluate and verify

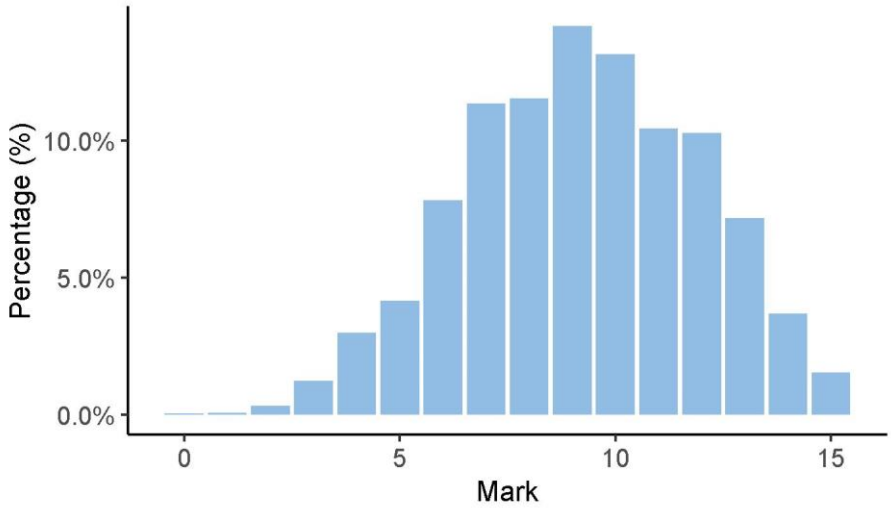


IA1 Criterion: Communicate

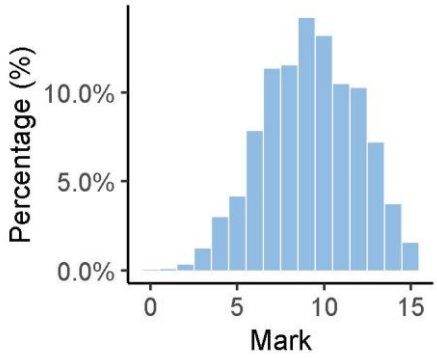


IA2 marks

IA2 total

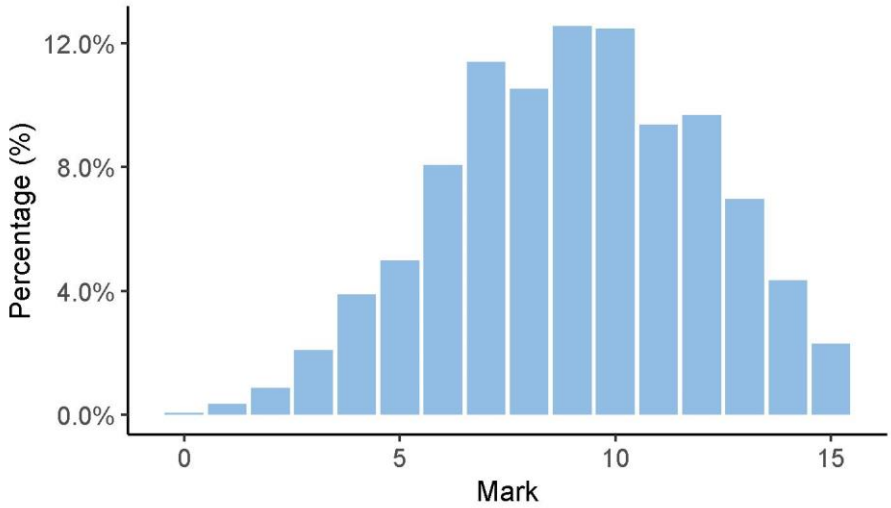


IA2 Criterion: Foundational knowledge and problem-solving

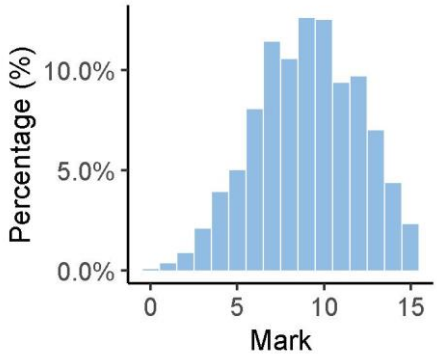


IA3 marks

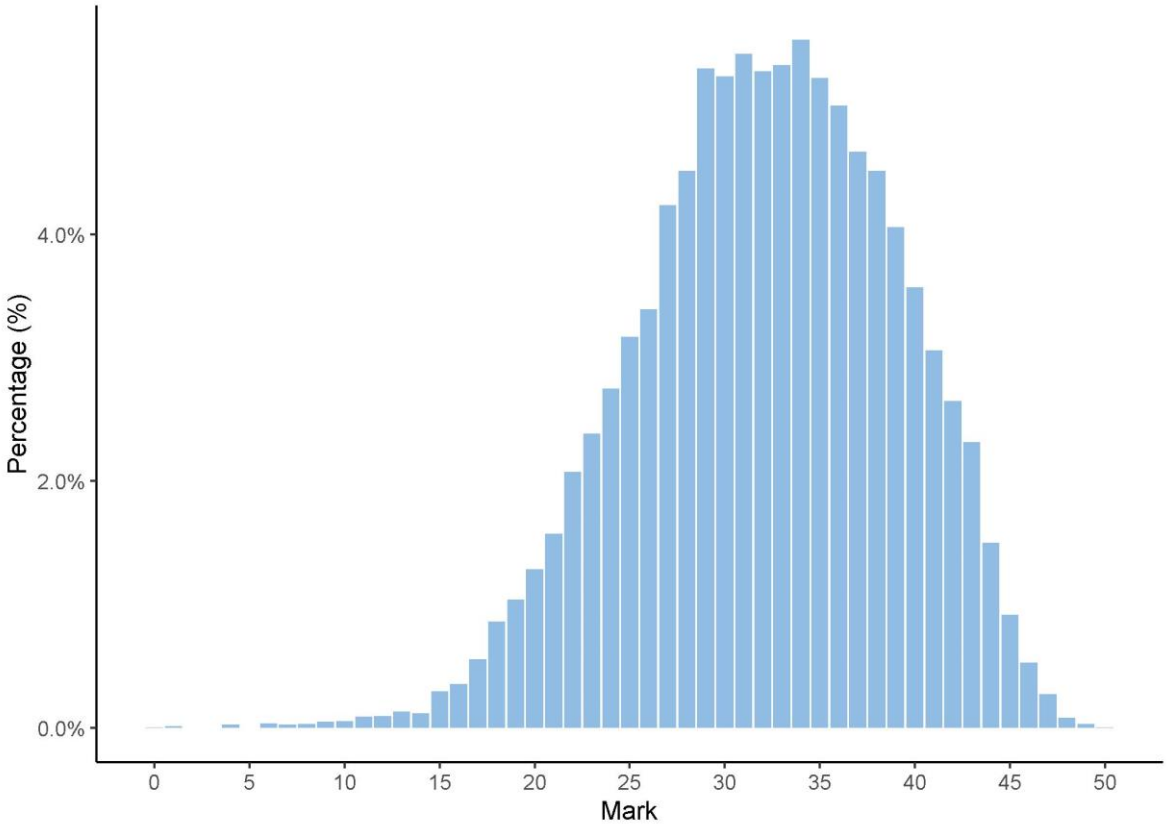
IA3 total



IA3 Criterion: Foundational knowledge and problem-solving

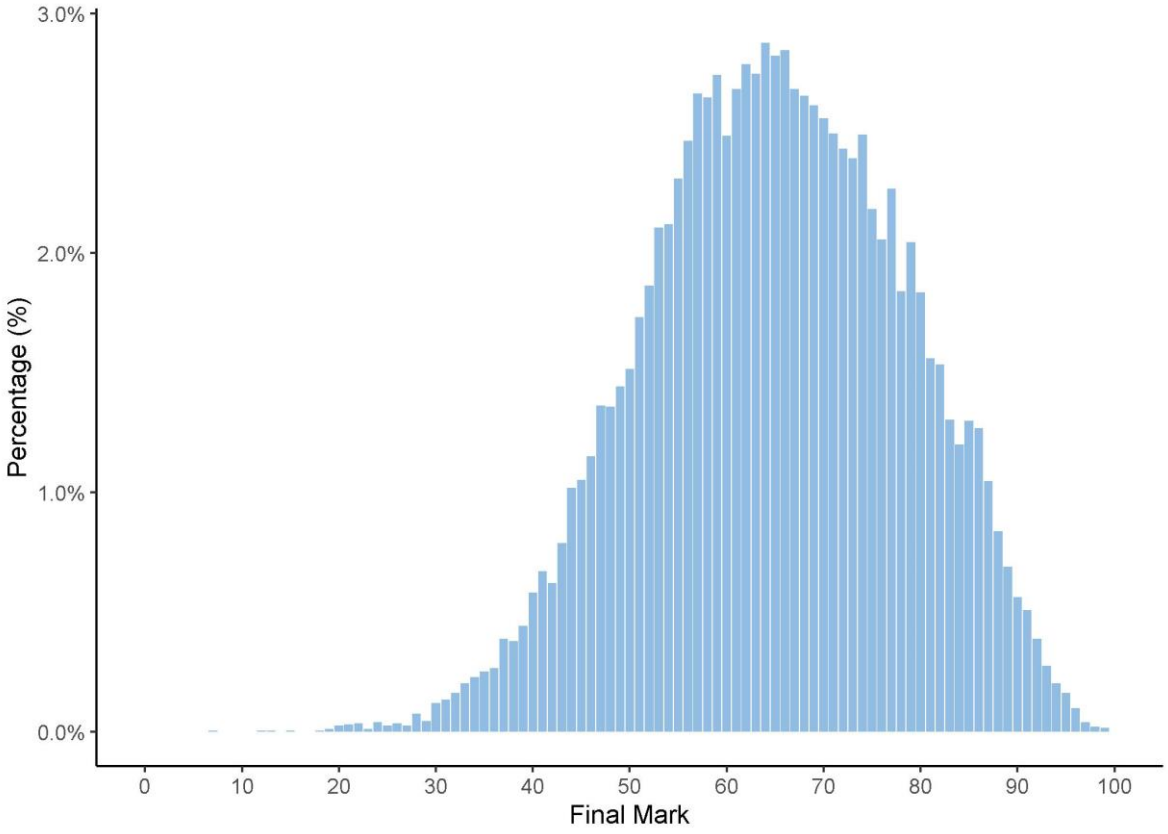


External assessment (EA) marks



Final subject results

Final marks for IA and EA



Grade boundaries

The grade boundaries are determined using a process to compare results on a numeric scale to the reporting standards.

Standard	A	B	C	D	E
Marks achieved	100–82	81–64	63–44	43–20	19–0

Distribution of standards

The number of students who achieved each standard across the state is as follows.

Standard	A	B	C	D	E
Number of students	2325	8658	8163	1130	7

Internal assessment

The following information and advice pertain to the assessment design and assessment decisions for each IA in Units 3 and 4. These instruments have undergone quality assurance processes informed by the attributes of quality assessment (validity, accessibility and reliability).

Endorsement

Endorsement is the quality assurance process based on the attributes of validity and accessibility. These attributes are categorised further as priorities for assessment, and each priority can be further broken down into assessment practices.

Data presented in the Assessment design section identifies the reasons why IA instruments were not endorsed at Application 1, by the priority for assessments. An IA may have been identified more than once for a priority for assessment, e.g. it may have demonstrated a misalignment to both the subject matter and the assessment objective/s.

Refer to the quality assurance tools for detailed information about the assessment practices for each assessment instrument.

Percentage of instruments endorsed in Application 1

Number of instruments submitted	IA1	IA2	IA3
Total number of instruments	443	443	442
Percentage endorsed in Application 1	64%	27%	46%

Confirmation

Confirmation is the quality assurance process based on the attribute of reliability. The QCAA uses provisional criterion marks determined by teachers to identify the samples of student responses that schools are required to submit for confirmation.

Confirmation samples are representative of the school's decisions about the quality of student work in relation to the ISMG and are used to make decisions about the cohort's results. If further information is required about the school's application of the ISMG to finalise a confirmation decision, the QCAA requests additional samples.

Schools may request a review where an individual student's confirmed result is different from the school's provisional mark in one or more criteria and the school considers this result to be an anomaly or exception.

The following table includes the percentage agreement between the provisional marks and confirmed marks by assessment instrument. The Assessment decisions section of this report for each assessment instrument identifies the agreement trends between provisional and confirmed marks by criterion.

Number of samples reviewed and percentage agreement

IA	Number of schools	Number of samples requested	Number of additional samples requested	Percentage agreement with provisional marks
1	440	3502	703	84.55%
2	440	3159	0	98.64%
3	439	3101	0	98.63%

Internal assessment 1 (IA1)

Problem-solving and modelling task (20%)

The problem-solving and modelling task must use subject matter from at least one of the following topics in Unit 3:

- Topic 1: Bivariate data analysis
- Topic 2: Time series analysis
- Topic 3: Growth and decay in sequences.

The problem-solving and modelling task is an assessment instrument developed in response to a mathematical investigative scenario or context. It requires students to respond with a range of understanding and skills, such as using mathematical language, appropriate calculations, tables of data, graphs and diagrams.

Students must provide a response to a specific task or issue that is set in a context that highlights a real-life application of mathematics. The task requires students to use relevant stimulus material involving the selected subject matter and must have sufficient scope to allow students to address all the stages of the problem-solving and modelling approach. Technology must be used.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	61
Authentication	25
Authenticity	33
Item construction	12
Scope and scale	32

*Each priority might contain up to four assessment practices.

Total number of submissions: 443.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- posed realistic contexts that were accessible and relevant to students, e.g. information relating to the use of smartphones or internet service providers (ISPs)
- established clear instructions to students about the requirements of the task, including identification of the topic/s being assessed
- provided opportunity for students to develop a unique response by providing an open-ended task, i.e. students made choices about how to use the data, what model to develop, and what concepts and techniques were relevant to investigate a relationship and/or solve the problem.

Practices to strengthen

It is recommended that assessment instruments:

- focus on subject matter within Unit 3, Topics 1, 2 and/or 3
- avoid scaffolding or task instructions that indicate to students how to solve the problem, as this interferes with students' ability to address all the stages of the problem-solving and modelling approach and to provide a unique, authentic response
- are sufficiently different from textbook practice assessments and QCAA sample assessment instruments so as not to compromise the authenticity of student responses.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	5
Language	39
Layout	2
Transparency	12

*Each priority might contain up to four assessment practices.

Total number of submissions: 443.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- were free from punctuation, grammatical, spelling and typographical errors
- provided a specific task or issue that was written in a straightforward manner and explicit about the nature of the problem
- used appropriate language, diagrams and images
- contained a scenario or context that was directly related to the task and accessible to students, e.g. environmental sustainability issues, such as carbon dioxide levels.

Practices to strengthen

It is recommended that assessment instruments:

- include clear, concise instructions and cues using language linked to the characteristics in the ISMG
- avoid repetition and contradictions in the context, task and scaffolding
- are free from spelling, grammatical and punctuation errors.

Additional advice

- It is recommended that schools develop problem-solving and modelling tasks significantly different to the QCAA sample tasks. This allows students to authentically address all the stages of the problem-solving and modelling approach (Syllabus section 4.7.1) to
  - translate all aspects of the problem themselves
  - be discerning in their application of mathematical concepts and techniques
  - make their own judgments about ideas and consider the strengths and limitations of their responses
  - organise and 'develop' the response, appropriate to the genre, which can be read independently of the task sheet.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Formulate	87.73%	10.91%	0.68%	0.68%
2	Solve	92.27%	5.68%	2.05%	0%
3	Evaluate and verify	91.82%	7.5%	0.68%	0%
4	Communicate	97.27%	1.82%	0.91%	0%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- judgments were made about the proficiency level for the use of technology, i.e. the descriptor 'accurate and appropriate use of technology' was well identified from the other descriptors such as 'use of technology' and 'superficial use of technology'
- students justified procedures and decisions by explaining mathematical reasoning based on the documentations of their assumptions and observations

- incorporating the correct use of technical vocabulary, procedural vocabulary and conventions added detail and fullness to the student's response
- appropriate judgments were made about the organisation and structure of the response with respect to its genre.

Samples of effective practices

The following are excerpts from responses that illustrate the characteristics for the criterion at the performance level indicated. The excerpts may provide evidence of more than one criterion. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

These student response excerpts have been included:

- to demonstrate 'documentation' of appropriate assumptions
- to provide evidence of accurate documentation of relevant observations
- to demonstrate the evaluation of the reasonableness of solutions by considering the results, assumptions and observations
- to provide documentation of the relevant strengths and limitations of the solution and/or model
- to provide evidence of the justification of decisions made using mathematical reasoning.

Formulate (3–4 marks) • accurate documentation of relevant observations	**Excerpt 1** • It is assumed that the variables are numeric, the association is linear and there are no outliers present in the data. This will ensure that the Correlation Coefficient (r) and Coefficient Determination (R) are suitable to draw conclusions on the measure of strength as identified by (Jones , Evans, Lipson, & Staggard, 2019) these are the three criteria needed to be met. • It is assumed that the calculation to find the missing data set for the year of 1946 will not affect the reliability or reasonableness of the findings. As seen in appendix 1 there was no data recorded, resulting in the gap being filled with the average of 1945 (31.8°C) and 1947 (31.9°C). Therefore making the average temperature for 1946, 31.85°C. $1946\bar{x} = \frac{31.8 + 31.9}{2}$ $= 31.85\ °C$ • It is assumed the time (years) will be the explanatory variable and the climate (°C) will be the response variable, this is due to the fact that a time series analysis requires the explanatory variable to be of a time value. This will then determine whether the response variable has an increasing or decreasing trend, needed to verify *The State of Climate Report* (BOM,2016).
Formulate (3–4 marks) • documentation of appropriate assumptions	• For each Y value there is an X value which is listed in time order, allowing for a time series analysis to be forced upon the data to determine whether there is a trend between the two variables. • The mean climate being recorded every year will comprise a time series analysis as it is measured at an equally spaced interval which allows for a trend to be determined. • Raw data of average climates are collected from www.bom.gov.au/climate/data therefore data is reliable and accurate ensuring any models and conclusions made on the data will be as accurate as possible.

Evaluate and verify (4–5 marks)

- evaluation of reasonableness of solutions by considering the results, assumptions and observations
- justification of decisions made using mathematical reasoning

Excerpt 2

3. Discussion

For both road areas in dry conditions, a linear relationship was present between the variables. The built-up area scatterplot demonstrated a moderate, positive, linear association, deduced by the correlation coefficient 0.6105. The Coefficient of determination however identified only 37% of the variation in stopping distance is explained by the variation in driving speeds, indicating driving speed is not very explanatory of the variation in the stopping distance. The linear equation, developed as y=−3.524+0.6633x, indicated that, if the Driving speed of the vehicle was 0km/h, the stopping distance would be a predicted -3.524m. As the speed of the vehicle increased, however, the stopping distance will increase at a corresponding 0.6633m. This model allowed the calculation of a suitable following distance for a built-up area in dry conditions is an estimated 28.75m from the car in front. Similarly, the scatterplot for a motorway in dry conditions demonstrated another moderate, positive linear association between the two variables provided by the correlation coefficient of 0.6969. The coefficient of determination identified that only 48% of the variation in stopping distance is explained by the variation in driving speeds in a dry motorway. This relatively low R^2 for both roadways in dry conditions is indicative of the relationship of low explanation of driving speed in the variation of stopping distance in dry conditions (Frost, 2020). However, the linear relationship between the variables in dry conditions is reinforced by the residual plots, which both demonstrate a random pattern and distribution. The regression equation of y=−83.192+1.6829x identified that, if the driving speed on a motorway during dry conditions was 0km/h, the stopping distance would be a predicted -83.192m and would increase at a rate of 1.6829m as driving speed increased. Finally, the motorway model identified that the safe following distance on a dry motorway is an estimated 49m.

For wet road conditions, similar linear relationships were identified. The scatterplot for a built-up area demonstrated a moderate, positive, linear association with a correlation coefficient of 0.5599. The coefficient of determination identified that only 35% of the variation in stopping distance is explained by the variation in driving speeds in this area. The residual plot, however, demonstrated a randomly distributed pattern of residual values, indicating a strong linear relationship. The regression equation for the data, $y = -15.4213 + 0.8985x$, identified that, if the speed of the vehicle was 0km/h, the stopping distance would be -15.4213m, and would increase at a rate of 0.8985m as the speed increased. This predicted value is 0.2217m higher than the estimated slope of the line for the dry built-up area, coinciding with the observations that reduced friction contributes to a longer stopping distance (Queensland Government, 2020). The safe following distance was determined to be 56.69m. For a wet motorway, the scatterplot and relationship demonstrated an unprecedented strong, positive linear association with an extremely high correlation coefficient value of 0.8615. This contributes to the coefficient of determination of 74%, demonstrating that most of the variation in stopping distance is explained by the variation in driving speeds. This R^2 value is 39% higher than the variability in the wet built-up area, representing the strongest linear model in this investigation. Furthermore, the regression equation of $y = -74.921 + 1.7247x$ identified that, at a speed of 0km/h, the stopping distance would be a predicted 74.921m and would increase at a rate of 1.7247m as the driving speed increased. This relationship between the variables additionally indicated that stopping distance increases a rate of 0.058m higher than dry motorways. The linear relationship of both models is reinforced by the residual plots, which equally demonstrated a random residual pattern. Finally, the model identified that the safe following distance on a wet motorway is an estimated 49m.

While this mathematical model allows the identification of a suitable following distance for each roadway and driving condition, it does not provide a recommendation for safe driving speed. The most appropriate recommendation is to follow the signs provided by the state government. In Queensland, for built-up areas, the legal driving speed is 50km/h, and for motorways, 80-100km/h (Queensland Government, 2020). For wet weather conditions, the government recommends that the driver lower their speeds from these limits by at least 10km/h (Queensland Government, 2020).

Evaluate and verify (4–5 marks) • documentation of relevant strengths and limitations of the solution and/or model	**4. Evaluation** Strengths: A major strength of this model is the conformity of the data to the assumptions of linear regression. Due to the absence of outliers, a linear relationship between the variables, homoscedasticity of the data, and random residual plots, the linear regression model was the appropriate method to investigate the bivariate relationship between driving speed and stopping distance (Stat Trek, 2020).. Instead of sole reliance on R^2 to determine the statistical variability, these mathematical models employed the use of residual plots to interpolate the reliability and accuracy of the models determined. Additionally, by maintaining a consistent speed bracket for both road areas that represents the speeds of the state, the models' recommendations apply effectively (Queensland Government, 2020). Furthermore, as the recommendations of this model are founded upon reliable road safe sources, the model recommendations coincide with the governments' guidelines, validifying their suitability in a modern context. Additionally, the provision of reliable data from the website 'It's a Drag' allowed the investigation stopping distance at various speeds in various driving conditions (Education Services Australia, 2016). Limitations: The linear regression model has several limitations that impacts its use in the modern context. Because there is no statistical basis to assume that the linear regression model will apply to future driving speeds and subsequent stopping distances, the limitation to only current sample data limits the model's suitability for proving future relationships (Aivaz Kamer-Ainur, 2020). If speeds increased and organisations needed to apply this model to recommend safe driving speed and following distance, the model will be statistically unreliable (Aivaz Kamer-Ainur, 2020). The relationships identified by the two variables are also heavily hindered by this investigations' inadequate sample space. As the sample space for each roadway and condition is limited to 20 data points obtained from the one website and the same person, the margin of error and power of the study is reduced in determining the relationship between the variables (Deziel, 2018). This limitation is exacerbated by the potential unreliability of the simulator, which could potentially invalidate the mathematical relationships identified in this report (Deziel, 2018). Additionally, by limiting the investigation to only two external conditions, set speeds, and not assessing other conditions such as road type or vehicle, this solutions' practicality in a wide range of road conditions that can impact driving speed and stopping distance is very limited.

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- within both the Formulate criterion and the Evaluate and verify criterion, a clear distinction is made between 'statements' and 'documentation' of observations, assumptions and/or strengths and limitations. In both these criteria
  - responses that clearly demonstrate 'documentation' of assumptions provide evidence of the logic of the mathematisation process, as identified in the flowchart (Syllabus section 4.6.1). While this can often take the form of references or citations, it can also be achieved with coherent reasoning
  - for 'documentation' of observations, supporting evidence could include the source of the observations, the reasoning for the method used to collect the data, or the validity of the observations with respect to the proposed solution and/or model
  - for 'documentation' of strengths and limitations, there needs to be supporting evidence as to why or how the elements identified are strengths or limitations. Where necessary, this will involve refinement of a model and/or solution
- within the Evaluate and verify criterion, a clear distinction is made between 'statements' and 'evaluation of reasonableness'. In this criterion
  - responses that clearly demonstrate an 'evaluation' include consideration of the original real-world problem to be solved, the effects of making the assumptions used to mathematise the problem as well as both strengths and limitations of the model and/or results

  - an ‘evaluation of reasonableness’ can take many forms but should involve making an appraisal or judgments supported by reasoning. It could include use of technology, further analysis of data, or algebraic calculations to refine the output of their model and/or check that it provides a valid solution to the problem

- within the Communicate criterion, a ‘coherent’ response has a logically consistent structure consisting of parts that connect harmoniously. In this criterion
  - responses that clearly demonstrate ‘coherent’ organisation can be read independently of the task sheet and use appropriate structure (e.g. headings and subheadings for a report, labels on diagrams or graphs) and language to make the formulation, solving and evaluation of a solution easy to perceive, understand and interpret. The reader should be free from confusion when following the response.

Additional advice

- It is advisable that student responses are checked against the QCAA high-level annotated sample as part of the process of authenticating student work.
- Spreadsheeting formulas should be shown to demonstrate the ‘use of technology’ (i.e. clearly stating the formulas used in the development of the spreadsheet), otherwise a table of values alone does not demonstrate technology use.

Internal assessment 2 (IA2)

Examination — short response (15%)

The examination assesses the application of a range of cognitions to a number of items using a representative sample of subject matter from all Unit 3 topics. Where relevant, this assessment should focus on subject matter not assessed in the problem-solving and modelling task. Subject matter from Units 1 and 2 is considered assumed knowledge. Student responses must be completed individually, under supervised conditions, and in a set timeframe (120 minutes plus 5 minutes perusal). The percentage allocation of marks must match the degree of difficulty specifications: ~20% complex unfamiliar; ~20% complex familiar; ~60% simple familiar.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	335
Authentication	0
Authenticity	27
Item construction	25
Scope and scale	112

*Each priority might contain up to four assessment practices.

Total number of submissions: 443.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- explicitly provided opportunities to address all objectives
- assessed a selection of subject matter that accurately reflected the intended learning of all Unit 3 topics
- were made up of an appropriate number of questions that matched the degree of difficulty specifications in the syllabus and allowed students to respond within the time conditions
- included a correct marking scheme that indicated clearly how marks have been allocated; this assisted schools to check the scope and scale of the assessment and promoted consistency in the awarding of marks.

Practices to strengthen

It is recommended that assessment instruments:

- assess all objectives; in particular, Assessment objective 4: evaluate the reasonableness of solutions
- provide appropriate questions that are complex and unfamiliar
- use questions that are sufficiently different from textbook questions and QCAA sample questions to ensure responses are authentic and not rehearsed.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	58
Language	109
Layout	41
Transparency	39

*Each priority might contain up to four assessment practices.

Total number of submissions: 443.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- clearly displayed identifiable characteristics in the simple familiar questions
- had limited use of bold and italics
- featured correct language conventions, and were free of punctuation, grammatical, spelling and typographical errors.

Practices to strengthen

It is recommended that assessment instruments:

- contain correct mathematical, statistical and everyday language, in addition to conventions and notations
- are reviewed using the Print Preview button, prior to submission, to ensure that the layout is clear and not distracting (e.g. avoiding misaligned text), and that items, such as tables, appear in their entirety on the page.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	98.64%	0.45%	0%	0.91%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- the marking schemes were prepared at the time of writing the examination to ensure questions were 'worked'
- schools used cross-marking strategies to ensure marking schemes were consistently applied across the cohort, especially where multiple teachers are timetabled across classes
- marking schemes clearly outlined where the marks were allocated
- the marking of the student responses followed the school supplied marking guide
- marks were totalled correctly.

Samples of effective practices

The following are excerpts from responses that illustrate the characteristics for the criterion at the performance level indicated. The excerpts may provide evidence of more than one criterion. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

These student response excerpts have been included:

- to highlight a student response that achieved full marks, demonstrating authoritative and accurate command of mathematical concepts and techniques, and use of mathematical reasoning to correctly justify procedures and decisions in complex situations
- to demonstrate a fluent application of the use of the most appropriate mathematical techniques in a complex unfamiliar situation
- to provide evidence of logical organisation communicating key steps.

Foundational knowledge and problem-solving (18 marks)

- accurate command of mathematical concepts
- mathematical reasoning to correctly justify procedures

Excerpt 1

The prices of two financial quantities were provided in a tabular form:

a) Assuming a linear association, calculate the correlation coefficient to 2 decimal places. Using calculator

$r \approx 0.78$

b) Justify the procedure used by listing your calculator input.

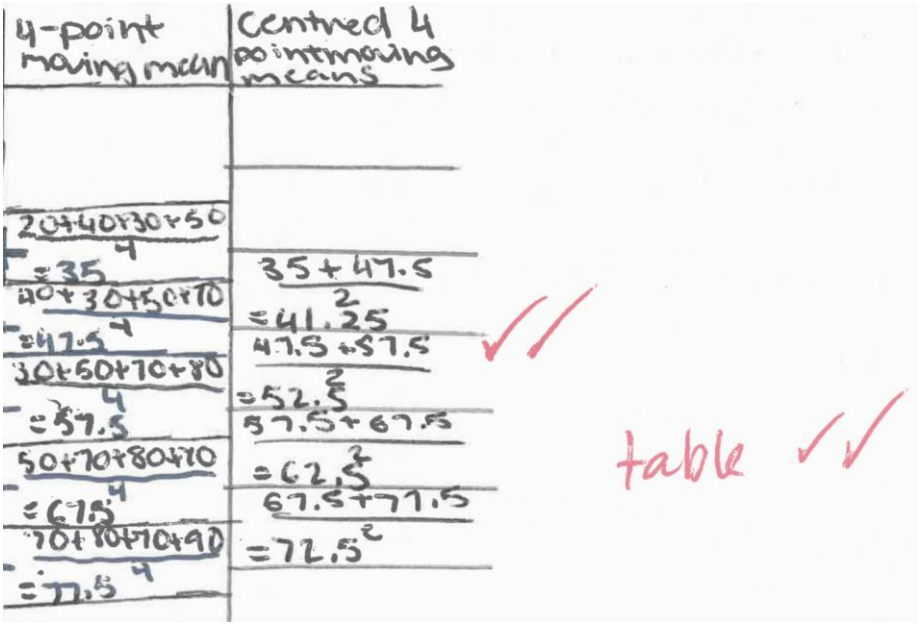
1. MODE → 2: STAT → 2: A+BX
2. | | x | y |
   |---|---|---|
   | 1 | | |
   | 2 | | |
   | 3 | | |

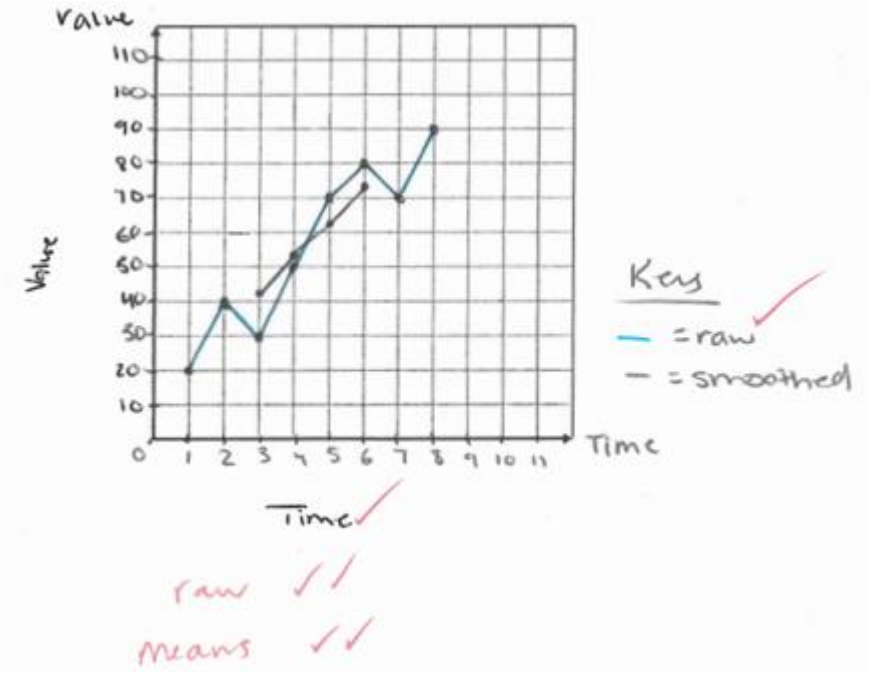
   HT share prices where entered into left column (x) and Q2 share prices were entered into right column (y)
3. SHIFT 1 → 5: Reg → 3: r → gives r value.

c) Describe the correlation between the two shares.

There is a moderate, positive, linear correlation between the HJ share prices and the Q2 share prices.

Sketch the graphs of the raw and smoothed data, using the centred 4-point moving means on the dataset provided:





The 2nd and 5th terms of a geometric sequence are 48 and $\frac{3}{4}$ respectively.

a) Find the first term and common ratio.

① $t_2 = 48$ $t_5 = \frac{3}{4}$

② Using $t_n = t_1 r^{(n-1)}$ for t_2

$48 = t_1 r$

$\therefore t_1 = \frac{48}{r}$ [1]

③ Using $t_n = t_1 r^{(n-1)}$ for t_5

$\frac{3}{4} = t_1 r^4$

$\therefore t_1 = \frac{3}{4r^4}$ [2]

④ sub [1] in [2]

$\frac{48}{r} = \frac{3}{4r^4}$

$\frac{48}{r} \times r^4 = \frac{3}{4}$

$\frac{48r^4}{r} = \frac{3}{4}$

$48r^3 = \frac{3}{4}$

$r^3 = \frac{3}{4} \div 48$

$r^3 = \frac{1}{64}$

$\therefore r = \frac{1}{4}$

⑤ sub $r = \frac{1}{4}$ in [1]

$t_1 = \frac{48}{1/4}$

$\therefore t_1 = 192$

⑥ $\therefore t_1 = 192$ and $r = \frac{1}{4}$ or 0.25

b) Evaluate the reasonableness of your solution.

For $t_n = t_1 r^{(n-1)}$, where $t_1 = 192$ and $r = 0.25$

$t_n = 192 \times 0.25^{(n-1)}$

sub in $t_2 = 48$ $\therefore t_n = 48$ and $n = 2$

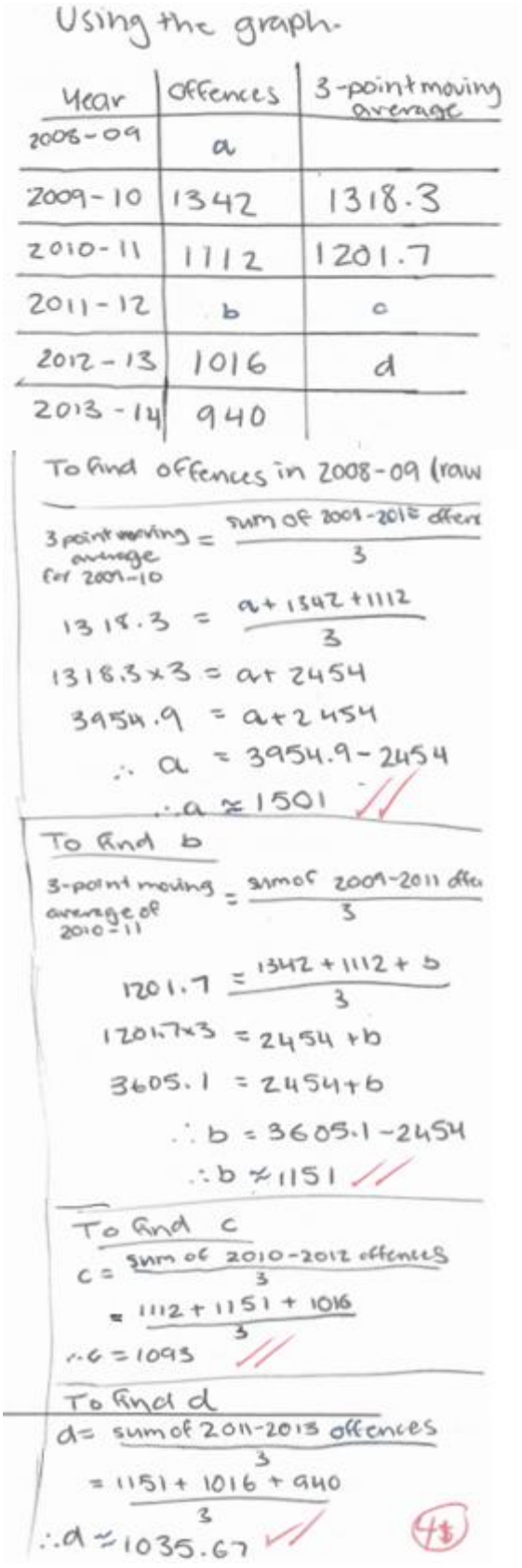
$\therefore 48 = 192 \times 0.25$

$48 = 48$

$\therefore$ LHS = RHS

$\Rightarrow$ The solution is reasonable.

The following partially completed graph shows the raw data and the three-point moving averages for two properties over a period of time. Use your mathematical reasoning to complete the graph.



Using the graph.

Year	Offences	3-point moving average
2008-09	a	
2009-10	1342	1318.3
2010-11	1112	1201.7
2011-12	b	c
2012-13	1016	d
2013-14	940	

To find offences in 2008-09 (raw

3 point moving average for 2009-10 = sum of 2009-2011 offen / 3

$1318.3 = \frac{a + 1342 + 1112}{3}$

$1318.3 \times 3 = a + 2454$

$3954.9 = a + 2454$

$\therefore a = 3954.9 - 2454$

$\therefore a \approx 1501$

To find b

3-point moving average of 2010-11 = sum of 2009-2011 offe / 3

$1201.7 = \frac{1342 + 1112 + b}{3}$

$1201.7 \times 3 = 2454 + b$

$3605.1 = 2454 + b$

$\therefore b = 3605.1 - 2454$

$\therefore b \approx 1151$

To find c

$c = \frac{\text{sum of 2010-2012 offences}}{3}$

$= \frac{1112 + 1151 + 1016}{3}$

$\therefore c = 1093$

To find d

$d = \frac{\text{sum of 2011-2013 offences}}{3}$

$= \frac{1151 + 1016 + 940}{3}$

$\therefore d \approx 1035.67$

4/4

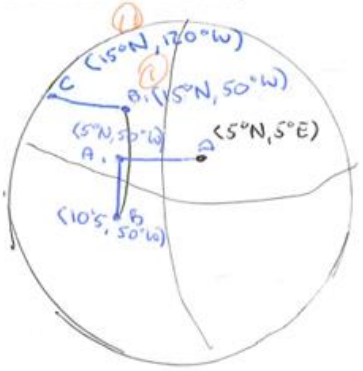
Foundational knowledge and problem-solving (12 marks)

- fluent application of mathematical concepts and techniques to solve problems in complex unfamiliar situations

Excerpt 2

A plane takes off from point A (coordinates given) and travels via point B along certain parallels of latitudes and longitudes then lands at point C.

Find the total distance flown between points A and C and find the time difference, stating which location is furthest ahead.



A - A₁ = 5 + 50 = 55°

distance = 111.2 × cosθ × ad

= 111.2 × cos5 × 55

= 6092.73km

A₁ - B = 5 + 10 = 15.

distance = 111.2 × ad

= 111.2 × 15

= 1668km

B - B₁ = *states 25° north wards is flown so goes up 25°N ~~making~~ from 10°S making it 15°N. 10 + 15 = 25

distance = ~~111.2 × cosθ × ad~~

= 111.2 × ad

= 111.2 × 25

= 2780km

Q13.

B₁ - C = 120 - 50

= 70°

distance = 111.2 × cosθ × ad

= 111.2 × cos15 × 70

= 7518.76

Total distance = all points added

= 6092.73 + 1668 + 2780 + 7518.76

~~1860~~ = 18059.49km.

∴ the total distance travelled by the plane was 18059.49km.

Time difference: A = (5°N, 5°E)

~~B~~ C = (15°N, 120°W)

add e & w 5 + 120 = 125°

125 ÷ 15 = 8.333....

8

8.333 = 0.333 × 60 = 20.

∴ Location A is 8 hours and 20 minutes ahead of location C

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- schools update their marking scheme for an endorsed instrument to appropriately allocate marks to the expected student responses and, when necessary, to
  - correct errors in the questions or sample responses
  - change mark allocations
  - accept alternative solutions
- changes to the marking scheme should be communicated to the QCAA in a timely manner, either through the amendment process or at the time of confirmation submission
- the correct marking scheme is used and submitted for a comparable assessment.

Additional advice

- When adapting an assessment from a previous year, update the marking scheme so it reflects any changes to questions, numerical values and/or the context.
- Follow the marking scheme closely, e.g. if 4 marks are allocated to a question, including 3 marks for mathematical justification, and a student only supplies the final answer then they should only be awarded 1 mark.
- When writing the marking scheme for sequences, take care to correctly use $t_0 \; or \; t_1$.
- When allocating a mark from 0–15, ensure the cut-offs are >__% and the ISMG is a whole number, e.g. a student who receives > 80% is awarded 13/15, whereas a student who receives exactly 80% is awarded 12/15.

Internal assessment 3 (IA3)

Examination — short response (15%)

The examination assesses the application of a range of cognitions to a number of items using a representative sample of subject matter from all Unit 4 topics. Subject matter from Units 1, 2 and 3 is considered assumed knowledge. Student responses must be completed individually, under supervised conditions, and in a set timeframe (120 minutes plus 5 minutes perusal). The percentage allocation of marks must match the degree of difficulty specifications: ~20% complex unfamiliar; ~20% complex familiar; ~60% simple familiar.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions*
Alignment	260
Authentication	0
Authenticity	7
Item construction	12
Scope and scale	66

*Each priority might contain up to four assessment practices.

Total number of submissions: 442.

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided questions that gave students opportunities to address all relevant assessment objectives
- contained an appropriate number of questions that matched the degree of difficulty specified in the syllabus and allowed students to respond within the time conditions
- featured realistic contexts where appropriate
- used stimulus that was relevant to the question and necessary to solve the problem.

Practices to strengthen

It is recommended that assessment instruments:

- explicitly assess subject matter from Unit 4 and do not include questions that solely assess subject matter from Units 1, 2 or 3
- include implicit opportunities for students to respond to Assessment objective 4: evaluate the reasonableness of solutions
- assess subject matter within the scope and scale of the syllabus, e.g. the syllabus requires students to 'use … the Hungarian algorithm (3 × 3) for larger problems' but does not require the use of 4 × 4 (or 5 × 5) matrices
- provide complex unfamiliar opportunities, i.e. relationships and interactions have a number of elements, and all the information to solve the problem is not immediately identifiable.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions*
Bias avoidance	10
Language	67
Layout	10
Transparency	15

*Each priority might contain up to four assessment practices.

Total number of submissions: 442.

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- clearly displayed identifiable characteristics in the simple familiar questions
- featured clear, relevant images where appropriate
- had adequate response space for each question.

Practices to strengthen

It is recommended that assessment instruments:

- contain correct mathematical, statistical and everyday language, as well as conventions and notations
- are reviewed using the Print Preview button, prior to submission, to ensure that the layout is clear and not distracting (e.g. avoiding misaligned text), and that items, such as tables, appear in their entirety on the page.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	98.63%	0.46%	0.46%	0.46%

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- the marking schemes were prepared at the time of writing the examination to ensure questions were ‘worked’
- schools used cross-marking strategies to ensure marking schemes were consistently applied across the cohort, especially where multiple teachers are timetabled across classes
- marking schemes clearly outlined where the marks were allocated
- the marking of the student responses followed the school supplied marking guide
- marks were totalled correctly.

Samples of effective practices

The following is an excerpt from a response that illustrates the characteristics for the criterion at the performance level indicated. The excerpt may provide evidence of more than one criterion. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

This student response excerpt has been included:

- to demonstrate the use of appropriate procedures
- to provide communication using mathematical and everyday language and conventions.

Foundational knowledge and problem-solving (10 marks)

- select, recall and use rules, and procedures
- communicate using mathematical, statistical and everyday language and conventions

Excerpt 1

A weighted graph of electricity connections for a regional town is shown.

Determine the minimum cost to connect the houses to electricity if the cost per metre of cabling is $45 per meter.

Total distance =
19 + 14 + 20 + 28 + 26 + 25
= 132m ✓

Minimum cost =
distance x price per meter

= 132 x 45
= $5940. ✓

3

The minimum cost to connect the houses to electricity is $5940.

Determine the amount of each contribution to an annuity needed to obtain $650 000 in 20 years time at 5.5% compounding monthly.

$$A = M\left[\frac{(1+i)^n - 1}{i}\right]$$

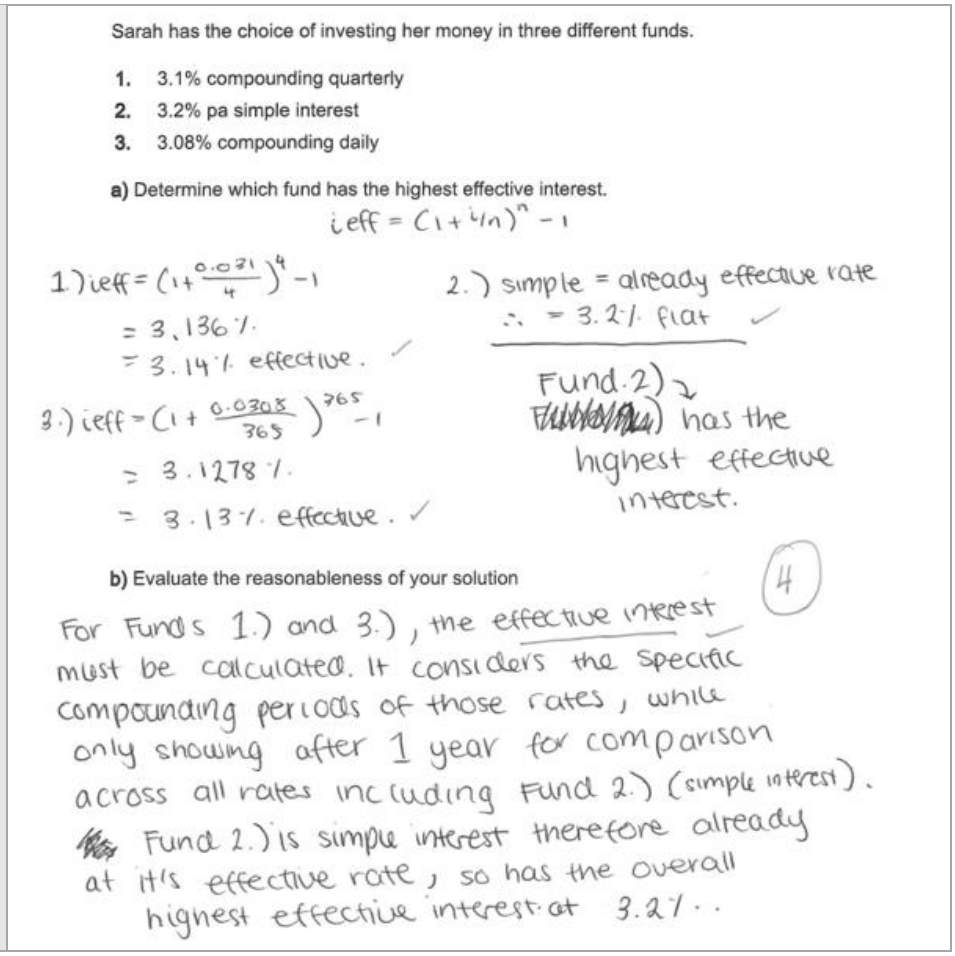
$$650\,000 = M\left[\frac{\left(1+\frac{0.055}{12}\right)^{20\times12} - 1}{\left(0.055/12\right)}\right]$$

$$M = 650\,000 \Big/ \left[\frac{\left(1+\frac{0.055}{12}\right)^{20\times12} - 1}{\left(0.055/12\right)}\right]$$

$M = \$1492.10$

3

The amount needed for each contribution is $1492.10 per month.



Sarah has the choice of investing her money in three different funds.

1. 3.1% compounding quarterly
2. 3.2% pa simple interest
3. 3.08% compounding daily

a) Determine which fund has the highest effective interest.

$i_{eff} = (1 + i/n)^n - 1$

1.) $i_{eff} = (1 + \frac{0.031}{4})^4 - 1$
$= 3.136\%$
$= 3.14\%$ effective. ✓

2.) simple = already effective rate
$\therefore = 3.2\%$ flat ✓

3.) $i_{eff} = (1 + \frac{0.0308}{365})^{365} - 1$
$= 3.1278\%$
$= 3.13\%$ effective. ✓

Fund 2) has the highest effective interest.

b) Evaluate the reasonableness of your solution

4

For Funds 1.) and 3.), the effective interest must be calculated. It considers the specific compounding periods of those rates, while only showing after 1 year for comparison across all rates including Fund 2.) (simple interest). Fund 2.) is simple interest therefore already at it's effective rate, so has the overall highest effective interest at 3.2%.

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- schools update their marking scheme for an endorsed instrument to appropriately allocate marks to the expected student responses and, when necessary, to
  - correct errors in the questions or sample responses
  - change mark allocations
  - accept alternative solutions
- changes to the marking scheme should be communicated to the QCAA in a timely manner, either through the amendment process or at the time of confirmation submission
- the correct marking scheme is used and submitted for a comparable assessment
- schools use internal practices to quality assure judgments and check mark totals, percentage calculations, and correct application of the ISMG
- if an error in an endorsed examination is discovered after implementation, evidence that the QCAA has granted permission to make changes to the examination has been provided.

Additional advice

- Schools should indicate on the ISMG the raw mark and percentage for the student response, as well as circling the mark (0–15).

- When marking, the use of the ticks or mark allocation on the marking scheme should be evident on the student annotated samples. It should be clear where the teacher has allocated marks and for what section of student work.
- When totalling marks, check that the total on the exam is correct, e.g. if the exam is marked out of 65, do the available marks add to 65? Sometimes errors occur when schools adapt a previous assessment.

External assessment

External assessment (EA) is developed and marked by the QCAA. The external assessment for a subject is common to all schools and administered under the same conditions, at the same time, on the same day.

Summative external assessment (EA) — Examination (50%)

Assessment design

The assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the syllabus. The examination assessed subject matter from Units 3 and 4.

The examination consisted of two papers:

- Paper 1, Section 1 consisted of 15 multiple choice questions (15 marks)
- Paper 1, Section 2 consisted of 10 short response questions (42 marks)
- Paper 2, Section 1 consisted of 7 short response questions (38 marks).

Assessment decisions

Assessment decisions are made by markers by matching student responses to the external assessment marking guide (EAMG). The external assessment papers and the EAMG are published in the year after they are administered.

Multiple choice item responses

There were 15 multiple choice items in Paper 1.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Question	A	B	C	D
1	9.05	4.39	**76.34**	8.66
2	9.97	**81.72**	4.15	3.4
3	7.73	11.46	5.53	**74.46**
4	9.69	28.06	23.22	**37.53**
5	2.47	1.33	**90.26**	5.07

Question	A	B	C	D
6	**39.67**	46.43	5.74	6.97
7	**36.94**	14.02	30.71	17.34
8	13.52	**48.44**	5.49	31.42
9	3.77	46.63	**17.45**	30.97
10	19.81	16.74	28.75	**33.21**
11	11.54	**74.25**	8.64	4.54
12	2.95	9.85	28.9	**57.15**
13	25.13	9.56	**61.37**	2.99
14	**51.42**	20.92	19.93	6.48
15	35.36	**33.95**	9.86	19.58

Effective practices

Overall, students responded well to:

- using a bipartite graph to represent an assignment/allocation problem
- identifying the meaning of the terms ‘planar graph’ and ‘face’
- smoothing time series data by using a simple moving average.

Samples of effective practices

The following excerpts have been selected to illustrate effective student responses in one or more of the syllabus assessment objectives. The characteristics identified may not be the only time the characteristics have occurred throughout a response.

Short response

Item: Question 17 — Paper 1

Assessment objectives: 1, 2 and 3 — simple familiar

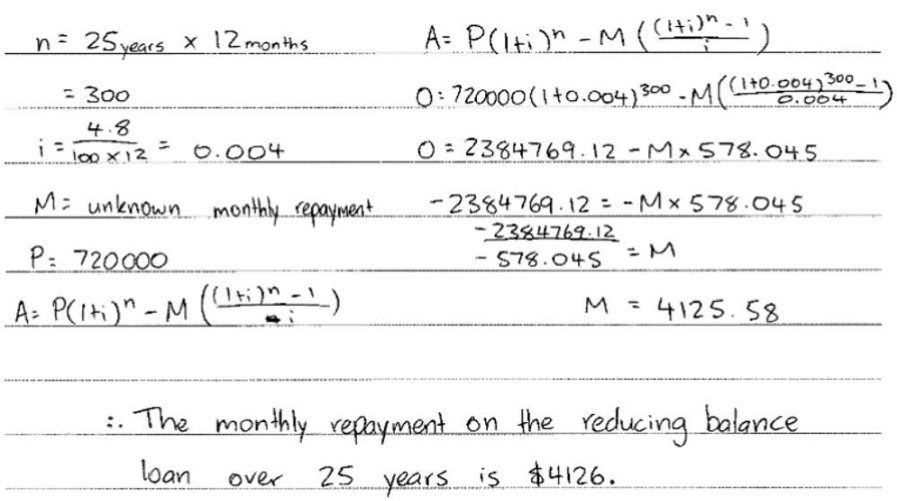
This question required students to determine the monthly repayment on a reducing balance loan.

Effective student responses:

- correctly determined the i and n values
- substituted into the appropriate annuity rule
- determined the monthly repayment and stated the solution to the nearest dollar.

This student response excerpt has been included:

- as it demonstrates a high-level response with an alternative method to using the given annuity formula from the provided QCAA formula book.

Foundational knowledge and problem-solving (4 marks)	**Excerpt 1** 

Item: Question 19 — Paper 1

Assessment objectives: 1, 2 and 3 — simple familiar

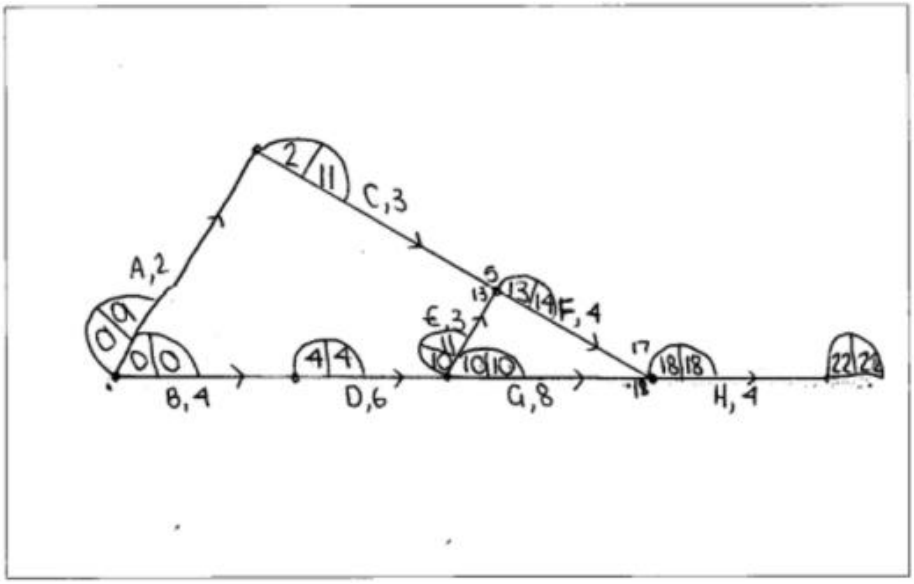
This question required students to construct a network diagram using an activity table for a project.

Effective student responses:

- correctly translated the information into a network diagram by labelling each activity letter and duration
- provided evidence of forward and backward scanning
- determined the critical path and shortest time.

This student response excerpt has been included:

- to provide evidence of a high-level response via an alternative technique of a network diagram construction.

Foundational knowledge and problem-solving (5 marks)	**Excerpt 1**  B-D-G-H 22 days minimum completion time.

Item: Question 5 — Paper 2

Assessment objectives: 1, 2, 3, 5 and 6 — complex unfamiliar

This question required students to understand the difference between actual and predicted values for a residual plot and to use these values to derive a least-squares regression model.

Effective student responses:

- correctly determined the required parameters for a least-squares regression on a residual plot
- predicted the actual systolic blood pressure of a person.

This student response excerpt has been included:

- to demonstrate a reverse understanding of the use of a residual plot
- to demonstrate logical organisation and communicating key steps.

Foundational knowledge and problem-solving (6 marks)	**Excerpt 1** To determine the blood pressure a least square line must be constructed which is $y = xb + a$ $b = r\frac{s_y}{s_x}$ $\approx 0.975\ \frac{6}{4}$ $\therefore b = 1.3125$, $y = x\,1.3125 + a$ as we have a & b for a 31 year old candidate we can substitute the figures in $119 = 31 \times 1.3125 + a$ $a = 78.8125$. However as the residual plot shows the predicted was 0.75 more a actually equals $78.3125 + 0.75$ $\therefore a = 79.0625$ Thus the equation is $y = x \times 1.3125 + 79.0625$, substitute 40 year old in $y = 40 \times 1.3125 + 79.0625$ $y = 131.5625$. However the residual plot shows this actual blood pressure is around 1.49 higher than the predicted $\therefore 131.5625 + 1.49$ $\approx$ ~~131~~ 133.0525 ✗ 133 $\therefore$ The actual blood pressure of the 40yr old is 133

Item: Question 7— Paper 2

Assessment objectives: 1, 2, 3, 4, 5 and 6 — complex unfamiliar

This question required students to develop a mathematical model and evaluate the reasonableness of a prediction.

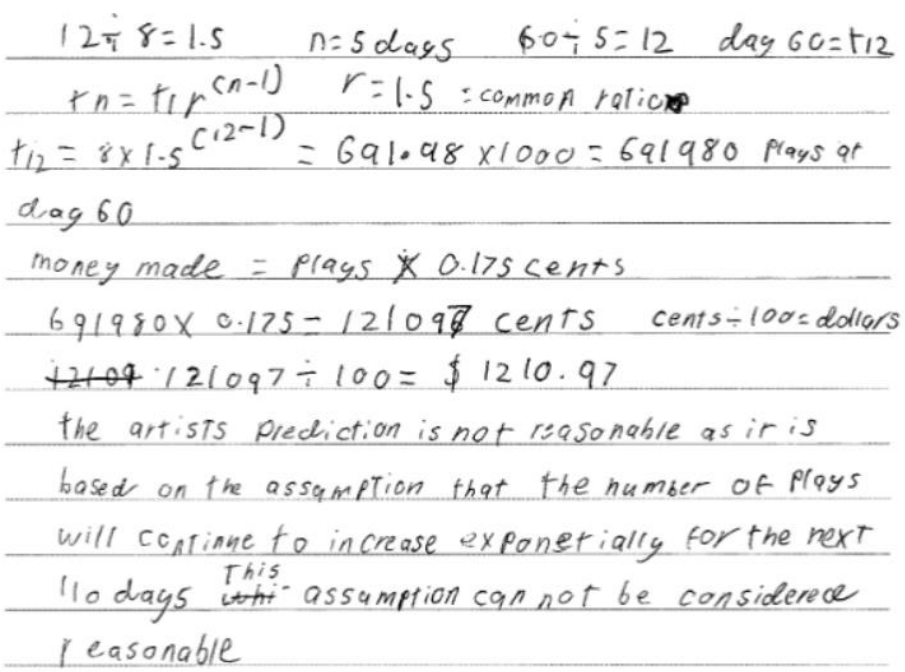
Effective student responses:

- correctly determined a geometric model and the total number of plays
- evaluated the reasonableness of solution.

This student response excerpt has been included:

- to demonstrate an algebraic approach to solving a complex unfamiliar problem
- to provide a different but acceptable evaluation of the prediction.

Foundational knowledge and problem-solving (6 marks)	**Excerpt 1**



Practices to strengthen

It is recommended that when preparing students for external assessment, teachers consider:

- defining variables, explaining with mathematical reasoning and rounding answers appropriately
- accurately identifying the latitude and longitude of a location on a map, selecting the appropriate distance rule, using 24-hour time to assist adding and subtracting flight durations, and converting between minutes and decimal values in Earth geometry
- developing a network diagram with a single start vertex, demonstrating directions through the network (e.g. arrows on edges), avoiding unnecessary vertices in the network graph as well as completing forward and backward scans
- modelling setting out to show logical organisation and communication of key steps when solving problems
- explaining what the word ‘evaluate’ means, i.e. the emphasis on supporting or refuting statements/claims/beliefs based on mathematical results or checking calculations related to the context
- different ways to explain and respond to annuity questions (e.g. recurrence relations for fewer payments) to enable students to relate to the annuity formula provided in the QCAA formula book.

Senior External Examination

The General Mathematics Senior External Examination (SEE) is a standalone examination offered to eligible Year 12 students and adult learners. It contributes 100% to a student's final subject result.

The assessment was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the General Mathematics Senior External Examination syllabus.

The SEE consisted of two assessments:

- SEE 1 contributed 50% of the marks
- SEE 2 contributed 50% of the marks.

Note: The SEE information should be read in conjunction with the rest of the subject report.

Number of students who completed the General Mathematics Senior External Examination: 33.

Distribution of standards

There were insufficient student enrolments in this subject to provide useful analytics.

Assessment decisions

Effective practices

Overall, students responded well to:

- accurately plotting points to construct scatter and residual plots from datasets
- using technology to generate least-squares regression models
- calculating simple moving averages.

Practices to strengthen

It is recommended that when preparing students for the Senior External Examination, teachers consider:

- determining a linear model based on smoothed data
- using a generated model to make predictions across a span of multiple years
- solving simultaneous linear equations
- determining an exponential model based on given data points
- exploring the strengths and limitations of a solution and/or model
- evaluating the reasonableness of a solution.